Math 71

REVIEW
Chapter 1 – The Real Number System

In class work: Solve all exercises.

(Sections 1.1 & 1.2)

Definition **A set** is a collection of objects (elements).

The Set of Natural Numbers $\mathbb{N}$ — $\mathbb{N}$ = **{ 1, 2, 3, 4, 5,…}**

The Set of Whole Numbers ***W*** — ***W*** = **{ 0, 1, 2, 3, 4, 5,…}** $\mathbb{N} \subset W$

The Set of Integers $\mathbb{Z}$ — $\mathbb{Z}$ = **{ …, -4, -3, -2, -1 ,0, 1, 2, 3, 4, 5,…}** $\mathbb{N} \subset W \subset \mathbb{Z}$

The Set of Rational Numbers $\mathbb{Q}$ — $\mathbb{Q} = \left\{ \frac{a}{b} \middle| a, b \in \mathbb{Z}, b \neq 0 \right\}$ $\mathbb{N} \subset W \subset \mathbb{Z} \subset \mathbb{Q}$

The Set of Irrational Numbers — Examples: $\sqrt{2}, -\sqrt{5}, p$

The Set of Real Numbers $\mathbb{R}$ — $\mathbb{R} = \{ x \mid x \text{ is rational or } x \text{ is irrational} \}$

$$\mathbb{N} \subset W \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

Mathematical Symbols

SYMBOL	MEANING	EXAMPLES
=	is equal to	
≠	is not equal to	
∈	belongs to (about an element)	
∉	it doesn't belong to	
<	is less than	
≤	is less than or equal to	
>	is greater than	
≥	is greater than or equal to	
∀	any	

Properties of Real Numbers

PROPERTIES	ADDITION +	MULTIPLICATION •
COMMUTATIVITY	$a+b=b+a, \quad \forall a,b\in \mathbb{R}$	$ab=ba \quad \forall a,b\in \mathbb{R}$
ASSOCIATIVITY	$(a+b)+c=a+(b+c), \forall a,b,c\in \mathbb{R}$	$(ab)c=a(bc), \quad \forall a,b,c\in \mathbb{R}$
IDENTITY ELEMENT	0 $a+0=0+a=a, \forall a\in \mathbb{R}$	1 $a\cdot 1=1\cdot a=a, \forall a\in \mathbb{R}$
INVERSE ELEMENT	$\forall a\in \mathbb{R}$, there is $-a\in \mathbb{R}$ such that $a+(-a)=(-a)+a=0$	$\forall a\in \mathbb{R}, a\neq 0$, there is $\frac{1}{a}\in \mathbb{R}$ such that $a\cdot\frac{1}{a}=\frac{1}{a}\cdot a=1$
DISTRIBUTIVITY	$a(b+c)=ab+ac$ → multiply out (remove parentheses) ← factor out the common factor	

Exercise #1 Find the opposite and the reciprocal (if any) of each number:

The Number	Its Opposite	Its Reciprocal

The Double Negative Rule

$$-(-a)=a$$

(Section 1.2)

The Absolute Value of a Number

Definition (1) **The absolute value of a number** is the distance between the number and 0 (the origin) on the number line.

$$|a| = dist(a,0)$$

Property $|a| \geq 0, \quad \forall a \in R$

Definition (2)

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

Properties (1) $|ab| = |a| \cdot |b|, \ \forall a, b \in \mathbb{R}$

(2) $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}, \forall a, b \in \mathbb{R}, b \neq 0$

Note: $|a+b| \neq |a| + |b|$ Example: ______________________

$|a-b| \neq |a| - |b|$ Example: ______________________

Exercise #2 Simplify the following:

a) $|-7| =$

b) $-(-7) =$

c) $-|-7| =$

d) $-|-(-7)| =$

Exercise #3 Simplify the following:

a) $(-5)^2 - 3^2 + |10 - 2 \cdot 3|$ (A: 20)

b) $\dfrac{(-4)^2 - |1 - 2^3|}{-(-2)^3 + (-1)^{125}}$ (A: $\dfrac{9}{7}$)

c) $\dfrac{9[4-(1+6)]-(3-9)^2}{5+\dfrac{12}{5-\dfrac{6}{2+1}}}$ (A: -7)

Exercise #4 Evaluate the following expressions if $x = 2, y = -3, z = -1$:

a) $\dfrac{3y^2 - x^2 + 1}{y|z|}$ (A: -8)

b) $yz^3 - (xy)^3$ (A: 219)

(Section 1.6)

Properties of Integral Exponents

Definition   If $n \in \mathbb{N}$, then $a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ times}}$

a is called **base** and n is called **power (exponent).**

PROPERTY		EXAMPLES
The Product Rule	$a^m \cdot a^n = a^{m+n}$	
The Quotient Rule	$\frac{a^m}{a^n} = a^{m-n}$	
The Zero-Exponent Rule	$a^0 = 1, \forall a \neq 0$	
The Negative-Exponent Rule	$a^{-n} = \frac{1}{a^n}$	
The Power Rule	$\left(a^m\right)^n = a^{m \cdot n}$	
Products to Power	$(ab)^n = a^n \cdot b^n$	
Quotients to Power	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	

Exercise #5 Simplify the following expressions:

a) $x-5[x-5(x-5)]$ (A: $21x-125$

b) $x^2y(xy-x)-7xy(x^2y-x^2)$ $\left(A: -6x^3y^2+6x^3y\right)$

c) $(-8xy)(x^5y^4)(-4xy)$ $\left(A: 32x^7y^6\right)$

d) $x\left[2x^2+x\left(x-3\left(x-1\right)\right)\right]$ $\left(A: 3x^2\right)$

Exercise #6 Simplify each expression .
Write answers without using parentheses or negative exponents.

a) $\dfrac{y^2}{yy^{-2}}$ $\left(A: y^3\right)$

b) $\left(\dfrac{a^2b^{-1}}{4a^3b^{-2}}\right)^{-3}$ $\left(A: \dfrac{64a^3}{b^3}\right)$

c) $\dfrac{a^0+b^0}{2\left(a+b\right)^0}$ $\left(A: 1\right)$

d) $\left(\dfrac{-2a^{-4}b^3c^{-1}}{3a^{-2}b^{-5}c^{-2}}\right)^{-4}$ $\left(A: \dfrac{81a^8}{16b^{32}c^4}\right)$

e) $\left(\dfrac{2x^{-4}y}{x^5y^5}\right)^{-3}\left(\dfrac{4x^{-2}y^0}{x^7y^2}\right)^2$ $\left(A: 2x^9y^8\right)$

f) $\dfrac{24x^2y^{13}}{-2x^5y^{-2}}$ $\left(A: -\dfrac{12y^{15}}{x^3}\right)$

g) $\left(-4x^{-4}y^5\right)^{-2}\left(-2x^5y^{-6}\right)$ $\left(A: -\dfrac{x^{13}}{8y^{16}}\right)$

Sets. Operations with Sets

Example#1 Let A and B be two sets of elements: $A = \{a, b, c\}$, $B = \{a, b, c, d\}$

$a \in A$ because a is an element of A

$d \notin A$ because d is not an element of A.

$\{a, b, c\} = \{b, a, c\}$

Definition $A \subset B$ **A is included in B** if any element of A is also in B.

Example #2 $\{a, b, c\} \subset \{a, b, c, d\}$

$\{1,2,3\} \not\subset \{1,2\}$

Operations with sets

$\bigcup$ **- "union"** $A \bigcup B = \{x \mid x \in A \text{ or } x \in B\}$

Examples:

$\bigcap$ **- "intersection"** $A \bigcap B = \{x \mid x \in A \text{ and } x \in B\}$

Examples:

The Empty Set $\varnothing$ **-** the set with no elements

<u>Definition</u> A number ***a* is less than a number *b*** $(a < b)$ if a is to the left of b on the number line.

<u>Exercise #7</u> Write equivalent statements:

a) $2 \le 3$ ______________

b) $2 > y$ ______________

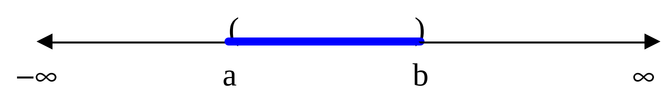
c) $5 > x \ge -2$ ______________

d) $-4 < -2$ ______________

Intervals of real numbers

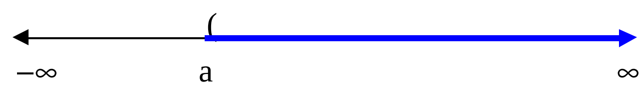
$[a,b] = \{x \mid a \le x \le b\}$

$(a,b) = \{x \mid a < x < b\}$

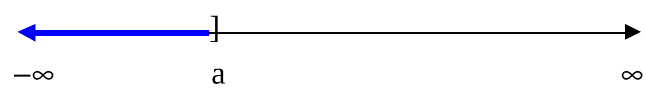


$[a,\infty) = \{x \mid x \ge a\}$

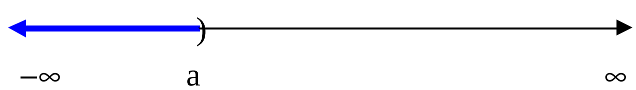
$(a,\infty) = \{x \mid x > a\}$



$(-\infty,a] = \{x \mid x \le a\}$



$(-\infty,a) = \{x \mid x < a\}$



<u>Exercise #8</u> Do the following operations and graph the solution set:

a) $[-2,5] \cup [-3,1] =$

b) $[-2,5] \cap [-3,1] =$

c) $(1,\infty) \cap (-3,4) =$

d) $(-\infty,2) \cup [0,\infty) =$

e) $(-4,-1) \cap (-1,2) =$

<u>Exercise #9</u> Graph the following sets and express them using interval notation:

a) $\{x \mid x \le -2\} =$

b) $\{x \mid 2 < x \le 3\} =$

c) $\{x \mid -3 \ge x \ge -7\} =$

(Sections 1.4 & 1.5)

Linear Equations

Definition An **equation** is a mathematical statement that two algebraic expressions are equal.

Examples:

Types of Equations

(1) **IDENTITY** = an equation which is always **true** regardless of the value of the variable.

Examples: $3 = 3$

$x+1 = x+1$

(2) **CONTRADICTION (INCONSISTENT)** = an equation which is always **false** regardless of the value of the variable.

Examples: $5 = 7$

$x+2 = x+4$

(3) **CONDITIONAL** = an equation whose truth or falsehood depends on the value of the variable.

Examples: $x+2 = 5$

Exercise #10 Determine the type of each of the following equations:

a) $2(x-3) = 2x-3$

b) $5(x+2) = 5x+10$

c) $3(w+1) = w+3$

Definition A **solution** of an equation is the value of the variable that **satisfies** the equation.

Definition The process of finding the values that satisfy an equation is called **solving the equation.**

Exercise #11 Determine which of the listed values satisfies the given equation:

a) $2x+3=6$, $x=0, x=\frac{3}{2}$

b) $6-2w=10-3w$, $w=-4, w=1$

Properties of Equality

$$\text{If } a=b\text{, then } \begin{cases} a+c=b+c, \forall c\in \mathbb{R} \\ a-c=b-c, \forall c\in \mathbb{R} \\ ac=bc, \forall c\in \mathbb{R} \\ \frac{a}{c}=\frac{b}{c}, \forall c\neq 0 \end{cases}$$

Exercise #12 Solve the following equations .

a) $9(6+x)=-7(2+x)$ $\left(A: x=-\frac{17}{4}\right)$

b) $\frac{3}{5}x+2=\frac{10}{3}$ $\left(A: x=\frac{20}{9}\right)$

c) $\frac{3(n-2)}{5}=\frac{3n+6}{6}$ $(A: n=22)$

d) $9(a+5)-10(1-a)=14$ $\left(A: a=-\frac{21}{19}\right)$

e) $\frac{x+1}{3}=5-\frac{x+2}{7}$ $\left(A: x=\frac{46}{5}\right)$

f) $\frac{x+4}{2}+\frac{x+1}{4}=3$ $(A: x=1)$

g) $0.8q-3.2=1.6$ $(A: q=6)$

h) $\frac{1}{4}m+\frac{2}{3}m=\frac{1}{6}$ $\left(m=\frac{2}{11}\right)$

i) Evaluate $x^2-(xy-y)$ for x satisfying $\frac{3(x+3)}{5}=2x+6$ and y satisfying $-2y-10=5y+18$. $(A: -7)$

j) $\frac{5}{6}=\frac{2u-3}{5}$ $\left(A: u=\frac{43}{12}\right)$

k) $\frac{2m-1}{2}-\frac{3m-1}{3}=\frac{4m-1}{4}$ $\left(A: m=\frac{1}{12}\right)$

l) $0.3r+1.2(20)=0.8(r+20)$ $(A: r=16)$

Exercise #13 Solve each formula for the specified variable:

a) $v = k + gt$, for t $\left(A: t = \dfrac{v-k}{g}\right)$

b) S=3pd+pa, for d $\left(A: d = \dfrac{S - p a}{3p}\right)$

c) $A = P(1 + rt)$, for r $\left(A: r = \dfrac{A-p}{pt}\right)$

d) $A = 2w^2 + 4lw$, for l $\left(A: l = \dfrac{A - 2w^2}{4w}\right)$

e) $A = \dfrac{1}{2}h(a+b)$ for a $\left(A: a = \dfrac{2A}{h} - b\right)$

f) $A = 2lw + 2lh + 2wh$ for l $\left(A: l = \dfrac{A - 2wh}{2(w+h)}\right)$

(Section 1.3)

A Review of the Rectangular Coordinate System
Graphing Equations

Definition The **Rectangular Coordinate System** (Cartesian coordinate system) is a system of two perpendicular number lines:

- the horizontal number line **(*x*-axis)**
- the vertical number line **(*y*-axis)**
- the point of intersection of the coordinate axes is called the **origin.**

Definition The **general form of a linear equation in two variables** is

$ax + by = c$, where a and b are not both zero.

Definition A **solution** of an equation in two variable is an **ordered pair** (x, y)

that satisfies the equation.

Definition The **graph** of an equation is the set of all points that **satisfy** the equation.

<u>Theorem</u> The graph of an equation of the form $ax + by = c$ is a **line** provided that a and b are not both zero.

<u>Property</u> i) The ordered pair (x, y) is a solution of an equation if and only if (x, y) belongs to the **graph of the given equation.**

ii) The ordered pair (x, y) is **not a solution** of an equation if and only if (x, y) doesn't belong to the **graph of the given equation.**

<u>Definition</u> The x- intercept of a line is the point where the line intersects the ***x*-axis.**

The y-intercept of a line is the point where the line intersects the ***y*-axis.**

<u>Exercise #19</u> Graph the following equations:

a) $y = x^3$

b) $y = |x|$

c) $y = \frac{1}{x}$

d) $y = 2x + 4$

e) $2x + 3y = 6$