Math 71 Spring 2006 Name:

TEST 4 @ 120 points

Show your work for credit. Write all responses on separate paper. Please write only on one side and clearly label the exercises.

1) $3x^2 + 3y^2 + 5x - 4y = 1$

a) What type of curve is this? Circle

b) What is the standard form of this equation? Show all work. $\left(x+\frac{5}{6}\right)^2 + \left(y-\frac{2}{3}\right)^2 = \frac{53}{36}$

c) Center? $\left(-\frac{5}{6}, \frac{2}{3}\right)$ d) Radius? $r = \frac{\sqrt{53}}{6} \approx 1.21$

e) Find the x- and y-intercepts (if any). Do not approximate. Give exact answers. Show all work

x-intercepts $\left(-\frac{5}{6} \pm \frac{\sqrt{37}}{6}, 0\right)$ y-intercepts $\left(0, \frac{2}{3} \pm \frac{\sqrt{7}}{3}\right)$

$3x^2 + 3y^2 + 5x - 4y = 1$

To find the coordinates of the center and the radius, we need to write the equation in standard form. For that, we need to complete the square on x and y.

Step 1: coefficient of x^2 and y^2 be 1 – divide both sides of the equation by 3.

$$x^2 + \frac{5}{3}x + y^2 - \frac{4}{3}y = \frac{1}{3}$$

Step 2: complete the square on x by adding $\left(\frac{1}{2}\text{coefficient of } x\right)^2$ and complete the square on y by adding $\left(\frac{1}{2}\text{coefficient of } y\right)^2$ to both sides of the equation.

$$x^2 + \frac{5}{3}x + \frac{25}{36} + y^2 - \frac{4}{3}y + \frac{4}{9} = \frac{1}{3} + \frac{25}{36} + \frac{4}{9}$$

$$\left(x+\frac{5}{6}\right)^2 + \left(y-\frac{2}{3}\right)^2 = \frac{53}{36}$$ - the standard equation of the circle.

To find the y-intercepts, make x=0 in the given equation (then use the quadratic formula to solve for y) or in the standard form (and use the square root property to solve the equation for y).

$$\left(0+\frac{5}{6}\right)^2 + \left(y-\frac{2}{3}\right)^2 = \frac{53}{36}$$

$$\frac{25}{36} + \left(y-\frac{2}{3}\right)^2 = \frac{53}{36}$$

$$\left(y-\frac{2}{3}\right)^2 = \frac{28}{36}$$

$y - \frac{2}{3} = \pm\frac{\sqrt{28}}{6}$, so $y_{1,2} = \frac{2}{3} \pm \frac{\sqrt{7}}{3}$

To find the x-intercepts make y=0 in the given equation (then use the quadratic formula to solve for x) or in the standard form (and use the square root property to so solve for x.

$$3x^2 + 3y^2 + 5x - 4y = 1$$

$$3x^2 + 5x - 1 = 0$$

$$x_{1,2} = \frac{-5 \pm \sqrt{5^2 - 4(3)(-1)}}{2 \cdot 3}$$

$$x_{1,2} = \frac{-5 \pm \sqrt{37}}{6}$$

2) $\begin{cases} x^2 - 4y^2 = -7 \\ 3x^2 + y^2 = 31 \end{cases}$

a) Solve the above system graphically; that is, graph each equation and identify the common points of the two graphs.

$$x^2 - 4y^2 = -7$$

$$\frac{x^2}{-7} - \frac{4y^2}{-7} = 1$$

$$\frac{4y^2}{7} - \frac{x^2}{7} = 1$$

$\dfrac{y^2}{\frac{7}{4}} - \dfrac{x^2}{7} = 1$ - hyperbola with a vertical transverse

To graph the hyperbola:

- plot on the x-axis $\pm\sqrt{7}$;
- plot on the y-axis $\pm\dfrac{\sqrt{7}}{2}$
- draw the box and the asyptotes
- using the vertices $\left(0, \pm\dfrac{\sqrt{7}}{2}\right)$, sketch the hyperbola

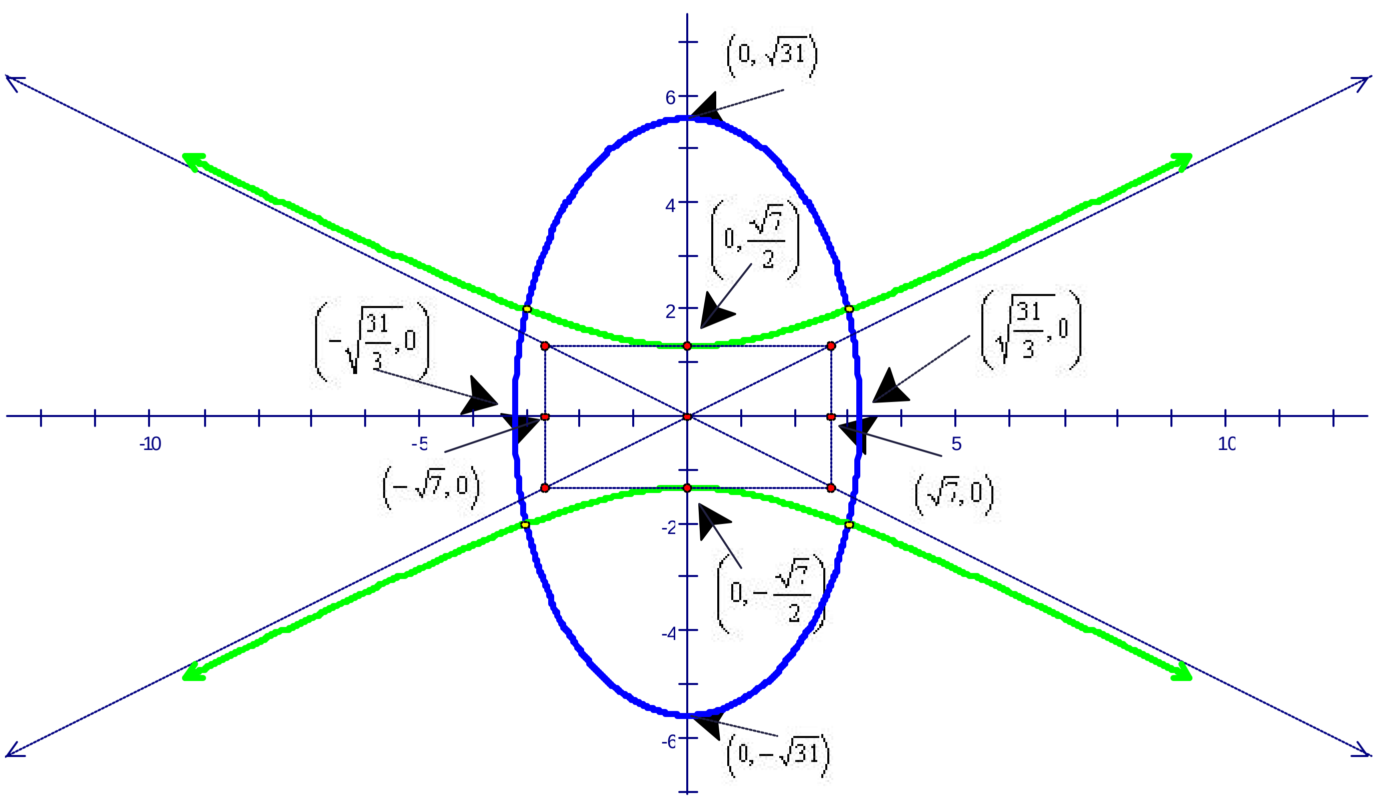
$$3x^2 + y^2 = 31$$

$$\frac{3x^2}{31} + \frac{y^2}{31} = 1$$

$\dfrac{x^2}{\frac{31}{3}} + \dfrac{y^2}{31} = 1$ - vertical ellipse

To graph the ellipse:

- plot on the x-axis $\pm\sqrt{\dfrac{31}{3}}$
- plot on the y-axis $\pm\sqrt{31}$
- sketch the ellipse



b) Solve the above system algebraically; that is, find the exact coordinates of the common points.

$$\begin{cases} x^2 - 4y^2 = -7 \\ 3x^2 + y^2 = 31 \end{cases}$$

If we multiply the first equation by -3 and add it to the second equation we obtain:

$$13y^2 = 52$$

$$y^2 = 4\,;\; y = \pm 2$$

If we substitute y into the second equation we obtain:

$$3x^2 = 27$$

$$x^2 = 9\,;\; x = \pm 3$$

Therefore, the solutions of the system are: (3,2), (3, -2), (-3, 2), and (-3, -2).

3)

a) Find the sum:

$$\sum_{i=1}^{3} \frac{(i+2)!}{i!} = \sum_{i=1}^{3} \frac{(i)!(i+1)(i+2)}{i!} = \sum_{i=1}^{3} (i+1)(i+2) = 2\cdot 3 + 3\cdot 4 + 4\cdot 5 = 6 + 12 + 20 = 38$$

b) Find the sum: $\sum_{k=3}^{7} \binom{7}{k} = \binom{7}{3} + \binom{7}{4} + \binom{7}{5} + \binom{7}{6} + \binom{7}{7} = \frac{7\cdot \cancel{6}\cdot 5}{1\cdot \cancel{2\cdot 3}} + 35 + \frac{7\cdot 6}{1\cdot 2} + 7 + 1 = 99$

Note: $\binom{7}{4} = \binom{7}{3}$, $\binom{7}{5} = \binom{7}{2}$, $\binom{7}{6} = \binom{7}{1}$

b) Express the sum using summation notation:

$$4 + \frac{4^2}{2} + \frac{4^3}{3} + \ldots + \frac{4^n}{n} = \sum_{i=1}^{n} \frac{4^i}{i}$$

c) Find a general term for the sequence:

$$\frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \ldots$$

$a_1 = \frac{3}{2}$

$a_2 = \frac{4}{3}$ $\qquad a_n = \frac{n+2}{n+1}$

$a_3 = \frac{4}{5}$

4) Use the Binomial Theorem to expand each binomial and express the result in simplified form:

a) $(3x+y)^5$ $(3x+y)^5 = {}_5C_0(3x)^5 + {}_5C_1(3x)^4 y + {}_5C_2(3x)^3 y^2 + {}_5C_3(3x)^2 y^3 + {}_5C_4(3x)y + {}_5C_5 y^5$

$$= 1\cdot 3^5 x^5 + 5\cdot 3^4 x^4 y + \frac{5\cdot 4}{1\cdot 2} 3^3 x^3 y^2 + \frac{5\cdot 4}{1\cdot 2} 3^2 x^2 y^3 + 5\cdot 3xy + 1\cdot y^5$$

Note: ${}_5C_3 = {}_5C_2$
${}_5C_4 = {}_5C_1$

b) $(x-1)^4 = {}_4C_0 x^4 + {}_4C_1 x^3(-1) + {}_4C_2 x^2(-1)^2 + {}_4C_3 x(-1)^3 + {}_4C_4(-1)^4 = x^4 - 4x^3 + \frac{4\cdot 3}{1\cdot 2}x^2 - \frac{4\cdot 3}{1\cdot 2}x + 1$

Note: ${}_4C_3 = {}_4C_1$

$$(x-1)^4 = x^4 - 4x^3 + 6x^2 - 4x + 1$$