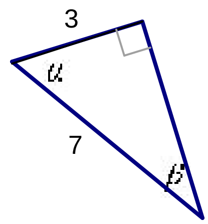
TEST #1 @ 150 points

Solve the problems on separate paper. Clearly label the problems. Show all steps in order to get credit. No proof, no credit given

1. Find $\sin\alpha$, $\cos\beta$, $\tan\alpha$, $\cot\beta$ if



2. Let *t* a real number and $P\left(\frac{2\sqrt{2}}{3},\frac{-1}{3}\right)$ a point on the unit circle that corresponds to *t*. Find the exact values of the six trigonometric functions of *t*.

3. Find θ if $0^\circ < \theta < 360^\circ$ and $\sin\theta = \frac{\sqrt{3}}{2}$ with θ in quadrant II.

4. Find the exact values of the following:

a) $\sin 45^\circ + \cos\left(-30^\circ\right)$ b) $\sin\left(240^\circ\right)$ c) $\tan\frac{\pi}{3}+\cos\frac{\pi}{3}$ d) $\cos\left(495^\circ\right)$

e) $\sin\left(-\frac{3\pi}{4}\right)$ f) $\tan\left(\frac{13\pi}{6}\right)$ g) $\cos\left(\frac{5\pi}{6}\right)$

5. Use the unit circle to find all the values of θ between 0 and 2π for which

a) $\sin\theta = \frac{1}{2}$ b) $\cos\theta = -\frac{\sqrt{2}}{2}$

6. Prove the following trigonometric identities:

a) $\csc\theta + \sin\left(-\theta\right) = \frac{\cos^2\theta}{\sin\theta}$ b) $\frac{\cos\alpha}{1+\sin\alpha} + \frac{1+\sin\alpha}{\cos\alpha} = 2\sec\alpha$

7. Show that tangent is an odd function.

8. A mixing blade on a food processor extends out 3 inches from its center. If the blade is turning at 600 revolutions per minute, what is the linear velocity of the tip of the blade in feet per minute?

9. A lawn sprinkler is located at the corner of a yard. The sprinkler is set to rotate through 90° and project water out 60 feet. What is the area of the yard watered by the sprinkler?

10. From a point on the ground 500 feet from the base of a building, it is observed that the angle of elevation to the top of the building is 24° and the angle of elevation to the top of a flagpole atop the building is 27°. Find the height of the building and the length of the flagpole.

(1) Let x = the other leg

$x^2 = 7^2 - 3^2$

$= 49 - 9$

$= 40$

$x = \sqrt{40} = 2\sqrt{10}$

$\sin\alpha = \frac{opp}{hyp} = \frac{x}{7} = \frac{2\sqrt{10}}{7}$

$\boxed{\sin\alpha = \frac{2\sqrt{10}}{7}}$

$\cos\beta = \frac{adj}{hyp} = \frac{x}{7} = \frac{2\sqrt{10}}{7}$

$\boxed{\cos\beta = \frac{2\sqrt{10}}{7}}$

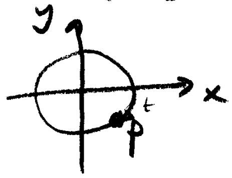
$\tan\alpha = \frac{opp}{adj} = \frac{x}{3} = \frac{2\sqrt{10}}{3}$

$\boxed{\tan\alpha = \frac{2\sqrt{10}}{3}}$

$\cot\beta = \frac{adj}{opp} = \frac{x}{3} = \frac{2\sqrt{10}}{3}$

$\boxed{\cot\beta = \frac{2\sqrt{10}}{3}}$

(2) $P\left(\frac{2\sqrt{2}}{3}, \frac{-1}{3}\right) \in \text{IV}$



$\cos t = \frac{2\sqrt{2}}{3}$

$\sin t = \frac{-1}{3}$

$\tan t = \frac{\sin t}{\cos t} = \frac{-\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = \frac{-1}{2\sqrt{2}} = \frac{-\sqrt{2}}{4}$

$\cot t = \frac{1}{\tan t} = -2\sqrt{2}$

$\sec t = \frac{1}{\cos t} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$

$\csc t = \frac{1}{\sin t} = -3$

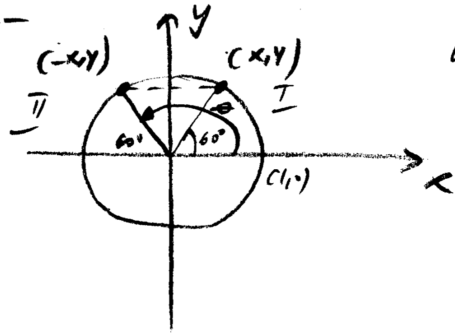
$\sin t = \frac{-1}{3}$
$\cos t = \frac{2\sqrt{2}}{3}$
$\tan t = \frac{-\sqrt{2}}{4}$
$\cot t = -2\sqrt{2}$
$\sec t = \frac{3\sqrt{2}}{4}$
$\csc t = -3$

(3) Given: $\theta \in (0°, 360°)$

$\theta \in II$

$\sin\theta = \frac{\sqrt{3}}{2}$

Find $\theta = ?$



We know $\sin 60° = \frac{\sqrt{3}}{2}$

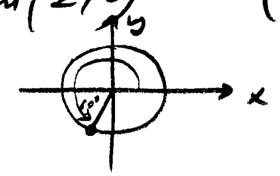
Therefore, $\theta = 180° - 60°$

$\boxed{\theta = 120°}$

(4) (a) $\sin 45° + \cos(-30°) = \sin 45° + \cos(30°)$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{2}+\sqrt{3}}{2}}$

(b) $\sin(240°) = \sin(180° + 60°) = -\sin 60° = \boxed{-\frac{\sqrt{3}}{2}}$



(c) $\tan\frac{\pi}{3} + \cos\frac{\pi}{3} = \frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}} + \cos\frac{\pi}{3}$

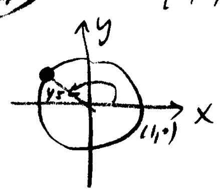
$= \frac{\sqrt{3}/2}{1/2} + \frac{1}{2} = \sqrt{3} + \frac{1}{2} = \boxed{\frac{2\sqrt{3}+1}{2}}$

(d) $\cos(495°) = \cos(360° + 135°)$

$= \cos(135°)$

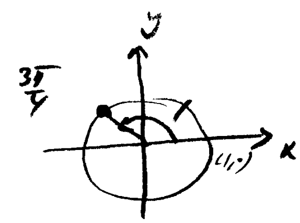
$= \cos(180° - 45°)$

$= -\cos 45° = \boxed{-\frac{\sqrt{2}}{2}}$



(e) $\sin\left(-\frac{3\pi}{4}\right) = -\sin\frac{3\pi}{4}$

$= -\sin\frac{\pi}{4} = \boxed{-\frac{\sqrt{2}}{2}}$



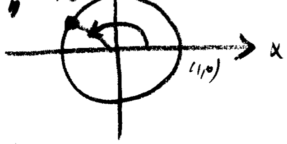
(f) $\tan\frac{13\pi}{6} = \tan\left(2\pi + \frac{\pi}{6}\right)$

$= \tan\frac{\pi}{6} = \frac{\sin\pi/6}{\cos\pi/6} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

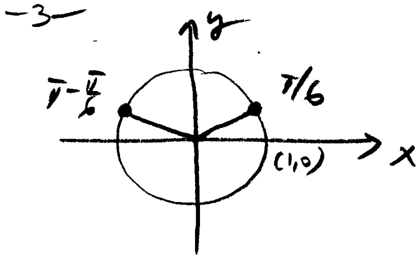
(g) $\cos\left(\frac{5\pi}{6}\right) = \cos\left(\pi - \frac{\pi}{6}\right)$

$= -\cos\frac{\pi}{6}$

$= \boxed{-\frac{\sqrt{3}}{2}}$



(5) $\theta \in (0, 2\pi)$

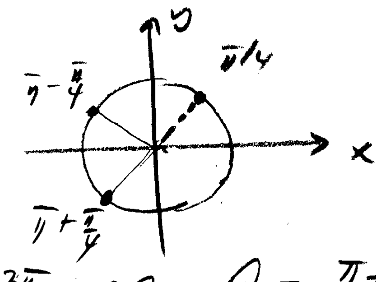


(a) $\sin\theta = -\frac{1}{2}$

We know $\sin\frac{\pi}{6} = \frac{1}{2}$

Therefore, $\theta = \pi/6$ OR $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$$\boxed{\theta \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}}$$



(b) $\cos\theta = -\frac{\sqrt{2}}{2}$

We know $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

Therefore, $\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ OR $\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$$\boxed{\theta \in \left\{\frac{3\pi}{4}, \frac{5\pi}{4}\right\}}$$

(6) (a) $\csc\theta + \sin(-\theta) = \frac{\cos^2\theta}{\sin\theta}$

Proof

$$\csc\theta + \sin(-\theta) = \frac{1}{\sin\theta} - \sin\theta$$

$$= \frac{1 - \sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$$

Therefore, $\csc\theta + \sin(-\theta) = \frac{\cos^2\theta}{\sin\theta}$

(b) $\frac{\cos a}{1+\sin a} + \frac{1+\sin a}{\cos a} = 2\sec a$

Proof

$$\frac{\cos a}{1+\sin a} + \frac{1+\sin a}{\cos a} = \frac{\cos^2 a + (1+\sin a)^2}{\cos a(1+\sin a)}$$

$$= \frac{\cos^2 a + 1 + 2\sin a + \sin^2 a}{\cos a(1+\sin a)}$$

$$= \frac{1+1+2\sin a}{\cos a(1+\sin a)} = \frac{2(1+\sin a)}{\cos a(1+\sin a)}$$

$$= \frac{2}{\cos a} = 2\sec a$$

Therefore, $\frac{\cos a}{1+\sin a} + \frac{1+\sin a}{\cos a} = 2\sec a$

(7) Need to show that $\tan(-\theta) = -\tan\theta$ for any $\theta \in \mathbb{R}$

Proof

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)}$$

$$= \frac{-\sin\theta}{\cos\theta}$$

$$= -\tan\theta$$

Therefore, tan is an odd function

(8) Given: $r = 3 in$

rate $= 600$ rev/min

Find $v = ?$ (ft/min)

Solution

$V = \omega r$

$\omega = 600 \frac{rev}{min} =$

$= 600 \cdot \frac{2\pi \; rad}{min}$

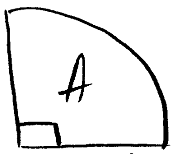
$= 1200\pi \frac{rad}{min}$

$V = 1200\pi \frac{rad}{min} \cdot 3in$

$= 1200\pi \frac{rad}{min} \cdot 3in \cdot \frac{1 \; ft}{12 \; in}$

$\boxed{V = 300\pi \frac{ft}{min}}$ $\quad V \approx 942.5$ ft/min

(9)



Given: $r = 60$ ft
Find: Area = ?

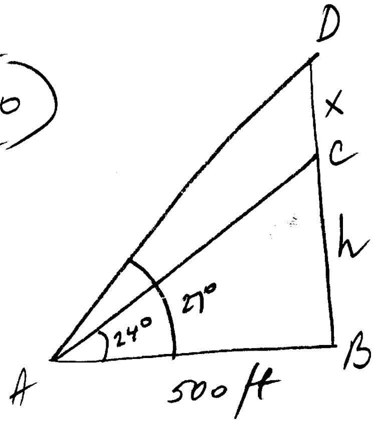
Solution

$A = \frac{1}{4} A_{circle}$

$A = \frac{1}{4} \cdot \pi r^2 = \frac{1}{4} \cdot \pi \cdot (60 ft)^2$

$A = 900\pi$ ft^2 $\quad A \approx 2827.4$ ft^2

(10)



Let h = height building
x = length flagpole

ΔABC: $\tan 24° = \frac{h}{500}$

$h = 500 \tan 24° \approx 223$ ft

ΔABD: $\tan 27° = \frac{h+x}{500}$

$h + x = 500 \tan 27° \approx 255$ ft

$x = 255 - 223$

$x \approx 32$ ft