Math 71

Homework # 3

Section 3.1

(#39) $\begin{cases} \frac{x}{4} - \frac{y}{4} = -1 \\ x + 4y = -9 \end{cases} \Big| \cdot 4$

$\begin{cases} x - y = -4 \\ x + 4y = -9 \end{cases}$ Elimination method

$(-) \quad -5y = 5$

$y = -1$

$x + 4y = -9$

$x + 4(-1) = -9$

$x = -5$

The solution set is $\{(-5,-1)\}$

(#42) $\begin{cases} y = \frac{1}{3}x + 4 \\ 3y = x + 12 \end{cases}$ ← Substitution method

$3\left(\frac{1}{3}x + 4\right) = x + 12$

$x + 12 = x + 12$ identity

There is an infinite number of solutions.

(the equations are equivalent; they represent the same line)

The solution set is

$\{(x,y) \mid 3y = x + 12\}$

(#66) $\begin{cases} \frac{1}{16}x - \frac{3}{4}y = -1 \\ \frac{3}{4}x + \frac{5}{2}y = 11 \end{cases} \Big| \begin{matrix} LCD = 16 \\ LCD = 8 \end{matrix}$

$\begin{cases} x - 12y = -16 \\ 3x + 10y = 44 \end{cases} \Big| \cdot -3$

$\begin{cases} -3x + 36y = +48 \\ 3x + 10y = 44 \end{cases}$

$(+) \quad 46y = 92 \Rightarrow y = 2$

$x - 12y = -16$

$x - 12(2) = -16$

$x - 24 = -16 \Rightarrow x = 8$

The solution set is $\{(8,2)\}$

(#83) $\begin{cases} \frac{x+2}{2} - \frac{y+4}{3} = 3 \\ \frac{x+y}{5} = \frac{x-y}{2} - \frac{5}{2} \end{cases} \Big| \begin{matrix} LCD = 6 \\ LCD = 10 \end{matrix}$

$\begin{cases} 3(x+2) - 2(y+4) = 18 \\ 2(x+y) = 5(x-y) - 25 \end{cases}$

$\begin{cases} 3x + 6 - 2y - 8 = 18 \\ 2x + 2y = 5x - 5y - 25 \end{cases}$

$\begin{cases} 3x - 2y = 20 \\ -3x + 7y = -25 \end{cases}$

$(+) \quad 5y = -5 \Rightarrow y = -1$

$3x - 2y = 20$

$3x - 2(-1) = 20$

$3x + 2 = 20$

$3x = 18 \Rightarrow x = 6$

Sol. set is $\{(6,-1)\}$

SECTION 3.2

(#4) Let x = first number
y = second number

$$\begin{cases} 3x + 2y = 8 \\ 2x - y = 3 \end{cases} \Big| 2$$

$$\begin{cases} 3x + 2y = 8 \\ 4x - 2y = 6 \end{cases}$$

$7x = 14 \Rightarrow x = 2$

$2x - y = 3$
$2(2) - y = 3$
$4 - 3 = y$ $\quad y = 1$

The first number is 2
The second number is 1.

(#14) Let x = amount (in \$) invested at 5%
y = amount (in \$) invested at 8%

Given
\$11,000 < x at 5%, y at 8%

Total interest = \$730

$$\begin{cases} x + y = 11{,}000 \\ 5\%x + 8\%y = 730 \end{cases}$$

$$\begin{cases} x + y = 11{,}000 \\ \frac{5}{100}x + \frac{8}{100}y = 730 \end{cases} \Big| \cdot 100$$

$$\begin{cases} x + y = 11{,}000 \\ 5x + 8y = 73{,}000 \end{cases} \Big| -5$$

$$\begin{cases} -5x - 5y = -55{,}000 \\ 5x + 8y = 73{,}000 \end{cases}$$

(+) $3y = 18{,}000$
$y = 6000$

$x + y = 11{,}000$
$x + 6000 = 11{,}000$
$x = 5000$

\$5000 were invested at 5%
\$6000 were invested at 8%

(#30)

	distance	rate	time
downstream	36 mi	$x+y$	1.5 hrs
upstream	36 mi	$x-y$	2 hrs

Let x = rate of boat in still water (in mi/h)
y = rate of current (mi/h)

distance = rate · time

$$\begin{cases} 36 = 1.5(x+y) \\ 36 = 2(x-y) \end{cases} \Big| \div 1.5 \Big| \div 2$$

$$\begin{cases} x + y = 24 \\ x - y = 18 \end{cases}$$

(+) $2x = 42$
$x = 21$ mi/h

$x + y = 24$
$21 + y = 24 \Rightarrow y = 3$ mi/h

The rate of the boat is 21 mi/h
The rate of the current is 3 mi/h

SECTION 3.3

#10
$$\begin{cases} 2x+3y+7z=13 & (1)\\ 3x+2y-5z=-22 & (2)\\ 5x+7y-3z=-28 & (3)\end{cases}$$

Eliminate x

$$\begin{cases} (1)\ 2x+3y+7z=13 \\ (2)\ 3x+2y-5z=-22 \end{cases} \Big| \begin{matrix}3\\-2\end{matrix}$$

$$\begin{cases} +6x+9y+21z=39 \\ -6x-4y+10z=44 \end{cases}$$

$$\boxed{5y+31z=83} \quad (4)$$

$$\begin{cases} (1)\ 2x+3y+7z=13 \\ (3)\ 5x+7y-3z=-28 \end{cases} \Big| \begin{matrix}5\\-2\end{matrix}$$

$$\begin{cases} 10x+15y+35z=65 \\ -10x-14y+6z=56 \end{cases}$$

$$\boxed{y+41z=121} \quad (5)$$

$$\begin{cases} (4)\ 5y+31z=83 \\ (5)\ y+41z=121 \end{cases} \Big| -5$$

$$\begin{cases} 5y+31z=83 \\ -5y-205z=-605 \end{cases}$$

$$-174z=-522$$

$$z=\frac{522}{174}=3 \qquad \boxed{z=3}$$

(5) $y+41z=121$

$y+41(3)=121$

$y+123=121$

$\boxed{y=-2}$

(1) $2x+3y+7z=13$

$2x+3(-2)+7(3)=13$

$2x-6+21=13$

$2x=-2 \Rightarrow \boxed{x=-1}$

The solution is $(-1,-2,3)$

#24 $y=ax^2+bx+c$

$(-2,7) \in$ graph $\Rightarrow$ when $x=-2$, $y=7$

$$\boxed{4a-2b+c=7} \quad (1)$$

$(1,-2) \in$ graph $\Rightarrow$ when $x=1$, $y=-2$

$$\boxed{a+b+c=-2} \quad (2)$$

$(2,3) \in$ graph $\Rightarrow$ when $x=2$, $y=3$

$$\boxed{4a+2b+c=3} \quad (3)$$

$$\begin{cases} (1)\ 4a-2b+c=7 \\ (2)\ a+b+c=-2 \\ (3)\ 4a+2b+c=3 \end{cases}$$

Eliminate c

$$\begin{cases} (1)\ 4a-2b+c=7 \\ (2)\ a+b+c=-2 \end{cases}$$

$$(-)\ 3a-3b=9 \quad /\div 3$$

$$\boxed{a-b=3} \quad (4)$$

$$\begin{cases} (1)\ 4a-2b+c=7 \\ (3)\ 4a+2b+c=3 \end{cases}$$

$$(-)\ -4b=4$$

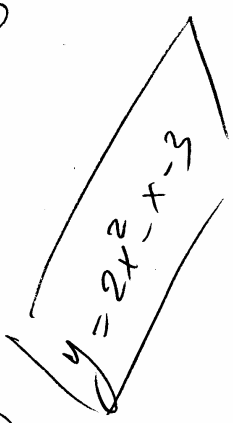
$$b=-1$$

(4) $a-b=3$

$a-(-1)=3$

$a+1=3 \Rightarrow \boxed{a=2}$

(2) $a+b+c=-2 \Rightarrow \boxed{c=-3}$



#46

Let x = the number of children desks
y = the number of office desks
z = the number of deluxe desks

	children's model (x)	office model (y)	deluxe model (z)	Availability
cutting	2 hr/unit	3 hr/unit	2 hr/unit	100 hours
Construction	2 hr/unit	1 hr/unit	3 hr/unit	100 hours
Finishing	1 hr/unit	1 hr/unit	2 hr/unit	65 hours

$$\begin{cases} 2x + 3y + 2z = 100 & (1) \\ 2x + y + 3z = 100 & (2) \\ x + y + 2z = 65 & (3) \end{cases}$$

Eliminate x

$$\begin{cases} 2x + 3y + 2z = 100 & (1) \\ 2x + y + 3z = 100 & (2) \end{cases}$$

$$2y - z = 0 \quad (4)$$

$$\begin{cases} x + y + 2z = 65 & (3) \\ 2x + 3y + 2z = 100 & (1) \end{cases} \times -2$$

$$\begin{cases} -2x - 2y - 4z = -130 \\ 2x + 3y + 2z = 100 \end{cases}$$

$$y - 2z = -30 \quad (5)$$

$$\begin{cases} 2y - z = 0 \Rightarrow z = 2y & (4) \\ y - 2z = -30 & (5) \end{cases}$$

$$y - 2(2y) = -30$$
$$-3y = -30 \Rightarrow y = 10$$

$$z = 2y \Rightarrow z = 20$$

$$x + y + 2z = 65$$
$$x + 10 + 40 = 65$$
$$x = 15$$

They should produce
15 children's models
10 office models
20 deluxe models

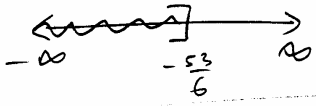
Section 4.1

#34 $18x + 45 \leq 12x - 8$

$18x - 12x \leq -8 - 45$

$6x \leq -53$

$\boxed{x \leq \frac{-53}{6}}$



$\boxed{x \in (-\infty, \frac{-53}{6}]}$

#40 $5(3-x) \leq 3x - 1$

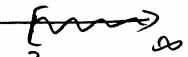
$15 - 5x \leq 3x - 1$

$15 + 1 \leq 3x + 5x$

$16 \leq 8x$

$8x \geq 16$

$\boxed{x \geq 2}$



$\boxed{x \in [2, \infty)}$

#46 $\frac{2}{\frac{4x-3}{6}} + \frac{12}{2} \geq \frac{2x-1}{12}$

$LCD = 12$

$2(4x-3) + 24 \geq 2x - 1$

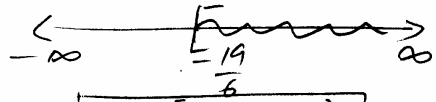
$8x - 6 + 24 \geq 2x - 1$

$8x + 18 \geq 2x - 1$

$8x - 2x \geq -1 - 18$

$6x \geq -19$

$\boxed{x \geq \frac{-19}{6}}$



$\boxed{x \in [-\frac{19}{6}, \infty)}$

#55 $6 - \frac{2}{3}(3x - 12) \leq \frac{2}{5}(10x + 50)$

$6 - 2x + 8 \leq 4x + 20$

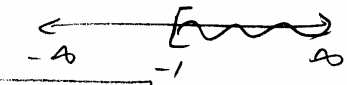
$14 - 2x \leq 4x + 20$

$14 - 20 \leq 4x + 2x$

$-6 \leq 6x$ or

$6x \geq -6$

$\boxed{x \geq -1}$



$\boxed{x \in [-1, \infty)}$

#60 $f(x) = 2x - 9$

$g(x) = 5x + 4$

find x such that $f(x) > g(x)$

$2x - 9 > 5x + 4$

$-9 - 4 > 5x - 2x$

$-13 > 3x$

$3x < -13$

$\boxed{x < \frac{-13}{3}}$

$\boxed{x \in (-\infty, \frac{-13}{3})}$