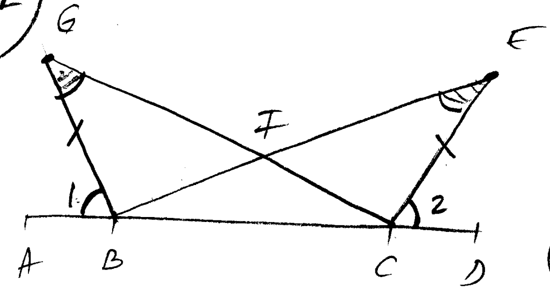
More applications (2.3)

#5 -2.3



Given: $\overline{BG} \cong \overline{CE}$

$\angle 1 \cong \angle 2$

Prove: $\angle G \cong \angle E$

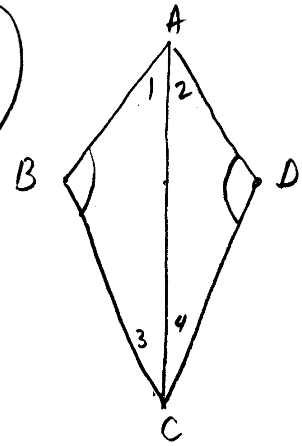
(we'll show $\triangle GBC \cong \triangle ECB$

$\left\{ \begin{array}{l} BC \text{ - common side} \\ \angle B = \angle C \text{ (supplem)} \\ BG = CE \end{array} \right\}$

Proof

1. $\angle 1$ and $\angle GBC$ = suppl.	1. given
2. $\angle 2$ and $\angle ECB$ = suppl.	2. given
3. $\angle 1 \cong \angle 2$	3. given
4. $\angle GBC \cong \angle ECB$ (1,2,3)	4 suppl. of eq. $\angle$'s are eq.
5 $\triangle GBC$, $\triangle ECB$ $\left\{ \begin{array}{l} \overline{BG} \cong \overline{CE} \\ \overline{BC} \cong \overline{BC} \\ \angle GBC \cong \angle ECB \end{array} \right.$	5. $\left\{ \begin{array}{l} \text{given} \\ \text{reflexive (common side)} \\ \text{(4) above} \end{array} \right.$
6. $\triangle GBC \cong \triangle ECB$	6. SAS
7. $\angle G \cong \angle E$	7. CPCTC

#10
2.3



Given $\overrightarrow{AC}$ bisects $\angle BAD$
$\overrightarrow{CA}$ bisects $\angle BCD$

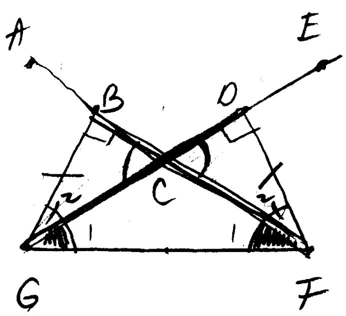
Prove $\angle B \cong \angle D$

(we'll show $\triangle ABC \cong \triangle ADC$)
ASA

Proof

1. $\overrightarrow{AC}$ bisects $\angle BAD$ — 1. given
2. $\angle 1 \cong \angle 2$ — 2. def. of $\angle$ bisector
3. $\overrightarrow{CA}$ bisects $\angle BCD$ — 3. given
4. $\angle 3 \cong \angle 4$ — 4. def. of $\angle$ bisector
5. $\triangle ABC$, $\triangle ADC$: $\angle 1 \cong \angle 2$, $\overline{AC} \cong \overline{AC}$, $\angle 3 \cong \angle 4$ — 5. (2) above; reflexive prop. (common side); (4) above
6. $\triangle ABC \cong \triangle ADC$ — 6. ASA
7. $\angle B \cong \angle D$ — 7. CPCTC

#15
2.3



Given $\overline{GB} \perp \overline{AF}$
$\overline{FD} \perp \overline{GE}$
$\overline{GD} \cong \overline{FB}$
$\overline{GB} \cong \overline{FD}$

Prove $\overline{BC} \cong \overline{DC}$

we'll show $\triangle DGF \cong \triangle BFG \Rightarrow \angle G \cong \angle F$, $\angle F_1 \cong \angle G_1$ $\Rightarrow \angle G_2 \cong \angle F_2$

$\triangle BCG \cong \triangle DCF$ ASA

Proof

	Statement	Reason
1.	$\triangle DGF$, $\triangle BFG$: $\overline{BG} \cong \overline{DF}$; $\overline{BF} \cong \overline{DG}$; $\overline{GF} \cong \overline{GF}$	given; given; reflexive (common side)
2.	$\triangle DGF \cong \triangle BFG$	SSS
3.	$\angle G \cong \angle F$	CPCTC
4.	$m\angle G = m\angle F$	def. of $\cong \angle$'s
5.	$\angle BFG \cong \angle DGF$	CPCTC
6.	$m\angle BFG = m\angle DGF$	def. of $\cong \angle$'s
7. (4,6)	$m\angle G - m\angle DGF = m\angle F - m\angle BFG$	Add/Subt. prop. of =
8.	$m\angle BGC = m\angle G - m\angle DGF$; $m\angle DFC = m\angle F - m\angle BFG$	Add. Post. for $\angle$'s
9. (7,8)	$m\angle BGC = m\angle DFC$	substitution
10.	$\angle BGC \cong \angle DFC$	def. of $\cong \angle$'s
11.	$\overline{GB} \perp \overline{AF}$; $\overline{FD} \perp \overline{GE}$	given
12.	$\angle B$, $\angle D$ = right $\angle$'s	$\perp$ lines form right $\angle$'s
13.	$\angle B \cong m\angle D$	right $\angle$'s are $\cong$
14.	$\triangle BCG$, $\triangle DCF$: $\angle B \cong \angle D$; $\overline{BG} \cong \overline{DF}$; $\angle BGC \cong \angle DFC$	(13) above; given; (10) above
15.	$\triangle BCG \cong \triangle DCF$	ASA
16.	$\overline{BC} \cong \overline{DC}$	CPCTC

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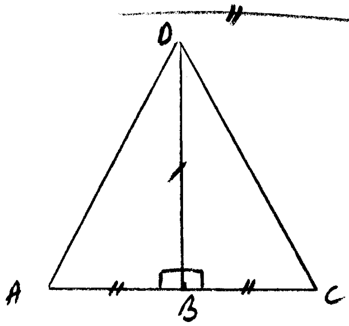
HOMEWORK # 3

SECTION 2.2

(14) Given $\overline{DB} \perp \overline{AC}$

$\overline{DB}$ bisects $\overline{AC}$

Prove $\triangle ABD \cong \triangle CBD$



Proof

Statements	Reasons
1. $\overline{DB} \perp \overline{AC}$	1. given
2. $\angle ABD, \angle CBD$ = right $\angle$'s	2. $\perp$ iff right $\angle$'s
3. $\angle ABD \cong \angle CBD$	3. right $\angle$'s are $\cong$
4. $\overline{DB}$ bisects $\overline{AC}$	4. given
5. $\overline{AB} \cong \overline{BC}$	5. def of segment bisector
6. $\triangle ABD$, $\triangle CBD$: $\overline{DB} \cong \overline{DB}$; $\overline{AB} \cong \overline{CB}$; $\angle ABD \cong \angle CBD$	6. reflexive prop of $\cong$; (5) above; (3) above
7. $\triangle ABD \cong \triangle CBD$	7. SAS

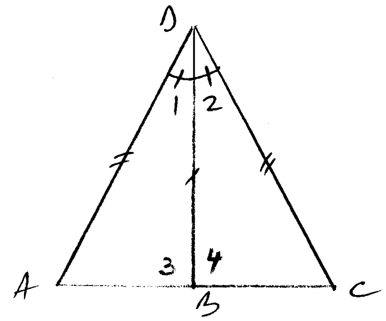
(Note - also, we could use
$\triangle ABD$, $\triangle CBD$ - right $\triangle$'s
$\triangle ABD \cong \triangle CBD$ (LL))

SECTION 2.3

(12) Given $\overrightarrow{DB}$ bisects $\angle ADC$

$\overline{AD} \cong \overline{CD}$

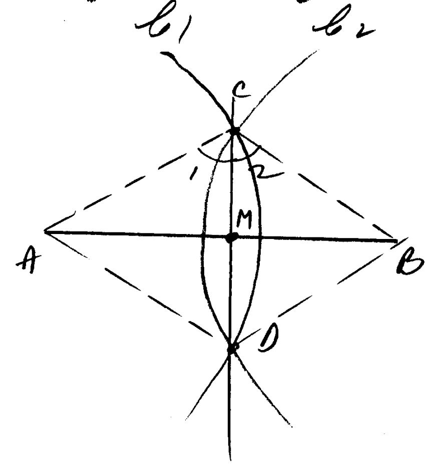
Prove $\overline{DB} \perp \overline{AC}$



Proof

Statements	Reasons
1. $\overline{DB}$ bisects $\angle ADC$	1. given
2. $\angle 1 \cong \angle 2$	2. def. $\angle$ bisector
3. $\triangle ADB$, $\triangle CDB$: $\overline{AD} \cong \overline{CD}$; $\overline{BD} \cong \overline{BD}$; $\angle 1 \cong \angle 2$	3. given; reflexive prop of $\cong$; (2) above
4. $\triangle ADB \cong \triangle CDB$	4. SAS
5. $\angle 3 \cong \angle 4$	5. CPCTC
6. $\angle 3, \angle 4$ adj $\angle$'s	6. given
7. $\overline{DB} \perp \overline{AC}$	7. $\perp$ iff $\cong$ adjacent $\angle$'s (def. of $\perp$ lines)

Construct the midpoint of a given segment



Given $\overline{AB}$ - segment

Construct M = midpct of $\overline{AB}$

(conditions: $M \in \overline{AB}$
$\overline{AM} \cong \overline{MB}$)

Solution

1. let $\overline{AB}$ - given segment
2. construct circle $\mathcal{C}_1$ - $\begin{cases} \text{center } A \\ \text{radius } r > \frac{AB}{2} \end{cases}$

   Construct circle $\mathcal{C}_2$ - $\begin{cases} \text{center } B \\ \text{radius } r \end{cases}$
3. $\mathcal{C}_1 \cap \mathcal{C}_2 = \{C, D\}$
4. connect C with D $\Rightarrow$ line $\overleftrightarrow{CD}$
   (two points determine exactly one line)
5. $\overleftrightarrow{CD} \cap \overline{AB} = \{M\}$
6. M = midpoint of $\overline{AB}$

We'll prove M = midpct of $\overline{AB}$

First, connect A with C and B with C (2 pts determine exactly one line)
A with D B with D

$\left.\begin{array}{l} \triangle ACD \\ \triangle BCD \end{array}\right\} \left.\begin{array}{ll} \overline{AC} \cong \overline{BC} & \text{(radii in } \cong \text{ circles)} \\ \overline{AD} \cong \overline{BD} & \text{(radii in } \cong \text{ circles)} \\ \overline{CD} \cong \overline{CD} & \text{(reflexive prop. } \cong) \end{array}\right| \Rightarrow$

$\triangle ACD \cong \triangle BCD$ (SSS) $\Rightarrow$
$\angle ACD \cong \angle BCD$ (CPCTC)
$(\angle 1 \cong \angle 2)$ (*)

$\triangle ACM$ $\triangle BCM$ $\left\{\begin{array}{ll} \overline{AC} \cong \overline{BC} & \text{(radii } \cong \text{ circles)} \\ \overline{CM} \cong \overline{CM} & \text{(reflexive prop. } \cong\text{)} \\ \angle ACM \cong \angle BCM & \text{(above *)} \end{array}\right| \Rightarrow$

$\triangle ACM \cong \triangle BCM$ (SAS) $\Rightarrow$

$\left.\begin{array}{ll} \overline{AM} \cong \overline{BM} & \text{(CPCTC)} \\ M \in \overline{AB} & \text{(by construction)} \end{array}\right| \Rightarrow$

M = midpoint of $\overline{AB}$ (def. of midpt)