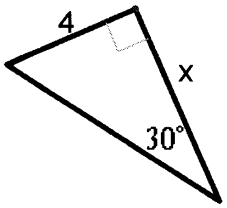
Math 150 Spring 07 Name:

QUIZ #2 @ 60 points

Solve the problems on separate paper. Clearly label the problems. Show all steps in order to get credit. No proof, no credit given

1. Find the side labeled x.



2. Sketch a right triangle that has one acute angle θ, and find the other five trigonometric ratios of θ.

$$\sin\theta = \frac{3}{7}$$

3. Find the exact value of each expression.

a) $\sin 30^\circ + \cos 60^\circ$

b) $\tan 45^\circ \left(\sin 60^\circ - \sec 45^\circ\right)$

4. From the top of a 200-ft lighthouse, the angle of depression to a ship in the ocean is 23°. How far is the ship form the base of the lighthouse?

5. A man standing on the roof of a building 60 feet high looks down to the building next door. He finds that the angle of depression to the roof of that building from the roof of his building to be 34.5°, while the angle of depression from the roof of his building to the bottom of the building next door is 63.2°. How tall is the building next door?

Solutions

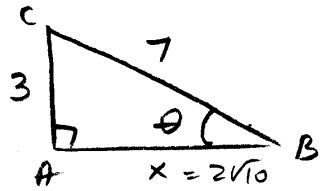
(1) $\tan 30^\circ = \frac{4}{x} \Rightarrow x = \frac{4}{\tan 30^\circ}$

$\tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

Therefore, $x = \frac{4}{\frac{1}{\sqrt{3}}}$

$$\boxed{x = 4\sqrt{3}}$$

(2) $\sin\theta = \frac{3}{7}$, then let $AC = 3$, $BC = 7$, $AB = x$



$x^2 + 3^2 = 7^2 \Rightarrow x^2 = 49 - 9$

$x^2 = 40$

$x = \sqrt{40}$

$x = 2\sqrt{10}$

Then, $\boxed{\cos\theta = \frac{2\sqrt{10}}{7}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{7}}{\frac{2\sqrt{10}}{7}} = \frac{3}{2\sqrt{10}} = \frac{3\sqrt{10}}{2 \cdot 10}$

$$\boxed{\tan\theta = \frac{3\sqrt{10}}{20}}$$

$\cot\theta = \frac{1}{\tan\theta} = \frac{2\sqrt{10}}{3}$

$$\boxed{\cot\theta = \frac{2\sqrt{10}}{3}}$$

$\sec\theta = \frac{1}{\cos\theta} = \frac{7}{2\sqrt{10}} = \frac{7\sqrt{10}}{2 \cdot 10}$

$$\boxed{\sec\theta = \frac{7\sqrt{10}}{20}}$$

$\csc\theta = \frac{1}{\sin\theta} = \frac{7}{3}$

$$\boxed{\csc\theta = \frac{7}{3}}$$

(3) a) $\sin 30^\circ + \cos 60^\circ =$

$= \frac{1}{2} + \frac{1}{2} = 1$

$\boxed{\sin 30^\circ + \cos 60^\circ = 1}$

	30°	60°	45°
$\sin$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$
$\cos$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$

b) $\tan 45^\circ (\sin 60^\circ - \sec 45^\circ) =$

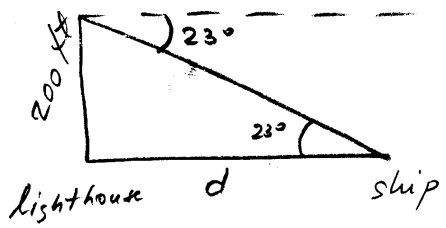
$= 1 \cdot \left(\frac{\sqrt{3}}{2} - \frac{1}{\cos 45^\circ} \right)$

$= \frac{\sqrt{3}}{2} - \frac{1}{\frac{\sqrt{2}}{2}} = \frac{\sqrt{3}}{2} - \frac{2}{\sqrt{2}}$

$= \frac{\sqrt{3}}{2} - \sqrt{2} = \frac{\sqrt{3} - 2\sqrt{2}}{2}$

$\boxed{\tan 45^\circ (\sin 60^\circ - \sec 45^\circ) = \frac{\sqrt{3} - 2\sqrt{2}}{2}}$

(4)



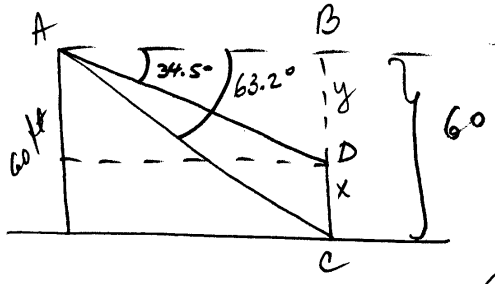
Let d = distance from ship to lighthouse

$\tan 23^\circ = \frac{200 \text{ ft}}{d}$

$d = \frac{200 \text{ ft}}{\tan 23^\circ}$

$\boxed{d \approx 471 \text{ ft}}$

(5)



Let $CD = x$ - the height of building next door

$BD = y$

$\triangle ABC$: $\tan 63.2^\circ = \frac{60 \text{ ft}}{AB} \Rightarrow AB = \frac{60 \text{ ft}}{\tan 63.2} \approx 30.3 \text{ ft} = AB$

$\triangle ABD$: $\tan 34.5^\circ = \frac{y}{AB} \Rightarrow y = AB \tan 34.5^\circ$

$y \approx 30.3 \tan 34.5^\circ$

$y \approx 20.8 \text{ ft}$

$x = 60 - y$

$x = 60 - 20.8$

$\boxed{x = 39.2 \text{ ft}}$ the height of the building next door