Math 61 Spring 2007 Name:

TEST #2 @ 130 points

Write in a neat and organized fashion. Use a straightedge and compass for your drawings.

1. Answer the following questions. Do not prove.

a) When is a quadrilateral a parallelogram?
To receive full credit, give a case involving the sides, one involving the angles, and one involving the diagonals of the quadrilateral.

sides { • when the opposite sides are parallel (definition)
OR
• when a pair of opposite sides are parallel and congruent

<'s • when the opposite angles are congruent

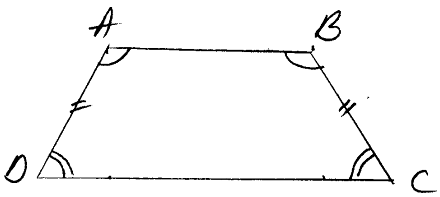
diag. • when diagonals bisect each other

b) How are the legs and the base angles of an isosceles trapezoid?
Make a drawing and state the answer using math notation pertinent to your drawing .

$\overline{AD} \cong \overline{BC}$ (legs)

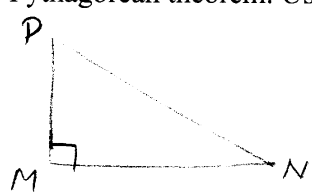
$\angle A \cong \angle B$ (base <'s)

$\angle D \cong \angle C$



c) Draw a right triangle and write the Pythagorean theorem. Use math notation pertinent to your drawing.

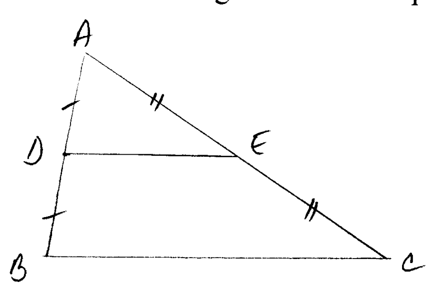
$(MN)^2 + (MP)^2 = (PN)^2$



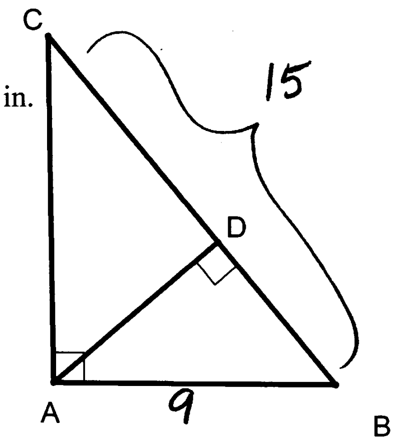
d) What do you know about the segment that joins the midpoints of two sides of a triangle?
Make a drawing and state the answer using math notation pertinent to your drawing .

$\overline{DE} \parallel \overline{BC}$

$DE = \frac{1}{2} BC$



2. Triangle ABC is a right triangle with hypotenuse $BC = 15$ in and leg $AB = 9$ in.
Find: a) BD
b) CD
c) AC
d) AD
Justify your answers.



Solution

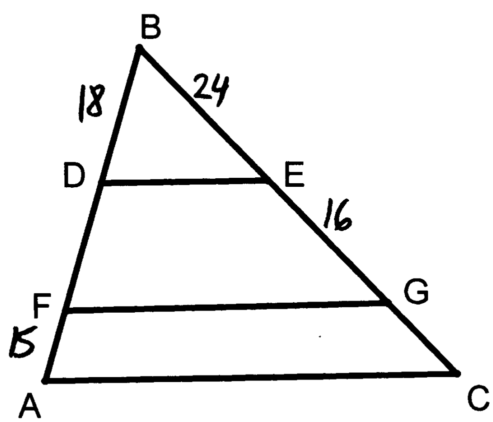
a)
$AB^2 = BC \cdot BD \Rightarrow BD = \frac{AB^2}{BC}$
(AB - leg)
$BD = \frac{81}{15} = \frac{27}{5} = 5.4$ in

b) $CD = BC - BD$
$= 15 - 5.4 = 9.6$ in

c) $AC^2 = BC^2 - AB^2$ (Pythagorean theorem in $\triangle ABC$)
$= 15^2 - 9^2$
$AC^2 = 144 \Rightarrow AC = 12$

d) $AD^2 = BD \cdot DC$
(AD - altitude to hyp)
$AD^2 = (5.4 \text{ in})(9.6 \text{ in})$ $AD = 7.2$ in

3. Given: $\triangle ABC$ with $\overline{DE} \parallel \overline{FG} \parallel \overline{AC}$ where $BE = 24, BD = 18, EG = 16, FA = 15$.
Find: DF and GC. Justify your answers.



Solution

$\triangle BFG$: $\overline{DE} \parallel \overline{FG} \Rightarrow$

$\frac{BD}{DF} = \frac{BE}{EG}$

$\frac{18}{DF} = \frac{24}{16} \Rightarrow DF = \frac{16 \cdot \cancel{18}^{3}}{\cancel{24}_{4}} = 4 \cdot 3$

$DF = 12$

$\triangle BAC$: $\overline{FG} \parallel \overline{AC} \Rightarrow$

$\frac{BF}{FA} = \frac{BG}{GC}$

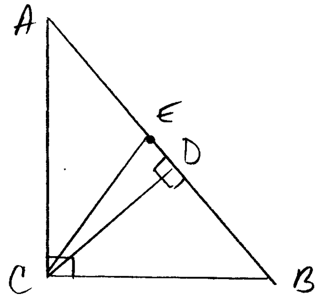
$\frac{18+12}{15} = \frac{24+16}{GC}$

$\frac{30}{15} = \frac{40}{GC} \Rightarrow$

$GC = \frac{40}{2} = 20$

$GC = 20$

4. a) Draw a right triangle with right angle C. Then draw the altitude $\overline{CD}$ and the median $\overline{CE}$.



b) If $AB = 20$, $AD = 4$ and [redacted], find CE, CD and AC. Justify your answers.

$\overline{CE}$ - median $\Rightarrow CE = \frac{1}{2} AB$

$CE = 10$

$\overline{CD}$ - altitude $\Rightarrow CD^2 = AD \cdot DB$

$AD = 4$

$DB = AB - AD$

$= 20 - 4 = 16$

$CD^2 = 4 \cdot 16$

$CD = 2 \cdot 4$

$CD = 8$

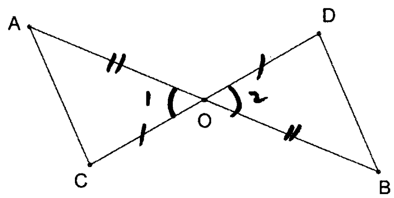
$\overline{AC}$ - leg $\Rightarrow AC^2 = AD \cdot AB$

$AC^2 = 4 \cdot 20$

$AC = 2 \cdot 2\sqrt{5}$

$AC = 4\sqrt{5}$

5.



Given $\overline{AB}$ bisects $\overline{CD}$
$\overline{CD}$ bisects $\overline{AB}$

Prove $\triangle AOC \cong \triangle BOD$ (Formal proof)

Proof

Statement	Reasons
1. $\overline{AB}$ bisects $\overline{CD}$	1. given
2. $\overline{CO} \cong \overline{DO}$	2. def. segment bisector
3. $\overline{CD}$ bisects $\overline{AB}$	3. given
4. $\overline{AO} \cong \overline{BO}$	4. def. segment bisector
5. $\triangle AOC$, $\triangle BOD$: $\overline{AO} \cong \overline{BO}$, $\overline{CO} \cong \overline{DO}$, $\angle 1 \cong \angle 2$	5. (4) above; (2) above; vertical $\angle$'s
6. $\triangle AOC \cong \triangle BOD$	6. SAS

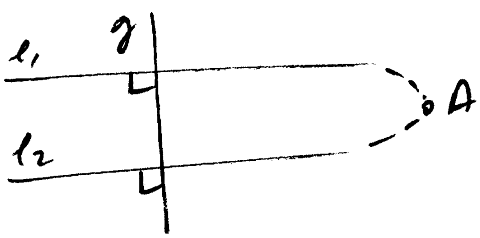
6.

Prove the following theorem using an <u>indirect proof.</u>
Make sure you make a drawing to illustrate the theorem; write the hypothesis and conclusion using math notation pertinent to your drawing.

If two coplanar lines are each perpendicular to a third line, then these lines are parallel to each other.

Given $l_1 \perp g$
$l_2 \perp g$
l_1, l_2 = coplanar

Prove: $l_1 \parallel l_2$



Proof

Assume $l_1 \nparallel l_2$, l_1, l_2 = coplanar $\Big\} \Rightarrow l_1 \cap l_2 \neq \emptyset$, let $l_1 \cap l_2 = A$

We have a line g
a point $A \notin g$
and two lines through A, both $\perp g$
Contradiction with uniqueness of a perpendicular to a line from a point outside line
Therefore, $l_1 \parallel l_2$

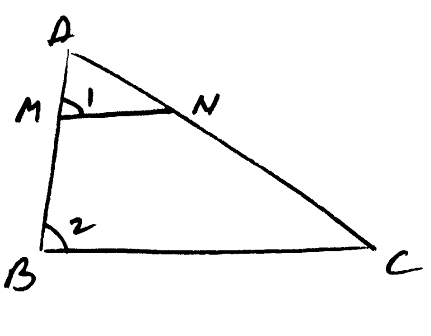
7.

Prove the following theorem using a <u>formal proof.</u>
Make a drawing to illustrate the theorem; write the hypothesis and conclusion using math notation pertinent to your drawing.

A line parallel to one side of a triangle that intersects the other two sides divides the two sides into proportional segments.

Given: $\triangle ABC$
$\overline{MN} \parallel \overline{BC}$
$M \in \overline{AB}$, $N \in \overline{AC}$

Prove: $\dfrac{AM}{MB} = \dfrac{AN}{NC}$



Proof

1. $\triangle ABC$, $\overline{MN} \parallel \overline{BC}$	1. given
2. $\angle 1 \cong \angle 2$	2. $\parallel$ iff. corresponding $\angle$'s $\cong$ ($\overline{MN} \parallel \overline{BC}$, $\overline{AB}$-trans)
3. $\angle A \cong \angle A$	3. reflexive
4. $\triangle AMN \sim \triangle ABC$	4. AA
5. $\dfrac{AM}{AB} = \dfrac{AN}{AC}$	5. CSSTP
6. $\dfrac{AM}{AB-AM} = \dfrac{AN}{AC-AN}$	6. Property proportions
7. $AB-AM = MB$, $AC-AN = NC$	7. Segm. addition prop.
8. (6,7) $\dfrac{AM}{MB} = \dfrac{AN}{NC}$	8. Substitution