Math 71 Spring 2007 QUIZ #4 @ 50 points Name:

Write in a neat and organized fashion. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given.

1. $\begin{cases} x^2 + y^2 = 9 \\ 4x^2 + 25y^2 = 100 \end{cases}$

   a) First, **solve the above system graphically**; that is, identify each equation and graph it. Highlight the solutions of the system on the graph. Make sure you label the axes and every point you are using in graphing the curves.

   b) Second, **solve the above system algebraically**, showing the exact solutions.

2. Find the **center** and **radius** of the following circle: $x^2 + y^2 + 6x + 2y + 6 = 0$

3. Find a formula for the ***n*th term** of each sequence, then find the **52nd term**:

   a) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$

   b) $2, -4, 8, -16, \ldots$

4. Find the **first three terms** of each sequence:

   a) $\begin{cases} a_n = 2(a_{n-1} - 3) \\ a_1 = 1 \end{cases}$

   b) $a_n = \dfrac{n^3}{(n-1)!}$

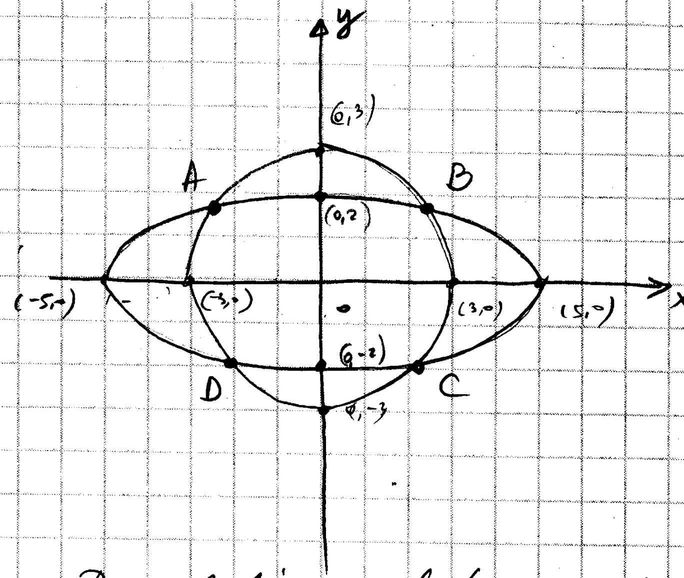
5. Find each sum:

   a) $\displaystyle\sum_{i=2}^{5} \frac{i!}{(i+1)!}$

   b) $\displaystyle\sum_{n=1}^{4} nx^{n-1}$

Quiz #4 - Solutions

① $\begin{cases} x^2+y^2=9 \\ 4x^2+25y^2=100 \end{cases}$



I graphical method

$x^2+y^2=9$

circle with center (0,0)
radius $r=\sqrt{9}=3$

$4x^2+25y^2=100 \quad / \div 100$

$\frac{4x^2}{100}+\frac{25y^2}{100}=1$

$\frac{x^2}{25}+\frac{y^2}{4}=1$

ellipse with center (0,0)
x-int: $(\pm 5, 0)$
y-int: $(0, \pm 2)$

The solutions of the system are the four common points: A, B, C, D

II algebraic method

$\begin{cases} x^2+y^2=9 \\ 4x^2+25y^2=100 \end{cases} \Big/ \cdot -4$

$\begin{cases} -4x^2-4y^2=-36 \\ 4x^2+25y^2=100 \end{cases}$

(+) $21y^2=64$

$y^2=\frac{64}{21}$

$y=\pm\sqrt{\frac{64}{21}}=\pm\frac{8}{\sqrt{21}}$

$x^2+y^2=9$

$x^2+\frac{64}{21}=9$

$x^2=9-\frac{64}{21}=\frac{189-64}{21}=\frac{125}{21}$

$x=\pm\sqrt{\frac{125}{21}}$

$x=\pm\frac{5\sqrt{5}}{\sqrt{21}}$

The solutions are

B $\left(\frac{5\sqrt{5}}{\sqrt{21}}, \frac{8}{\sqrt{21}}\right)$

C $\left(\frac{5\sqrt{5}}{\sqrt{21}}, \frac{-8}{\sqrt{21}}\right)$

A $\left(\frac{-5\sqrt{5}}{\sqrt{21}}, \frac{8}{\sqrt{21}}\right)$

D $\left(\frac{-5\sqrt{5}}{\sqrt{21}}, \frac{-8}{\sqrt{21}}\right)$

(2) $x^2+y^2+6x+2y+6=0$

$x^2+6x \quad + y^2+2y \quad = -6$

$\left(\frac{1}{2}\operatorname{coef}.x\right)^2 = \left(\frac{1}{2}6\right)^2 = 9$

$\left(\frac{1}{2}\operatorname{coef}y\right)^2 = \left(\frac{1}{2}2\right)^2 = 1$

$(x^2+6x+9)+(y^2+2y+1) = -6+9+1$

$(x+3)^2 + (y+1)^2 = 4$

Circle with center $(-3,-1)$

radius $r = \sqrt{4} = 2$

(3) a) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$

$a_1 = \frac{1}{2}$

$a_2 = \frac{2}{3}$

$a_3 = \frac{3}{4}$

$a_4 = \frac{4}{5}$

$\boxed{a_n = \frac{n}{n+1}}, n \in \mathbb{N}$

$\boxed{a_{52} = \frac{52}{53}}$

b) $2, -4, 8, -16, \ldots$

$a_1 = 2$

$a_2 = -4 = -2^2$

$a_3 = 8 = 2^3$

$a_4 = -16 = -2^4$

$a_5 = 32 = 2^5$

$\boxed{a_n = 2^n(-1)^{n+1}}, n \in \mathbb{N}$

$a_{52} = 2^{52}(-1)^{53}$

$a_{52} = -2^{52}$

(4) a) $\begin{cases} a_n = 2(a_{n-1}-3) \\ a_1 = 1 \end{cases}$

$a_1 = 1$ given

$a_2 = 2(a_1-3)$

$= 2(1-3) = -4$

$a_3 = 2(a_2-3)$

$= 2(-4-3) = -14$

b) $a_n = \frac{n^3}{(n-1)!}$

$a_1 = \frac{1^3}{(1-1)!} = \frac{1}{0!} = \frac{1}{1} = 1$

$a_2 = \frac{2^3}{(2-1)!} = \frac{8}{1!} = \frac{8}{1} = 8$

$a_3 = \frac{3^3}{(3-1)!} = \frac{27}{2!} = \frac{27}{2}$

(5) a) $\sum_{i=2}^{5} \frac{i!}{(i+1)!} = \sum_{i=2}^{5} \frac{i!}{i!(i+1)}$

$= \sum_{i=2}^{5} \frac{1}{i+1}$

$= \frac{1}{2+1} + \frac{1}{3+1} + \frac{1}{4+1} + \frac{1}{5+1}$

$= \overset{20}{\frac{1}{3}} + \overset{15}{\frac{1}{4}} + \overset{12}{\frac{1}{5}} + \overset{10}{\frac{1}{6}} = \frac{57}{60} = \frac{19}{20}$

LCD = 60

b) $\sum_{n=1}^{4} n x^{n-1} = x^{1-1} + 2x^{2-1} + 3x^{3-1} + 4x^{4-1}$

$= 1 + 2x + 3x^2 + 4x^3$