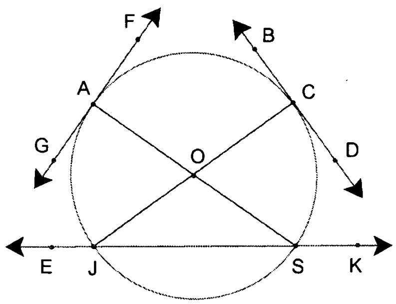
Math 61 Spring 2007 Name:

QUIZ #3 @ 85 points

Write in a neat and organized fashion. Use a pencil. Show all work to get credit.

1. In the given figure, name:



a) four radii $\overline{OA}, \overline{OC}, \overline{OS}, \overline{OJ}$

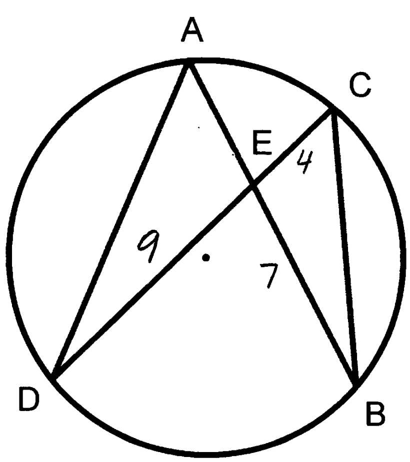
b) two diameters $\overline{JC}, \overline{AS}$

c) three chords $\overline{JS}, \overline{AS}, \overline{JC}$

d) two tangents $\overleftrightarrow{GF}, \overleftrightarrow{BD}$

e) one secant $\overleftrightarrow{EK}$

2.



Given: DE = 9, EC = 4, EB= 7

Find: AB

Solution

$\overline{AB}, \overline{DC}$ = secants

$\overline{AB} \cap \overline{DC} = E$

Then, $AE \cdot EB = CE \cdot DE$

$AE \cdot 7 = 4 \cdot 9$

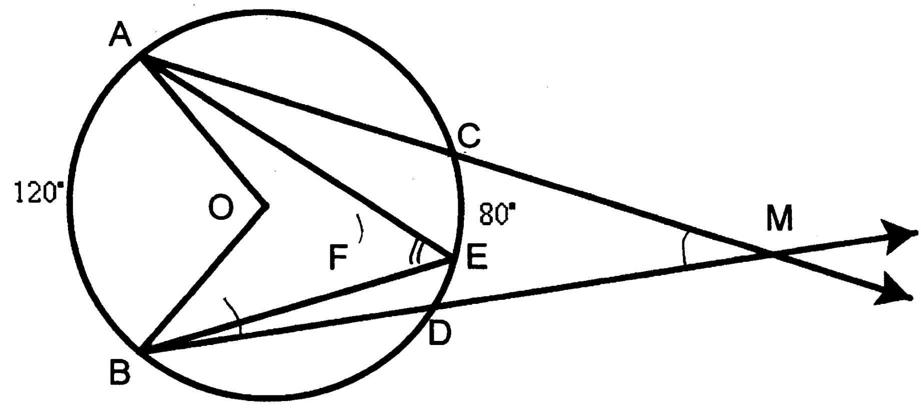
$AE = \frac{36}{7}$

$AB = AE + EB$

$= \frac{36}{7} + 7 = \frac{36+49}{7} = \frac{85}{7}$

$\boxed{AB = \frac{85}{7} = 12\frac{1}{7}}$

3.



Given arcs: $m\overset{\frown}{AB} = 120^{\circ}$ and $m\overset{\frown}{CD} = 80^{\circ}$

Find:

a) $m\angle AOB$ $= m\overset{\frown}{AB} = 120^{\circ}$

b) $m\angle CFD$ $= \frac{1}{2}\left(m\overset{\frown}{CD} + m\overset{\frown}{AB}\right) = \frac{1}{2}\left(120^{\circ}+80^{\circ}\right) = 100^{\circ}$

Name another angle that is congruent with $\angle CFD$: $\angle AFB$

c) $m\angle CBD$ $= \frac{1}{2}\, m\overset{\frown}{CD} = \frac{1}{2}(80^{\circ}) = 40^{\circ}$

Name another angle that is congruent with $\angle CBD$: $\angle CAD$

d) $m\angle AEB$ $= \frac{1}{2}\, m\overset{\frown}{AB} = \frac{1}{2}(120^{\circ}) = 60^{\circ}$

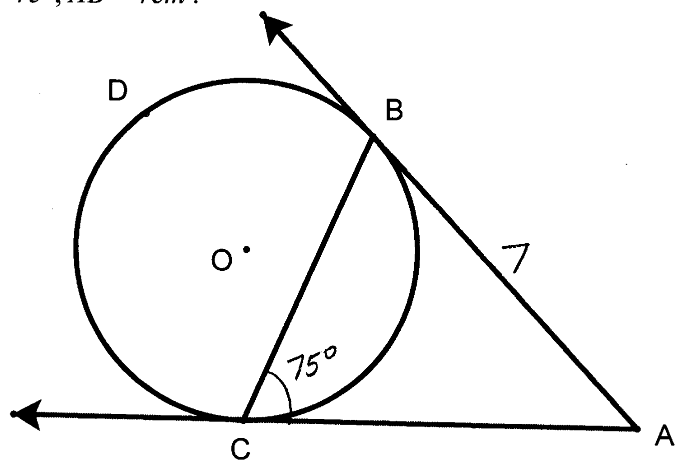
Name two other angles that are congruent with $\angle AEB$: $\angle ACB$ and $\angle ADB$

e) $m\angle AMB$ $= \frac{1}{2}\left(m\overset{\frown}{AB} - m\overset{\frown}{CD}\right)$

$= \frac{1}{2}\left(120^{\circ} - 80^{\circ}\right)$

$= 20^{\circ}$

4. Given: $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are tangents to $\odot O$, $m\angle ACB = 75^\circ$, $AB = 7cm$.



Find:

a) $m\widehat{BC}$

$m\angle ACB = \frac{1}{2} m\widehat{BC} \Rightarrow m\widehat{BC} = 2\, m\angle ACB$

$= 2(75^\circ) = 150^\circ$

b) $m\widehat{BDC}$

$m\widehat{BDC} = 360^\circ - m\widehat{BC}$

$= 360^\circ - 150^\circ = 210^\circ$

c) $m\angle ABC$

$m\angle ABC = \frac{1}{2}\, m\widehat{BC} = \frac{1}{2} 150^\circ = 75^\circ$

d) $m\angle A$ $= \frac{1}{2}\left(m\widehat{BDC} - m\widehat{BC}\right)$

$= \frac{1}{2}\left(210^\circ - 150^\circ\right) = \frac{1}{2}(60^\circ) = 30^\circ$

(OR: $\triangle ABC$: $m\angle A = 180^\circ - m\angle B - m\angle C$)

e) $AC = AB = 7\,cm$ because $\overline{AB}$ and $\overline{AC}$ are tangent segments and tangents to $\odot$ are $\cong$.

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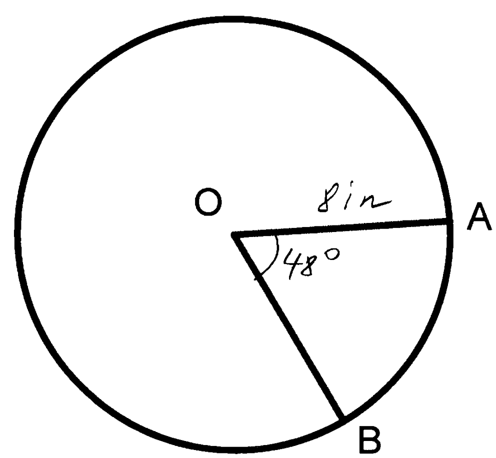
5. Given $\odot O$ with $m\angle AOB = 48^\circ$

$OA = 8in$

Find and use correct units:



a) $m\widehat{AB} = m\angle AOB = 48^\circ$

b) $l\widehat{AB}$

$\frac{l\widehat{AB}}{48^\circ} = \frac{2\pi r}{360^\circ} \Rightarrow l\widehat{AB} = \frac{2\pi(8in)48^\circ}{360^\circ}$

$l\widehat{AB} = \frac{32\pi}{15}\, in$

c) Circumference of the circle

$C = 2\pi r = 2\pi 8in = 16\pi\ in$

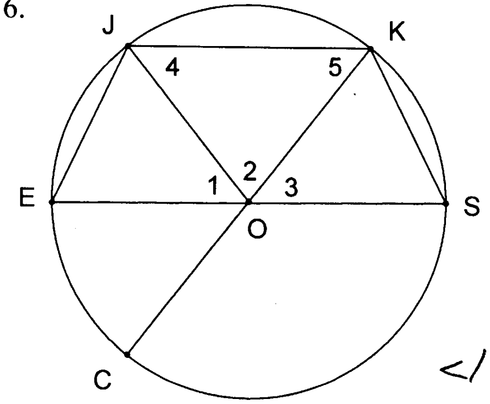
d) Area of the circle

$A = \pi r^2 = \pi(8in)^2 = 64\pi\ in^2$

e) Area of the sector AOB $\frac{A(\text{sector } AOB)}{48^\circ} = \frac{\pi r^2}{360^\circ} \Rightarrow A(\text{sector } AOB) = \frac{\pi\cdot(8in)^2\cdot 48^\circ}{360^\circ}$

$A(\text{sector } AOB) = \frac{128\pi}{15}\, in^2$

6.



Given: $\odot O$
$\overline{ES} \parallel \overline{JK}$
$m\widehat{JK} = 78^\circ$
$JE = 5$ cm

Find:
a) $\angle s 1-5$
b) $m\widehat{JE}$, $m\widehat{KS}$, KS, $m\widehat{JC}$

Solution

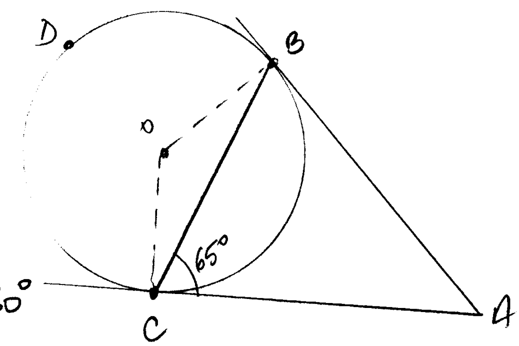
$\angle 1 \cong \angle 4 \cong \angle 5 \cong \angle 3$
(alt. int ∡'s) (isosc △) (alt. int. ∡'s)

(a) $m\angle 2 = m\widehat{JK} = 78^\circ$
$\angle EOS$ = straight ∡
Let $m\angle 1 = x$
Then, $x + m\angle 2 + x = 180^\circ$
$x = 51^\circ$
Therefore, $m\angle 1 = 51^\circ$
$m\angle 2 = 78^\circ$
$m\angle 3 = 51^\circ$
$m\angle 4 = 51^\circ$
$m\angle 5 = 51^\circ$

(b) $m\widehat{JE} = m\angle 1 = 51^\circ$
$m\widehat{KS} = m\angle 3 = 51^\circ$
$\Rightarrow KS = JE = 5$ cm
(≅ chords iff ≅ central ∡'s iff ≅ arcs)

7.Given: $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are tangents to $\odot O$, with B and C on the circle and $m\angle ACB = 65^\circ$.

Find: a) $m\widehat{BC}$
b) $m\widehat{BDC}$
c) $m\angle ABC$
d) $m\angle A$



Solution

(a) $m\angle ACB = 65^\circ$ (given)
$65^\circ = m\angle ACB = \frac{1}{2}\, m\widehat{BC} \Rightarrow m\widehat{BC} = 2(65^\circ) = 130^\circ$

(b) $m\widehat{BDC} = 360^\circ - m\widehat{BC} = 360^\circ - 130^\circ = 230^\circ$

(c) $m\angle ABC = m\angle ACB = 65^\circ$ (intercept same arc $\widehat{BC}$)

(d) $\triangle ABC$: $m\angle A = 180^\circ - m\angle B - m\angle C$
$= 180^\circ - 2(65^\circ)$
$= 180^\circ - 130^\circ$
$= 50^\circ$