Math 71 Spring 06 Name: SOLUTIONS

TEST 3 @ 120 points

Write in a neat and organized fashion. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given!

1. Find the domain and graph the function $f(x)=\sqrt{6-2x}$.

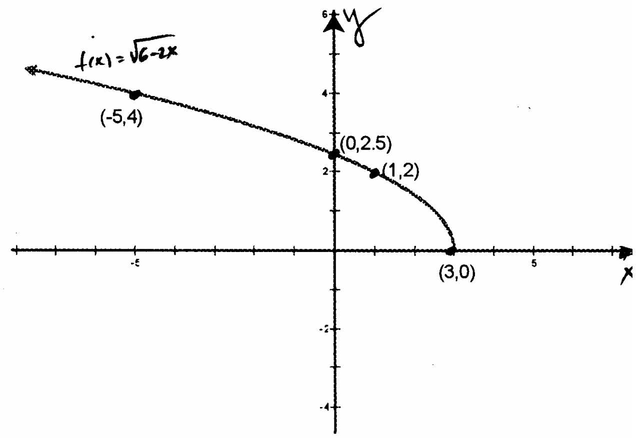
Domain

$6-2x \geq 0$

$6 \geq 2x$

$\boxed{x \leq 3}$

x	y
3	0
1	2
0	$\sqrt{6} \approx 2.5$
-5	4



2. Find the domain of each function:

a) $f(x)=\dfrac{\sqrt{x-2}}{\sqrt{5-x}}$

Conditions

$\begin{cases} ① \ x-2 \geq 0 \\ \text{AND} \\ ② \ 5-x > 0 \end{cases} \iff \begin{cases} x \geq 2 \\ \text{AND} \\ x < 5 \end{cases}$

$\iff \boxed{x \in [2,5)}$

b) $f(x)=\log_3(2-5x)$

Condition

$2-5x > 0$

$2 > 5x$

$\boxed{x < \frac{2}{5}}$

3. Let $f(x)=x^2-5x-3$. Find $f(1+i)$.

$f(1+i) = (1+i)^2 - 5(1+i) - 3$

$= 1+2i+i^2-5-5i-3$

$= \cancel{1} - 3i - \cancel{1} - 8$

$= \boxed{-8-3i}$

4. Simplify the following expressions. Show all steps. Do not just write down an answer.

a) $\left(\sqrt{2-\sqrt{3}}+\sqrt{2-\sqrt{3}}\right)^2 =$

$= \left(\sqrt{2-\sqrt{3}}\right)^2 + 2\sqrt{(2-\sqrt{3})(2-\sqrt{3})} +$

$+ \left(\sqrt{2-\sqrt{3}}\right)^2$

$= 2-\sqrt{3} + 2\sqrt{(2-\sqrt{3})^2} + 2-\sqrt{3}$

$= 4-2\sqrt{3} + 2(2-\sqrt{3})$

$= 4-2\sqrt{3} + 4-2\sqrt{3}$

$= \boxed{8-4\sqrt{3}}$

b) $\log_3 \frac{1}{27} = \log_3 (27)^{-1} =$

$= -\log_3 27$

$= \boxed{-3}$

c) $\log_{10}\left(\log_3\left(\log_5 125\right)\right) =$

$= \log_{10}(\log_3 3)$

$= \log_{10} 1$

$= \boxed{0}$

d) $3^{\log_3 5} - 7\log_5 \sqrt[7]{5} =$

$= 5 - \log_5 (\sqrt[7]{5})^7$

$= 5 - \log_5 5$

$= 5-1$

$= \boxed{4}$

5. Perform the indicated operations and write the result in the form $a+bi$.

$$\frac{1+i}{1+3i}\cdot\frac{1-3i}{1-3i}+\frac{1-i}{1-3i}\cdot\frac{1+3i}{1+3i} =$$

$$= \frac{(1+i)(1-3i) + (1-i)(1+3i)}{(1+3i)(1-3i)}$$

$$= \frac{1 \cancel{-3i} \cancel{+i} - 3i^2 + 1 \cancel{+3i} \cancel{-i} - 3i^2}{1^2 - (3i)^2}$$

$$= \frac{2-6i^2}{1-9i^2} = \frac{2-6(-1)}{1-9(-1)}$$

$$= \frac{2+6}{1+9} = \frac{8}{10} = \boxed{\frac{4}{5}+0i}$$

6. Solve each equation in $\mathbb{C}$ (the set of complex numbers) by the indicated method.

a) $5(x-2)^2+38=0$ by the square root property.

$5(x-2)^2=-38$

$(x-2)^2=\frac{-38}{5}$

$\sqrt{(x-2)^2}=\sqrt{\frac{-38}{5}}$

$x-2=\pm\sqrt{\frac{38}{5}}\,i$

$\boxed{x=2\pm\sqrt{\frac{38}{5}}\,i}$

b) $-4x^2-36x-61=0$ by completing the square.

$4x^2+36x=-61$

$x^2+9x=\frac{-61}{4}$

$\left(\frac{1}{2}\text{coef } x\right)^2=\left(\frac{9}{2}\right)^2=\frac{81}{4}$

$x^2+9x+\frac{81}{4}=\frac{-61}{4}+\frac{81}{4}$

$\left(x+\frac{9}{2}\right)^2=5$

$\sqrt{\left(x+\frac{9}{2}\right)^2}=\sqrt{5}$

$\left|x+\frac{9}{2}\right|=\sqrt{5}$

$x+\frac{9}{2}=\pm\sqrt{5}$

$\boxed{x=\frac{-9}{2}\pm\sqrt{5}}$

c) $-x^2+\frac{x}{2}=1$ by the quadratic formula.

$-2x^2+x=2$

$2x^2-x+2=0$

$\begin{cases} a=2 \\ b=-1 \\ c=2 \end{cases}$

$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-(-1)\pm\sqrt{(-1)^2-4(2)(2)}}{2(2)}$

$=\frac{1\pm\sqrt{1-16}}{4}$

$=\frac{1\pm\sqrt{-15}}{4}$

$\boxed{x_{1,2}=\frac{1\pm\sqrt{15}\,i}{4}}$

d) Solve $2x^2+xy+y^2=5$ for y.

$y^2+xy+2x^2-5=0$

quadratic in y

$y_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

where $\begin{cases} a=1 \\ b=x \\ c=2x^2-5 \end{cases}$

$y_{1,2}=\frac{-x\pm\sqrt{x^2-4(2x^2-5)}}{2}$

$=\frac{-x\pm\sqrt{x^2-8x^2+20}}{2}$

$\boxed{y_{1,2}=\frac{-x\pm\sqrt{20-7x^2}}{2}}$

7. Solve the following equations:

a) $x^4 - 3x^2 = -2$

$x^4 - 3x^2 + 2 = 0$

Let $x^2 = t$

then $x^4 = t^2$

$t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$ $\Rightarrow$ $t = 2$ OR $t = 1$

if $t = 2$: $x^2 = 2$, $x = \pm\sqrt{2}$

if $t = 1$: $x^2 = 1$, $x = \pm 1$

Therefore, $\boxed{x \in \{\pm\sqrt{2}, \pm 1\}}$

b) $x^{\frac{2}{3}} - 3 = 2x^{\frac{1}{3}}$

$x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 3 = 0$

Let $x^{\frac{1}{3}} = t$

then $x^{\frac{2}{3}} = t^2$

$t^2 - 2t - 3 = 0$

$(t-3)(t+1) = 0$ $\Rightarrow$ $t = 3$ OR $t = -1$

if $t = 3$: $x^{\frac{1}{3}} = 3$, $x = 3^3$, $x = 27$

if $t = -1$: $x^{\frac{1}{3}} = -1$, $x = (-1)^3$, $x = -1$

Therefore, $\boxed{x \in \{-1, 27\}}$

c) $\log_5(x+4) = 2$

$\Leftrightarrow$

$5^2 = x + 4$

$25 = x + 4$

$\boxed{x = 21}$

check

$\log_5(21+4) = \log_5 25 = 2$

d) $2^x = 7$ | $\log_2$

$\log_2 2^x = \log_2 7$

$\boxed{x = \log_2 7}$

OR

$2^x = 7$

$\ln 2^x = \ln 7$

$x \ln 2 = \ln 7$

$\boxed{x = \frac{\ln 7}{\ln 2}} \approx 2.8$

e) $\log_8(x+5) - \log_8 2 = 1$

$\log_8 \frac{x+5}{2} = 1$

$\Leftrightarrow$

$8^1 = \frac{x+5}{2}$

$16 = x + 5$

$\boxed{x = 11}$

check

$\log_8(16) - \log_8 2 = 1$

$\log_8 \frac{16}{2} = 1$

$\log_8 8 = 1$ true

f) $5^x = 3^{2x-1}$ | $\ln$

$\ln 5^x = \ln 3^{2x-1}$

$x \ln 5 = (2x-1)\ln 3$

$x \ln 5 = 2x \ln 3 - \ln 3$

$\ln 3 = 2x \ln 3 - x \ln 5$

$\ln 3 = x(2\ln 3 - \ln 5)$

$x = \frac{\ln 3}{2\ln 3 - \ln 5} = \frac{\ln 3}{\ln 9 - \ln 5}$

$\boxed{x = \frac{\ln 3}{\ln \frac{9}{5}}} \approx 1.87$

8.

a) Write a quadratic equation with rational coefficient has 4 and -5 as solutions.
Write the equation in standard form.

$x = 4$ solutions
$x = -5$

Then $(x-4)(x+5) = 0$

$x^2 + 5x - 4x - 20 = 0$

$\boxed{x^2 + x - 20 = 0}$

b) Write a quadratic equation with real coefficients that has $1-i$ as a solution.
Write the equation in standard form

$x = 1-i$ solution

Then
$x = 1+i$ is also a solution

$(x-(1-i))(x-(1+i)) = 0$

$(x-1+i)(x-1-i) = 0$

$(x-1)^2 - i^2 = 0$

$x^2 - 2x + 1 - (-1) = 0$

$\boxed{x^2 - 2x + 2 = 0}$.

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9. The total profit Kiyoshi makes from producing and selling "x" floral arrangements is

$$P = -0.3x^2 + 30x$$ parabola that opens down

a) How many floral arrangements should Kiyoshi produce and sell to maximize his profit?

$x_v = \frac{-b}{2a} = \frac{-30}{2(-0.3)} = \frac{30}{0.6} = \frac{300}{6} = 50$

He should produce and sell $\boxed{\text{50 arrangements}}$

b) What is his maximum profit? Explain how do you know for sure you have found the maximum profit.

$P_{max} = P_v = -0.3(50)^2 + 30(50)$

$= 750$ \$

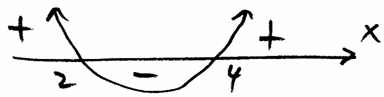
His maximum profit is $\boxed{750 \$}$.

The equation represents a parabola that opens downward, therefore the maximum occurs at the vertex.

10. Solve each inequality. Show clearly how you get the answers.

a) $x^2 - 6x + 8 \le 0$

$y = x^2 - 6x + 8$
parabola opens upward



x-n: $x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0 \quad \begin{cases} x = 4 \\ x = 2 \end{cases}$

So, $x^2 - 6x + 8 \le 0$ if

$\boxed{x \in [2,4]}$

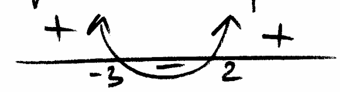
b) $\dfrac{1}{x+3} < \dfrac{1}{x-2}$

$\dfrac{1}{x-2} - \dfrac{1}{x+3} > 0$

$\dfrac{x+3-(x-2)}{(x-2)(x+3)} > 0$

$\dfrac{\cancel{x}+3-\cancel{x}+2}{(x-2)(x+3)} > 0$

$\dfrac{5}{(x-2)(x+3)} > 0 \iff (x-2)(x+3) > 0$
parabola opens up



So, $\boxed{x \in (-\infty, -3) \cup (2, \infty)}$

11. The number of bacteria present in a culture after t hours is given by the formula $N = 1000e^{0.69t}$.

a) How many bacteria will be there after ½ hour?

$t = 0.5$, find N

$N = 1000\, e^{0.69(0.5)} \approx \boxed{1412 \text{ bacteria}}$

b) How long will it be before there are 1,000,000 bacteria?

$N = 1{,}000{,}000$, find t

$1{,}000{,}\cancel{000} = \cancel{1000}\, e^{0.69t}$

$1000 = e^{0.69t} \quad |\ln$

$\ln 1000 = \ln e^{0.69t}$

$\ln 1000 = 0.69t$

$t = \dfrac{\ln 1000}{0.69} \approx \boxed{10 \text{ hours}}$

c) What is the doubling time?

The initial population was 1000.
If $N = 2000$, find t

$2\cancel{000} = \cancel{1000}\, e^{0.69t}$

$2 = e^{0.69t}$

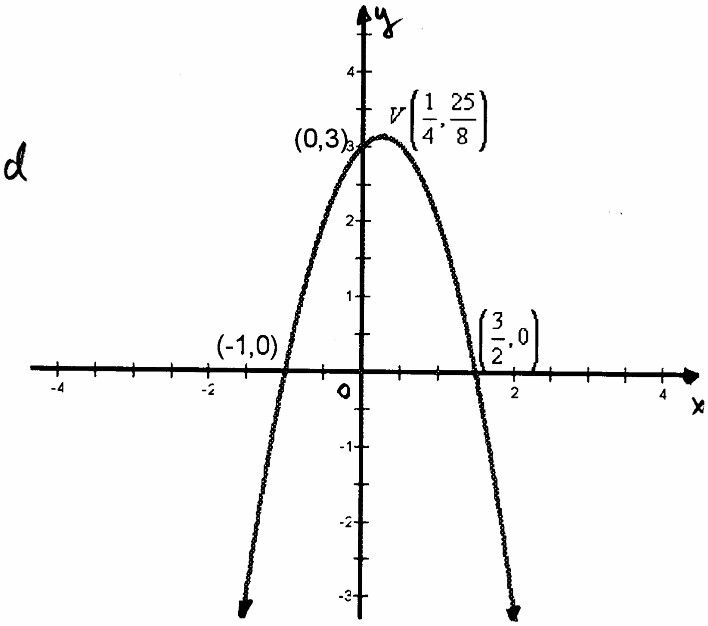
$\ln 2 = \ln e^{0.69t}$

$\ln 2 = 0.69t$

$t = \dfrac{\ln 2}{0.69} \approx \boxed{1 \text{ hour}}$

12. For the equation given below, fill in the blanks and graph (Be sure to label your axes). SHOW ALL WORK!

$$y=-2x^2+x+3$$



What type of curve is this?

parabola that opens downward

y-intercept?

$x=0,\ y=3$ $\boxed{(0,3)}$

Vertex

$V(x_v, y_v)$

$x_v = \frac{-b}{2a} = \frac{-1}{2(-2)} = \frac{1}{4}$ $\boxed{V\left(\frac{1}{4},\frac{25}{8}\right)}$

$y_v = -2\cdot\frac{1}{16} + \frac{1}{4} + 3$

$= -\frac{1}{8} + \frac{1}{4} + 3 = \frac{1}{8} + 3 = \frac{25}{8}$

x- intercept(s)?

$y=0,\ -2x^2+x+3=0$

$2x^2-x-3=0$

$x_{1,2} = \frac{1\pm\sqrt{1-4(2)(-3)}}{2(2)}$

$= \frac{1\pm\sqrt{1+24}}{4} = \frac{1\pm5}{4}$ ⟨ -1, $\frac{3}{2}$

$\boxed{(-1,0) \text{ and } \left(\frac{3}{2},0\right)}$

What is the vertex form of the equation?

$y = a(x-x_v)^2 + y_v$

$\boxed{y = -2\left(x-\frac{1}{4}\right)^2 + \frac{25}{8}}$

What is the domain?

$\boxed{x \in \mathbb{R}}$

What is the range?

$\boxed{y \in \left(-\infty, \frac{25}{8}\right]}$

Using the graph above, solve the following inequality:

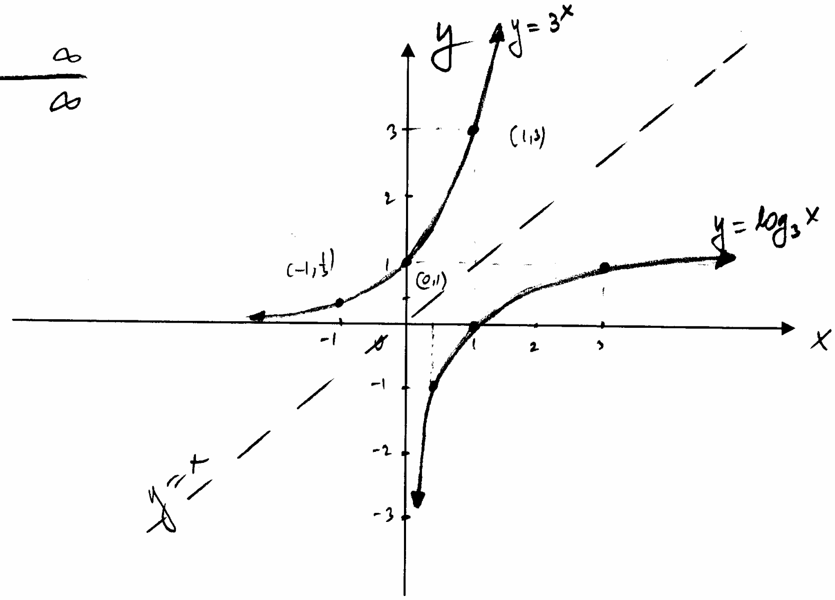
$$-2x^2+x+3<0$$

$\boxed{x \in (-\infty,-1) \cup \left(\frac{3}{2},\infty\right)}$

13. Graph the function $f(x) = 3^x$. Label the axes and show clearly how you graph (label all the points you use).

x	$-\infty$	-1	0	1	∞
$y=3^x$	0	$\frac{1}{3}$	1	3	∞

H.A. $y=0$



Answer the following questions:

What is the domain of f?

$x \in \mathbb{R}$

What is the range of f?

$y \in (0, \infty)$

What is the y-intercept?

$x = 0,\quad y = 0$ $\quad (0,1)$

What is the x-intercept (if any)?

none

Does the graph have an asymptote? What kind? What is its equation?

yes, horizontal asymptote. $y = 0$

Does f have an inverse? Explain

Yes, because it is a one-to-one function.

What is the inverse function (Do not prove).

$f(x) = \log_3 x$

Show on the above coordinate system how you obtain the graph of f^{-1} from the graph of f. Sketch the graph of f^{-1}.

The graphs of f and f^{-1} are symmetric about the line $y = x$

What is the domain of f^{-1}?

$x \in (0, \infty)$

What is the range of f^{-1}?

$y \in \mathbb{R}$

Does the graph of f^{-1} have an asymptote? What kind? What is its equation?

yes, a vertical asymptote $x = 0$.

Extra Credit @ 7 points

The fish population in a certain lake rises and falls according to the formula:

$$F = 2000\left(15 + \frac{17}{2}t - \frac{1}{2}t^2\right).$$

Here "F" is the number of fish at the time "t" where "t" is measured in years since January 1,1997 when the fish population was first estimated.

a) On what date will the fish population again be the same as on January 1,1998?

in 1998, $t=1 \Rightarrow F = 2000\left(15 + \frac{17}{2} - \frac{1}{2}\right)$

$F = 30{,}000 + 17{,}000 - 1000 = 46{,}000$ the fish population in 1998

if $F = 46{,}000$, find t

b) By what date will all the fish in the lake have died? (Approximate your answer in years to one decimal place).

$\cancel{46{,}000} = \cancel{2000}\left(15 + \frac{17}{2}t - \frac{1}{2}t^2\right)$

$46 = 30 + 17t - t^2$

$t^2 - 17t - 16 = 0$, $t = 1$ (1998) or $t = 16$

$(t-1)(t-16) = 0$

So, the fish population will again be the same as on Jan 1,1998, after 16 years, in 2013.

(b) $F = 0$, find t.

$2000\left(15 + \frac{17}{2}t - \frac{1}{2}t^2\right) = 0$

$15 + \frac{17}{2}t - \frac{1}{2}t^2 = 0$

$30 + 17t - t^2 = 0$

$t^2 - 17t - 30 = 0$

$t = \frac{17 \pm \sqrt{(17)^2 - 4(1)(-30)}}{2}$

$= \frac{17 \pm \sqrt{409}}{2} = \frac{17 \pm 20.2}{2}$ 18.6 or $\cancel{-1.6}$ no meaning

So, the fish will have died by end of 2015

$(1997 + 18.6 = 2015.6)$