Section 8.3
Quadratic Functions and Their Graphs

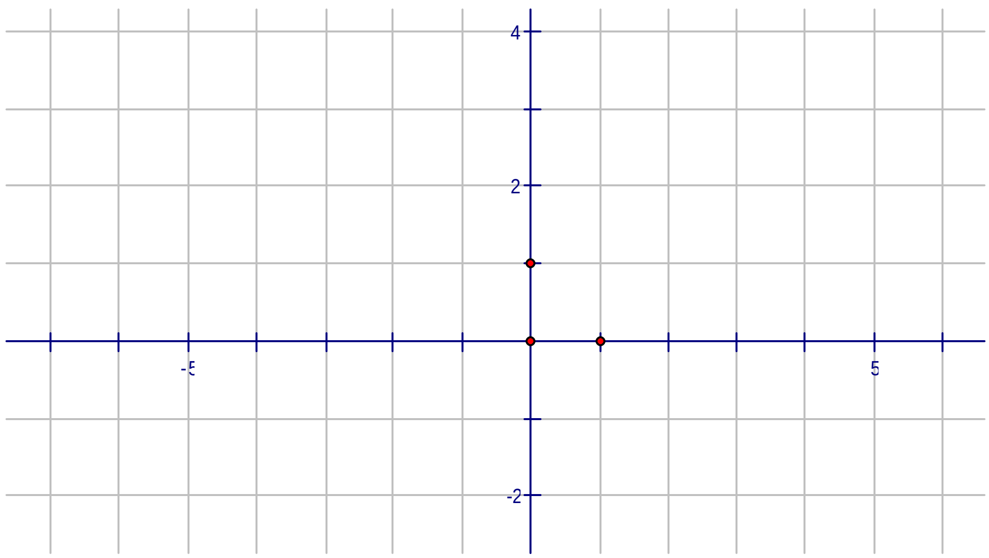
Quadratic Function: $f(x) = ax^2 + bx + c \quad (a \neq 0)$

The graph of a quadratic function is called a **parabola.**

Graphing Parabolas: Special Cases

The "basic" parabola is the graph of the simplest quadratic function $y = x^2$.

x	y
-3	
-2	
-1	
0	
1	
2	
3	



All parabolas share certain features.

Vertex – the lowest point (if the parabola opens up) or the highest point (if the parabola opens down).

The vertex of the basic parabola is ___________________.

Axis of symmetry – the parabola is symmetric about the vertical line that runs through the vertex.

The axis of symmetry of the basic parabola is ___________________.

***y*-intercept** – the point where the parabola intersects the *y*-axis.

***x*-intercept(s)** – the point(s) where the parabola intersects the *x*-axis.

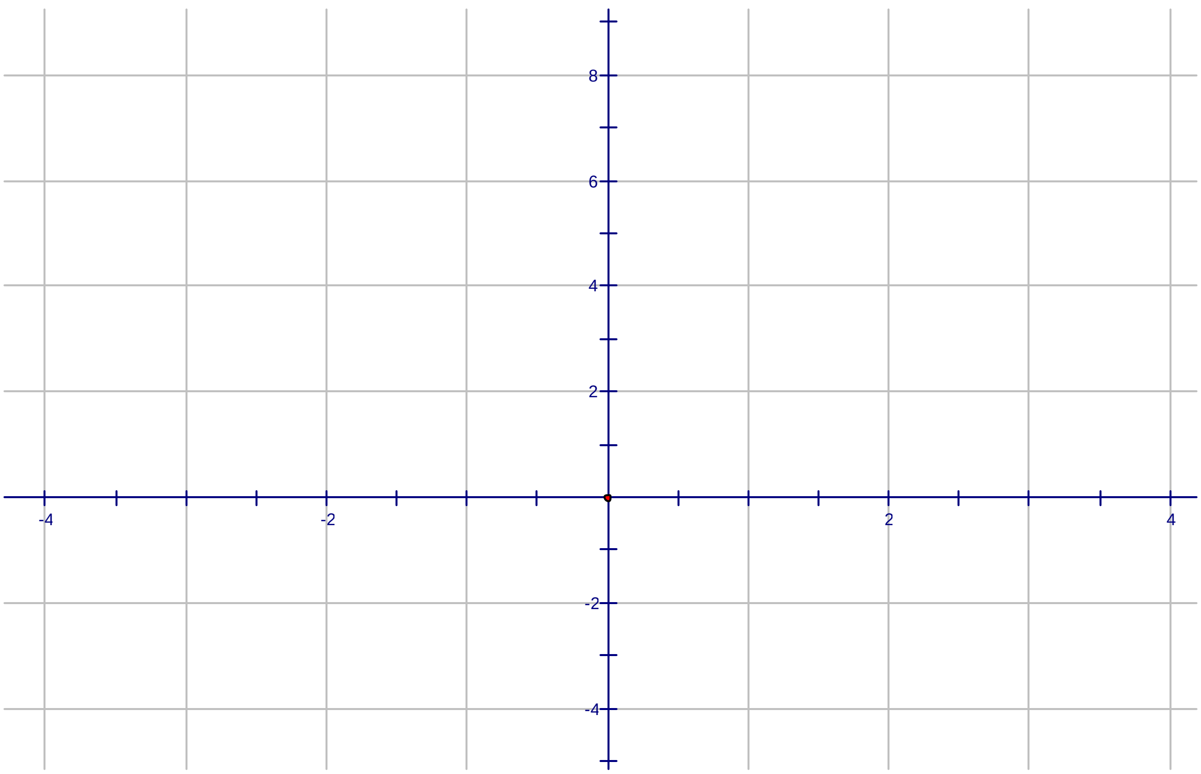
The x- and y-intercept of the basic parabola is ___________________.

Example #1 Graph the following parabolas on the same coordinate system:

1) $y = x^2$ 2) $y = 2x^2$ 3) $y = \frac{1}{2}x^2$ 4) $y = -x^2$ 5) $y = -2x^2$

Investigate the effect of the coefficient of x^2 on the graph.

x	$y = x^2$	$y = 2x^2$	$y = \frac{1}{2}x^2$	$y = -x^2$	$y = -2x^2$
-2					
-1					
0					
1					
2					



What are the effects of the coefficient *a* of x^2 on the graph?

If $a > 0$, the parabola opens ___________________________.

If $a < 0$, the parabola opens ___________________________.

If $a > 1$, the graph of $y = ax^2$ is __________________ than the graph of $y = x^2$.

($y = ax^2$ increases more quickly than the basic parabola).

If $0 < a < 1$, the graph of $y = ax^2$ is __________________ than the graph of $y = x^2$.

($y = ax^2$ increases more slowly than the basic parabola).

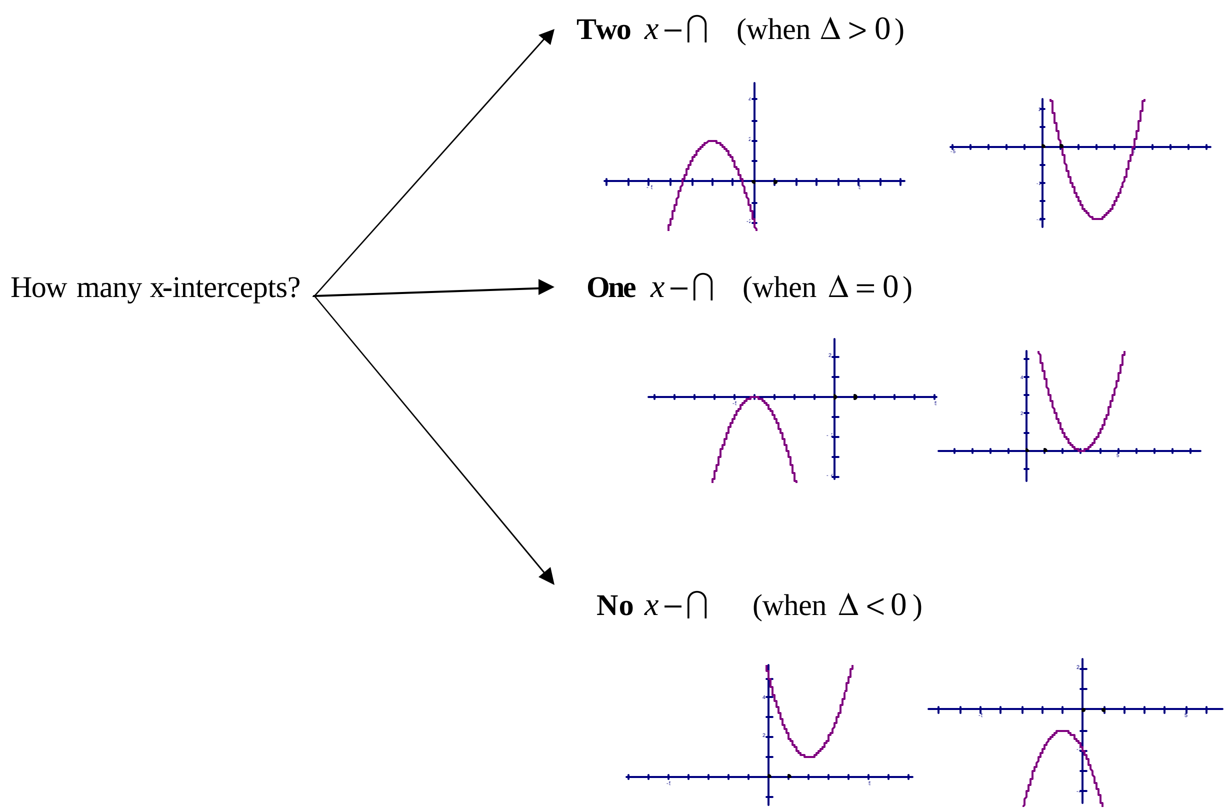
<u>How to Graph a Parabola</u> $y = ax^2 + bx + c \quad (a \neq 0)$

Note that if $a > 0$, the parabola opens upward, and if $a < 0$, the parabola opens downward.

Vertex $V(x_v, y_v)$ $\boxed{x_v = \frac{-b}{2a}}$ To find y_v substitute the value of x_v in the equation and solve for *y*.

y-intercept To find the *y*-intercept make *x*=0 and solve for *y*.

x-intercept(s) (if any) To find the *x*-intercept(s) make *y*=0 and solve for *x*.

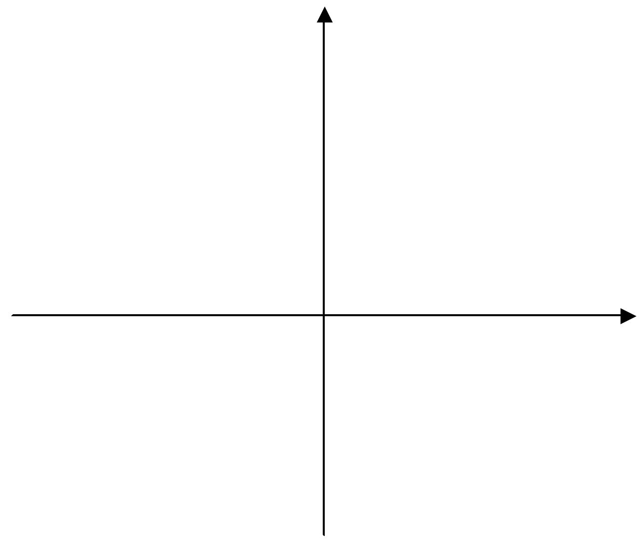


Note: The parabola is symmetric about the vertical axis that passes through the vertex. If no x-intercept, use the symmetric of the y-intercept about the axis of symmetry to graph the parabola.

Exercise #1:

(a) Graph the following parabola: $y = x^2 + 3x + 2$

Vertex:

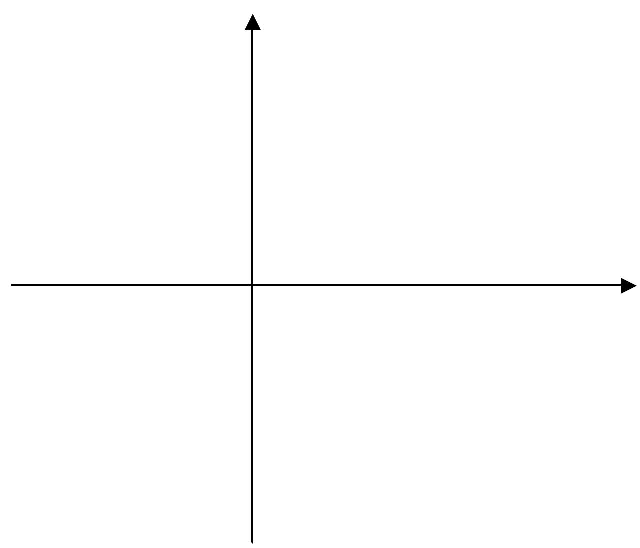


y-intercept:

x-intercepts:

(b) Graph the following parabola: $y = -2x^2 + 4x + 1$

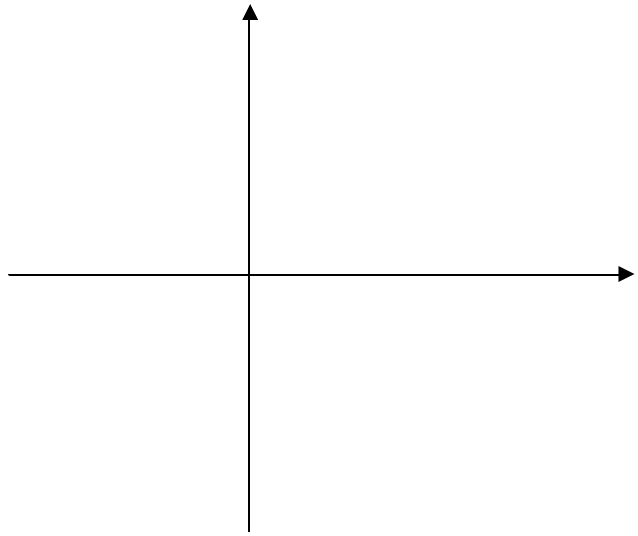
Vertex:



y-intercept:

x-intercepts:

(c) Graph the following parabola: $y = x^2 + x - 6$
Vertex:



y-intercept:

x-intercepts:

The Vertex Form of a Parabola

Example $y = -2x^2 + 4x + 1$

Complete the square on x:

The Vertex Form of a Parabola: $\boxed{y = a(x - x_v)^2 + y_v}$, where $V(x_v, y_v)$ is the vertex and a is the coefficient of x^2.

Exercise #2: Find the vertex of each parabola. Decide whether the vertex is a maximum or a minimum point.

(a) $y = 2(x-3)^2 + 4$

(b) $y = -3(x+3)^2 - 5$

(c) $y = 3x^2 + 4x + 2$

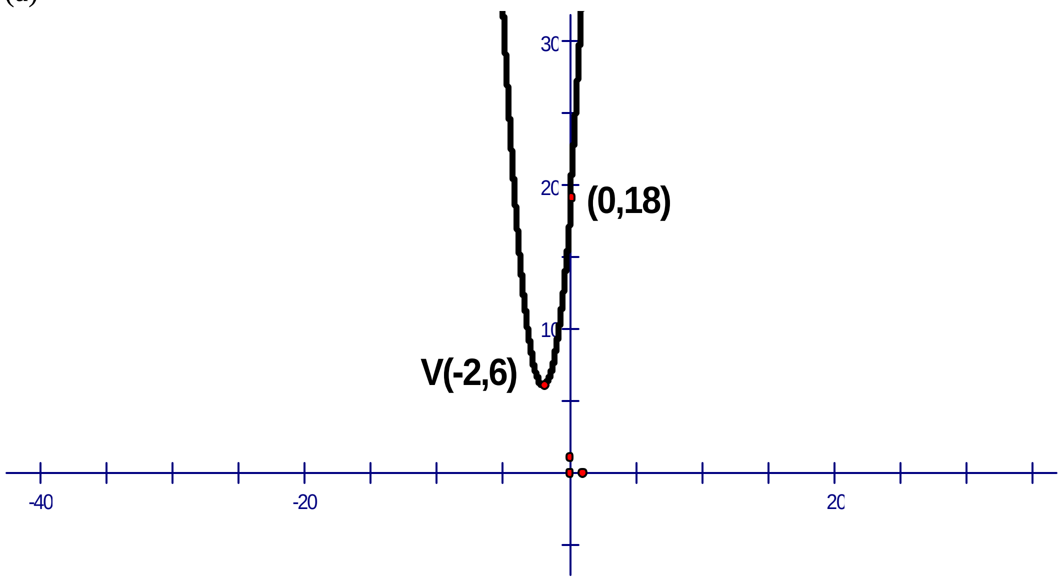
Method I – using the vertex formula	Method II – completing the square on x

Exercise #3: Write an equation for a parabola with vertex $V(1,-3)$.

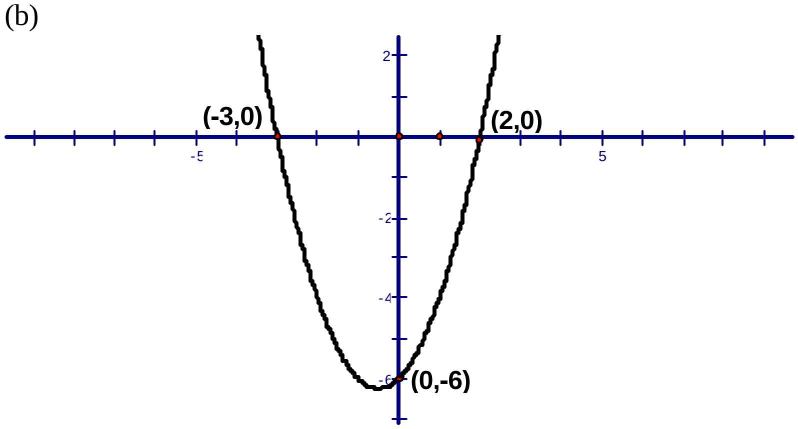
Exercise #4: Write an equation for the parabola with vertex $V(-1,1)$ that passes through the point $(2,3)$.

Exercise #5: i) Write an equation for each graph.
ii) What is the domain and the range of the function?
iii) Using the graph, solve the following: $f\left(x\right)>0$; $f\left(x\right)=0$; $f\left(x\right)<0$.

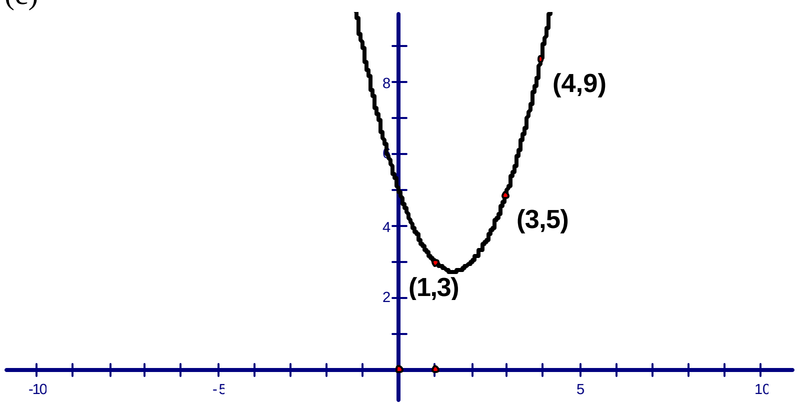
(a)



(b)



(c)



Applications:

(1) A person standing close to the edge on the top of a 160-foot building throws baseball vertically upward. The quadratic function $s(t) = -16t^2 + 64t + 160$ models the ball's height above the ground, *s(t),* in feet, *t* seconds after it is thrown.

(a) After how many seconds does the ball reach its maximum height? What is the maximum height?

(b) How many seconds does it take until the ball finally hits the ground?

(c) Find *s*(0) and describe what it means.

(2) You have 600 feet of fencing to enclose a rectangular plot that borders on a river. If you do not fence the side along the river, find the length and width of the plot that will maximize the area. What is the largest area that can be enclosed?

(3) Virtual Fido is a company that makes electronic virtual pets. The fixed weekly cost is $3000, and variable costs for each pet is $20.

(a) Let x represent the number of virtual pets made and sold each week. Write the weekly cost function, C, for Virtual Fido.

(b) The function $R(x) = -x^2 + 1000x$ describes the money that Virtual Fido takes in each week from the sale of x virtual pets. Use this revenue function and cost function from part (a) to write the weekly profit function P.

(c) Use the profit function to determine the number of virtual pets that should be made and sold each week to maximize the profit. What is the maximum weekly profit?

More applications

(4) The total profit Kiyoshi makes from producing and selling "x" floral arrangements is

$$P = -0.4x^2 + 36x .$$

a) How many floral arrangements should Kiyoshi produce and sell to maximize his profit?
b) What is his maximum profit? Explain how you know for sure you found the maximum profit.

(5) The sum of twice one number and three times another number is 5. Find two such numbers so to maximize their product.

(6) During a single day, the Roll-It shop will rent 35 rollerblades if they charge $5 per rental. They find that for every 20 cents they increase the charge, they lose one rental.

a) Find an expression for the total revenue "R" in terms of the number of $0.20 price increases "x".
b) What is the amount they can charge in order to maximize their revenue?

(7) The owners of a small fruit orchard decide to produce gift baskets as a sideline. The cost per basket for producing x baskets is $C = 0.01x^2 - 2x + 120$. How many baskets should they produce in order to minimize the cost per basket? What will their total cost be at that production level?