MATH 150

Quiz #1 - Solutions

(1) $\triangle BCD$: $DC^2 = BD^2 - BC^2$

$DC^2 = 5^2 - 4^2$

$\underline{DC = 3}$

Then $AC = AD + DC = \underline{5 = AC}$

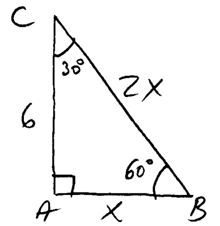
$\triangle ABC$: $AC^2 + BC^2 = AB^2$

$AB^2 = 5^2 + 4^2$

$AB^2 = 41$

$\boxed{AB = \sqrt{41}}$

(2)



Given: $\triangle ABC$ $(30°-60°-90°)$

$AC = 6$

Find: $AB = ?,\ BC = ?$

Solution

AB opposes 30° angle $\Rightarrow AB = \frac{BC}{2}$

Let $AB = x$

Then $BC = 2x$

$\triangle ABC$: $AB^2 + AC^2 = BC^2$

$x^2 + 6^2 = (2x)^2$

$x^2 + 36 = 4x^2$

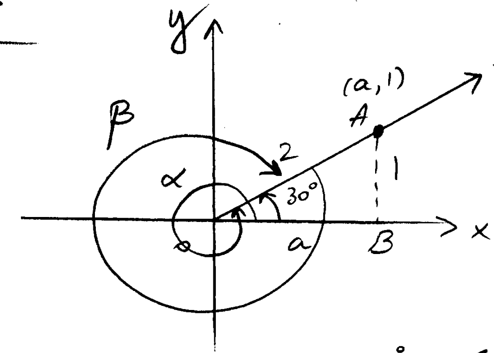
$3x^2 = 36$

$x^2 = 12$

$x = 2\sqrt{3}$

$\boxed{\begin{array}{l} AB = 2\sqrt{3} \\ BC = 4\sqrt{3} \end{array}}$

(3)



a) $\triangle OAB$: AB opposes 30° angle

$\Rightarrow AB = \frac{1}{2} OA \Rightarrow$

$OA = 2$

Then: $OB^2 + AB^2 = OA^2$

$a^2 + 1 = 4$

$a^2 = 3 \Rightarrow \boxed{a = \sqrt{3}}$ (I quadrant)

b) $\boxed{OA = 2}$ (see above)

c) coterminal ∡'s

$\boxed{\begin{cases} \alpha = 360° + 30° = 390° \\ \beta = -330° \end{cases}}$

(5)

(a) $\frac{\sin\theta}{\cos\theta} + \frac{1}{\sin\theta} =$

$= \frac{\sin^2\theta + \cos\theta}{\cos\theta \sin\theta}$

(6) $x = 3\sec\theta$

$\sqrt{9x^2 - 81} = \sqrt{9(3\sec\theta)^2 - 81}$

$= \sqrt{9(9\sec^2\theta) - 81}$

$= \sqrt{81\sec^2\theta - 81}$

$= \sqrt{81(\sec^2\theta - 1)}$

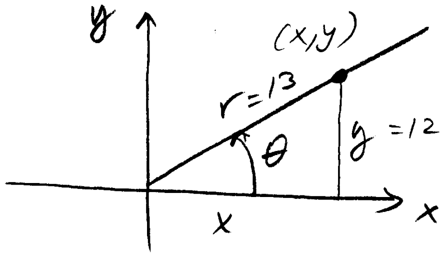
$= 9\sqrt{\sec^2\theta - 1}$

$= 9\sqrt{\tan^2\theta} = \boxed{9|\tan\theta|}$

(4) Method I

$\sin\theta = \frac{12}{13}$

$\theta \in I$ quadrant



if $\sin\theta = \frac{12}{13}$ / $\sin\theta = \frac{y}{r}$ / $\Rightarrow$ set $y=12$, $r=13$

Then $x^2 = r^2 - y^2$

$x^2 = 13^2 - 12^2$

$x^2 = 25$

$x = \pm 5$ / I quadrant / $\Rightarrow \boxed{x=5}$.

$\sin\theta = \frac{12}{13}$ - given

$\cos\theta = \frac{x}{r} = \frac{5}{13}$

$\tan\theta = \frac{y}{x} = \frac{12}{5}$

$\cot\theta = \frac{x}{y} = \frac{5}{12}$

$\sec\theta = \frac{r}{x} = \frac{13}{5}$

$\csc\theta = \frac{r}{y} = \frac{13}{12}$

Method II

$\sin\theta = \frac{12}{13}$

$\theta \in I$ quadrant

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{12}{13}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = 1 - \frac{144}{169}$

$\cos^2\theta = \frac{25}{169} \Rightarrow \cos\theta = \pm\frac{5}{13}$ / $\theta \in I$ quadrant / $\Rightarrow$

$\Rightarrow \cos\theta = \frac{5}{13}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{12}{\cancel{13}}}{\frac{5}{\cancel{13}}} = \frac{12}{5}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{5}{12}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{13}{5}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{13}{12}$

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(6) Prove $(\sin\theta - \cos\theta)^2 - 1 = -2\sin\theta\cos\theta$

Proof

$(\sin\theta - \cos\theta)^2 - 1 =$

$= \underline{\sin^2\theta} - 2\sin\theta\cos\theta + \underline{\cos^2\theta} - 1$

$= 1 - 2\sin\theta\cos\theta - 1$

$= -2\sin\theta\cos\theta$