Math 71

TEST #1 - SOLUTIONS

① (a) $y = f(x)$
y is a function of x if for every input x there is only one output y

(b) Domain of f = the set of all permissible values of x
$\{x \mid f(x) \in \mathbb{R}\} = D$

(c) Range of f = the set of all y-values
$R = \{y \mid y = f(x),\ x \in D\}$

(d) examples

$f(x) = 3x+1$ $\qquad D_f = \mathbb{R}$

oR $g(x) = x^2 - 2x + 3$ $\qquad D_g = \mathbb{R}$

oR $h(x) = \dfrac{3x}{x+5}$ $\qquad D_h = \mathbb{R} \setminus \{-5\}$

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② $f(x) = \dfrac{2x+1}{x+3}$

(a) $f(1) = \dfrac{2(1)+1}{1+3} = \boxed{\dfrac{3}{4} = f(1)}$

(b) $f(a+h) = \dfrac{2(a+h)+1}{(a+h)+3}$

$\boxed{f(a+h) = \dfrac{2a+2h+1}{a+h+3}}$

(c) Condition: $x+3 \neq 0$
$x \neq -3$

Domain $= \boxed{\mathbb{R} \setminus \{-3\}}$

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③ $f(x) = \begin{cases} 3x^2, & x \leq 1 \\ 2x-1, & x > 1 \end{cases}$

(a) $f(-1) = 3(-1)^2 = 3$ $\quad \boxed{f(-1) = 3}$
$x = -1 \leq 1$

(b) $f(5) = 2(5) - 1 = 9$ $\quad \boxed{f(5) = 9}$
$x = 5 > 1$

(c) $\boxed{D = \mathbb{R}}$

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④ (a) yes, because the graph passes the vertical line test.

(b) $\boxed{\begin{array}{l}\text{Domain: } x \in \mathbb{R} \\ \text{Range: } y \in [-0.25, 1]\end{array}}$

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⑤ line passes through $(4,-7)$
$\perp$ to $x - 2y - 3 = 0$

$x - 2y - 3 = 0$
find m for this line

$2y = x - 3$

$y = \frac{1}{2}x - \frac{3}{2}$

$m = \frac{1}{2}$

Then $m_{\perp} = -2$ (slope of a line $\perp$ $x - 2y - 3 = 0$)

$\begin{cases} m = -2 \\ (4,-7) \end{cases}$

$y - y_1 = m(x - x_1)$

$y - (-7) = -2(x-4)$

$\boxed{y + 7 = -2(x-4)}$

(7)
$$\begin{cases} 2x+3y+7z=13 & (1)\\ 3x+2y-5z=-22 & (2)\\ 5x+7y-3z=-28 & (3)\end{cases}$$

Eliminate x:

$$\begin{array}{l} (1)\\(2)\end{array}\begin{cases} 2x+3y+7z=13 & \cdot(-3)\\ 3x+2y-5z=-22 & \cdot 2\end{cases}$$

$$\begin{cases} -6x-9y-21z=-39\\ 6x+4y-10z=-44\end{cases}$$

(+) $-5y-31z=-83$

$$\boxed{5y+31z=83} \quad (4)$$

$$\begin{array}{l} (1)\\(3)\end{array}\begin{cases} 2x+3y+7z=13 & \cdot(-5)\\ 5x+7y-3z=-28 & \cdot 2\end{cases}$$

$$\begin{cases} -10x-15y-35z=-65\\ 10x+14y-6z=-56\end{cases}$$

(+) $-y-41z=-121$

$$\boxed{y+41z=121} \quad (5)$$

$$\begin{array}{l} (4)\\(5)\end{array}\begin{cases} 5y+31z=83 & \\ y+41z=121 & \cdot(-5)\end{cases}$$

Eliminate y

$$\begin{cases} 5y+31z=83\\ -5y-205z=-605\end{cases}$$

(+) $-174z=-522$

$$z=\frac{522}{174}=3$$

$$z=3$$

(5) $y+41z=121$, $z=3$ $\Rightarrow$

$y+41(3)=121 \Rightarrow y=-2$

(1) $2x+3y+7z=13$, $y=-2$, $z=3$ $\Rightarrow$

$2x+3(-2)+7(3)=13$

$2x=-2 \Rightarrow x=-1$

$$\boxed{\text{The solution is } (-1,-2,3)}$$

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(8) (a) Is t a function of P?

t is <u>not</u> a function of P because there are P values for which there is more than one t value.

(b) Is P a function of t?

P <u>is</u> a function of t because for every t value there is exactly one P value

$P=f(t)$

(c) Domain: $t\in[8am, 2pm]$

(d) Range: $P\in[\$2, \$12]$

(e) $P(t)=8$, $t=?$

$t=?$ when $P=8$

$t=11$ am

$t\approx 12{:}10$ pm

$t\approx 1{:}30$ pm

$P(t) = 8$ shows the time when the price of the stock was 8 dollars:

11 am, 12:10 pm, 1:30 pm

(7) $P(2) = ?$

when $t = 2$, find P

$P(2) = 12$ \$

$P(2)$ shows the price of the stock at 2 pm.

The price was \$12 at 2 pm

(9) (a) $\left|2x + \frac{1}{3}\right| = \frac{4}{5}$

$2x + \frac{1}{3} = \pm \frac{4}{5}$

I $2x + \frac{1}{3} = \frac{4}{5}$ OR II $2x + \frac{1}{3} = \frac{-4}{5}$

I:

$2x = \frac{4}{5} - \frac{1}{3}$

$2x = \frac{7}{15}$ $\Big/ \cdot \frac{1}{2}$

$x = \frac{7}{30}$

II:

$2x = \frac{-4}{5} - \frac{1}{3}$

$2x = \frac{-17}{15}$ $\Big/ \cdot \frac{1}{2}$

$x = \frac{-17}{30}$

The solution set is $\boxed{\left\{\frac{7}{30}, \frac{-17}{30}\right\}}$

(b) $\left|x + \frac{1}{2}\right| = |x - 3|$

I $x + \frac{1}{2} = x - 3$ OR

II $x + \frac{1}{2} = -(x - 3)$

I $\cancel{x} + \frac{1}{2} = \cancel{x} - 3$

$\frac{1}{2} = -3$ contradiction

No solution

II $x + \frac{1}{2} = -(x - 3)$

$x + \frac{1}{2} = -x + 3$

$x + x = +3 - \frac{1}{2}$

$2x = \frac{+5}{2}$ $\Big/ \cdot \frac{1}{2}$

$x = \frac{+5}{4}$

The solution set is $\boxed{\left\{+\frac{5}{4}\right\}}$

(c) $|3x + 14| + 7 = -2$

$|3x + 14| = -9$

not possible

The absolute value of any real number is ≥ 0

Therefore, $\boxed{x \in \emptyset}$

(d) $1 - |x - 3| = 1$

$1 - 1 = |x - 3|$

$|x - 3| = 0 \Rightarrow$

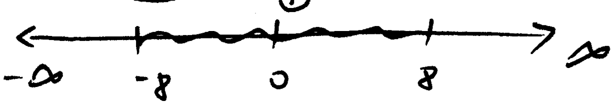
$x - 3 = 0$

$x = 3$

The solution set is $\boxed{\{3\}}$

(10) (a) $|2x-1| + 3 \le 11$

$|2x-1| \le 8$

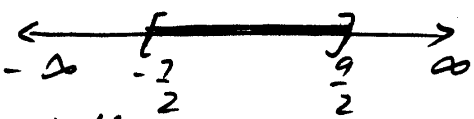


$$-8 \le 2x-1 \le 8$$
$$+1 \qquad +1 \qquad +1$$

$$-7 \le 2x \le 9 \quad / \div 2$$

$$\boxed{-\frac{7}{2} \le x \le \frac{9}{2}}$$

Graph:

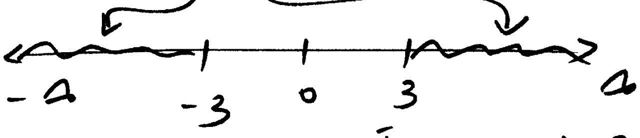


interval notation:

$$\boxed{x \in \left[-\frac{7}{2}, \frac{9}{2}\right]}$$

(b) $4|3x-5| > 12$

$|3x-5| > 3$



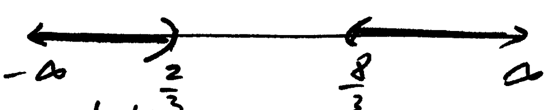
I $3x-5 < -3$ OR II $3x-5 > 3$

$3x < 2$ $\qquad$ $3x > 8$

$x < \frac{2}{3}$ $\qquad$ $x > \frac{8}{3}$

Therefore, $\boxed{x < \frac{2}{3} \text{ OR } x > \frac{8}{3}}$

Graph:



interval notation:

$$\boxed{x \in \left(-\infty, \frac{2}{3}\right) \cup \left(\frac{8}{3}, \infty\right)}$$

(11) $\begin{cases} x \ge 0 \\ y \ge 0 \\ 2x + 5y < 10 \\ 3x + 4y \le 12 \end{cases}$ | quadrant I

$\boxed{2x + 5y < 10}$

Boundary line: $2x + 5y = 10$ (1)

x	y	
0	2	(0,2)
5	0	(5,0)

Test point: $(0,0) \notin$ line

$2(0) + 5(0) < 10$

$0 < 10$ true $\Rightarrow$

$(0,0)$ = solution

$\boxed{3x + 4y \le 12}$ (2)

Boundary line: $3x + 4y = 12$

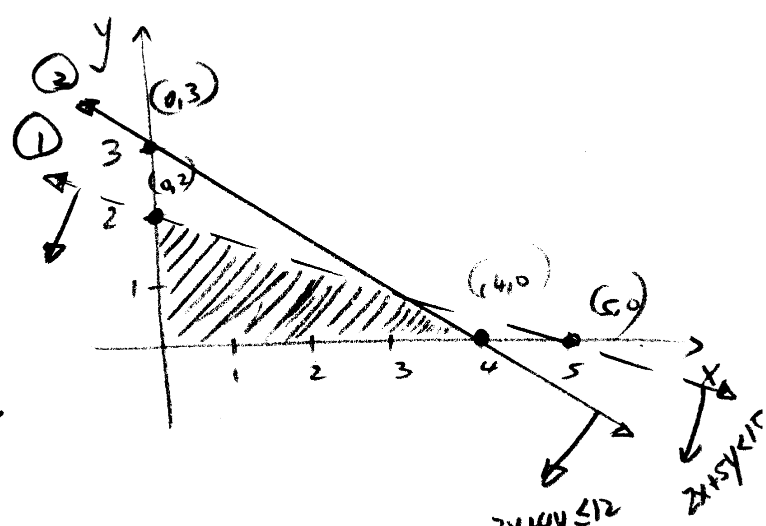
x	y	
0	3	(0,3)
4	0	(4,0)

Test point: $(0,0) \notin$ line

$3(0) + 4(0) \le 12$

$0 \le 12$ true $\Rightarrow$

$(0,0)$ = solution



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A) \$7000 { ACCOUNT I @ 6% / ACCOUNT II @ 8% }

TOTAL INTEREST \$520

How much was invested at each rate?

Solution

Let x = amount (\$) invested at 6% interest

y = amount (\$) invested at 8% interest

Then,

$$\begin{cases} x+y=7000 \\ 6\%x+8\%y=520 \end{cases}$$

$$\begin{cases} x+y=7000 \\ \frac{6}{100}x+\frac{8}{100}y=520 \end{cases} \Big| \cdot 100$$

$$\begin{cases} x+y=7000 \\ 6x+8y=52000 \end{cases} \Big| -6$$

$$\begin{cases} -6x-6y=-42{,}000 \\ 6x+8y=52{,}000 \end{cases}$$

$$(+)\quad 2y=10{,}000 \Rightarrow y=5000\ \$$$

$$\left.\begin{matrix} x+y=7000 \\ y=5000 \end{matrix}\right| \Rightarrow x=2000\ \$$$

2000 \$ were invested at 6% and 5000 \$ at 8%

B)

	Distance	Rate	Time
with wind	800 mi	$x+y$	5 hr
against wind	800 mi	$x-y$	8 hr

Let x = rate of plane in still air (mi/hr)

y = rate of wind (mi/hr)

Distance = rate · time

$$\begin{cases} 5(x+y)=800 & \text{(with wind)} \quad \div 5 \\ 8(x-y)=800 & \text{(against wind)} \quad \div 8 \end{cases}$$

$$\begin{cases} x+y=160 \\ x-y=100 \end{cases}$$

$$(+)\quad 2x=260 \Rightarrow x=130$$

$$\left.\begin{matrix} x+y=160 \\ x=130 \end{matrix}\right| \Rightarrow y=30$$

Therefore, the rate of the plane in still air is 130 mi/hr and the rate of the wind is 30 mi/hr.

(c) D = the number of days
W = the weight (kg)

W depends on D, so
D = independent variable
W = dependent variable

D	W	
2	3	(2,3)
31	9	(31,9)

Find a formula that gives W in terms of D = find the equation of the line that passes through (2,3) and (31, 9)

$$m = \frac{\Delta W}{\Delta D} = \frac{9-3}{31-2} = \frac{6}{29}$$

$$\left\{ \begin{array}{l} m = \frac{6}{29} \\ (2,3) \end{array} \right. \Big/ \quad y - y_1 = m(x - x_1)$$

$$\boxed{W - 3 = \frac{6}{29}(D-2)}$$

b) $m = \frac{6}{29}$ kg/day

and it shows the rate at which the pumpkin grows.

The pumpkin grows at a rate of $\frac{6}{29}$ kg/day or 6 kg every 29 days

(D) Let x = grade on the 5th exam

We want

$80 \leq$ average grade < 90

$$80 \leq \frac{70+75+87+92+x}{5} < 90$$

$$80 \leq \frac{324+x}{5} < 90 \quad \Big/ \cdot 5$$

$$\begin{array}{rcccl} 400 & \leq & 324 + x & < & 450 \\ -324 & & -324 & & -324 \\ \hline 76 & \leq & x & < & 126 \end{array}$$

The grade on the 5th exam must be between 76 and 100

$x \in [76, 100]$

(E) x = number nights spent in large resorts

y = number nights in small inns

- stay at least 5 nights:

$$x + y \geq 5$$

- at least one night at a large resort

$$x \geq 1$$

- Budget \$700 with \$200/night for large resorts and \$100/night for small inns

$$200x + 100y \leq 700$$

Therefore, the system of inequalities is

$$\begin{cases} x+y \geq 5 \\ x \geq 1 \\ 200x + 100y \leq 700 \\ y \geq 0 \end{cases}$$

(1) $\boxed{x+y \geq 5}$

Boundary line: $x+y=5$

x	y	
0	5	(0,5)
5	0	(5,0)

Test point: $(0,0) \notin$ line

$0+0 \geq 5$ false

$\Rightarrow (0,0) \neq$ solution

(2) $\boxed{x \geq 1}$

Boundary line: $x=1$ vertical line

Half-plane to the right of $x=1$

(3) $200x + 100y \leq 700$

or

$\boxed{2x + y \leq 7}$

Boundary line:

$2x+y=7$

x	y	
0	7	(0,7)
3.5	0	(3.5,0)

Test point: $(0,0) \notin$ line

$2(0)+0 \leq 7$ true

$\Rightarrow (0,0) =$ solution

(4) $\boxed{y \geq 0}$ $\Rightarrow$ quadrants I and II