Math 71 Spring 06 Name: SOLUTIONS

# TEST 2 @ 120 points

Write in a neat and organized fashion. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given!

---

1. Find the quadratic function $y = ax^2 + bx + c$ whose graph passes through $(-1,6),(1,4),(2,9)$. (6 points)

$(-1,6) \in$ graph $\Rightarrow$ when $x=-1, y=6$

(1) $a - b + c = 6$

$(1,4) \in$ graph $\Rightarrow$ when $x=1, y=4$

(2) $a + b + c = 4$

$(2,9) \in$ graph $\Rightarrow$ when $x=2, y=9$

(3) $4a + 2b + c = 9$

$$\begin{cases} a - b + c = 6 & (1) \\ a + b + c = 4 & (2) \\ 4a + 2b + c = 9 & (3) \end{cases}$$

Eliminate c:

eq.(2) − eq.(1): $2b = -2 \Rightarrow \boxed{b = -1}$

eq.(3) − eq(1): $3a + 3b = 3$

$3a + 3(-1) = 3$

$3a = 6 \Rightarrow \boxed{a = 2}$

eq.(2): $a + b + c = 4$

$2 + (-1) + c = 4$

$\boxed{c = 3}$

Therefore, the quadratic function whose graph passes through $(-1,6)$, $(1,4)$, and $(2,9)$ is

$$\boxed{y = 2x^2 - x + 3}$$

2. Solve the following equations and inequalities with absolute value:

a) $\left|2x+\frac{1}{3}\right|=\frac{4}{5}$ (6 points)

$2x+\frac{1}{3}=\frac{4}{5}$ OR $2x+\frac{1}{3}=-\frac{4}{5}$

$2x=\frac{4}{5}-\frac{1}{3}$ $\qquad$ $2x=-\frac{4}{5}-\frac{1}{3}$

$2x=\frac{12-5}{15}$ $\qquad$ $2x=\frac{-12-5}{15}$

$2x=\frac{7}{15}\ \Big/\cdot\frac{1}{2}$ $\qquad$ $2x=\frac{-17}{15}\ \Big/\cdot\frac{1}{2}$

$\boxed{x=\frac{7}{30}}$ $\qquad$ $\boxed{x=\frac{-17}{30}}$

The solution set is $\left\{\frac{7}{30},\frac{-17}{30}\right\}$

b) $\left|x+\frac{1}{2}\right|=|x-3|$ (6 points)

$x+\frac{1}{2}=x-3$ OR $x+\frac{1}{2}=-(x-3)$

$\frac{1}{2}=-3$ false $\qquad$ $x+\frac{1}{2}=-x+3$

$\Rightarrow$ no solutions $\qquad$ $2x=3-\frac{1}{2}$

$2x=\frac{5}{2}\ \Big/\cdot\frac{1}{2}$

$\boxed{x=\frac{5}{4}}$

The solution set is $\left\{\frac{5}{4}\right\}$

c) $|3x+14|+7=2$ (5 points)

$|3x+14|=2-7$

$|3x+14|=-5$

$\boxed{x\in\emptyset}$

d) $1-|x-3|=1$ (5 points)

$-|x-3|=0$

$|x-3|=0$

$x-3=0$

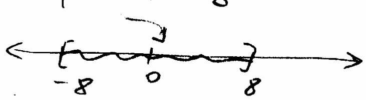
$\boxed{x=3}$

The solution set is $\{3\}$

e) $|2x-1|+3\le 11$ (6 points)

Graph the solution set and write it in interval notation.

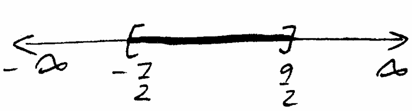
$|2x-1|\le 8$




$-8\le 2x-1\le 8$

$-7\le 2x\le 9$

$\boxed{-\frac{7}{2}\le x\le\frac{9}{2}}$

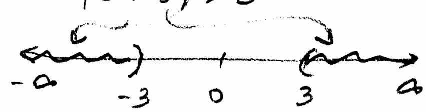



$\boxed{x\in\left[-\frac{7}{2},\frac{9}{2}\right]}$

f) $4|3x-5|>12$ (6 points)

Graph the solution set and write it in interval notation.

$|3x-5|>3$

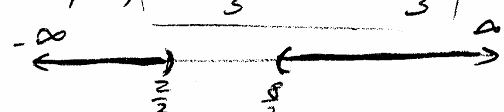



$3x-5<-3$ OR $3x-5>3$

$3x<2$ $\qquad$ $3x>8$

$x<\frac{2}{3}$ $\qquad$ $x>\frac{8}{3}$

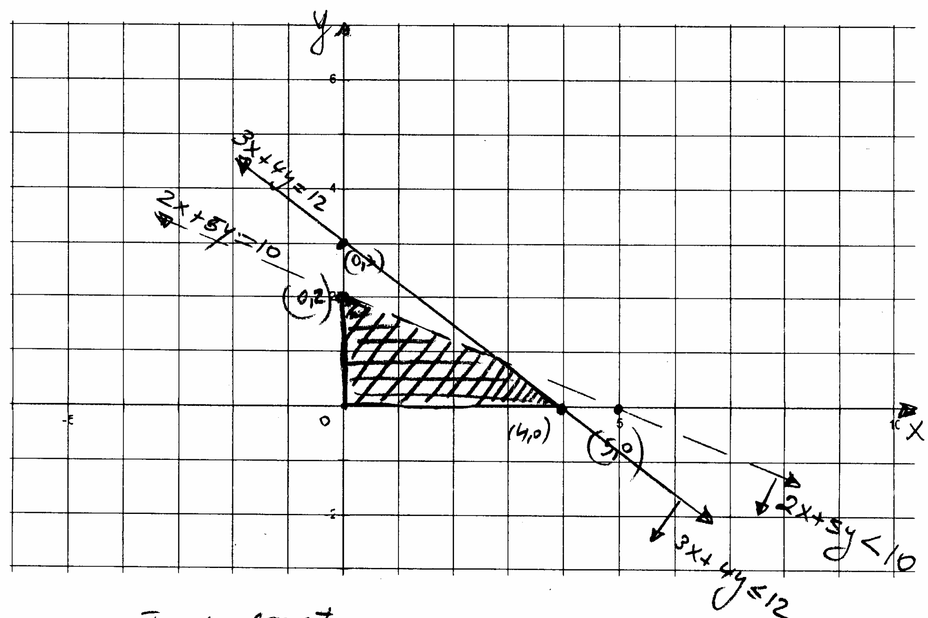
Therefore, $\boxed{x<\frac{2}{3}\text{ OR }x>\frac{8}{3}}$




$\boxed{x\in\left(-\infty,\frac{2}{3}\right)\cup\left(\frac{8}{3},\infty\right)}$

3. Graph the solution set of the following system of inequalities. Show clearly how you graph the lines and what test points you're using. Clearly label the axes, the lines, and the points used. (6 points)

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ 2x + 5y < 10 \\ 3x + 4y \le 12 \end{cases}$$




$x \geq 0$ and $y \geq 0$ $\iff$ I quadrant

$2x+5y<10$

Boundary line: $2x+5y=10$

| x | y |
|---|---|
| 0 | 2 |
| 5 | 0 |

Test point: $(0,0) \notin$ line

$0+0<10$ true

$\Rightarrow (0,0) =$ solution.

$3x+4y \le 12$

Boundary line: $3x+4y=12$

| x | y |
|---|---|
| 0 | 3 |
| 4 | 0 |

Therefore, the intersection of all half-planes is the shaded region.

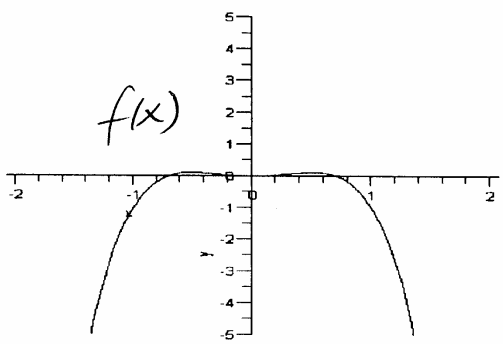
4. Two graphs are given. Using the end behavior of a polynomial, match each graph with one of the given functions. Show clearly your reasoning. (6 points)

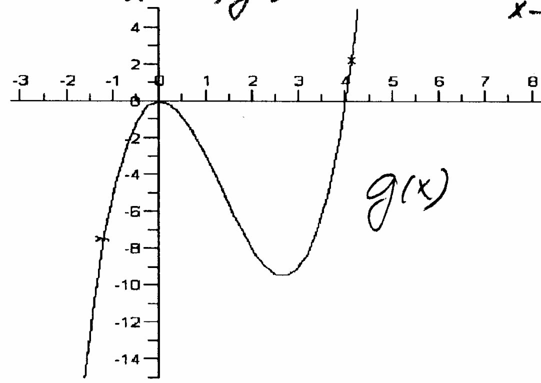
$f(x) = -2x^4 + x^2$
when $x \to \infty$, $y \to -\infty$
$x \to -\infty$, $y \to -\infty$

$g(x) = x^3 - 4x^2$
when $x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to -\infty$

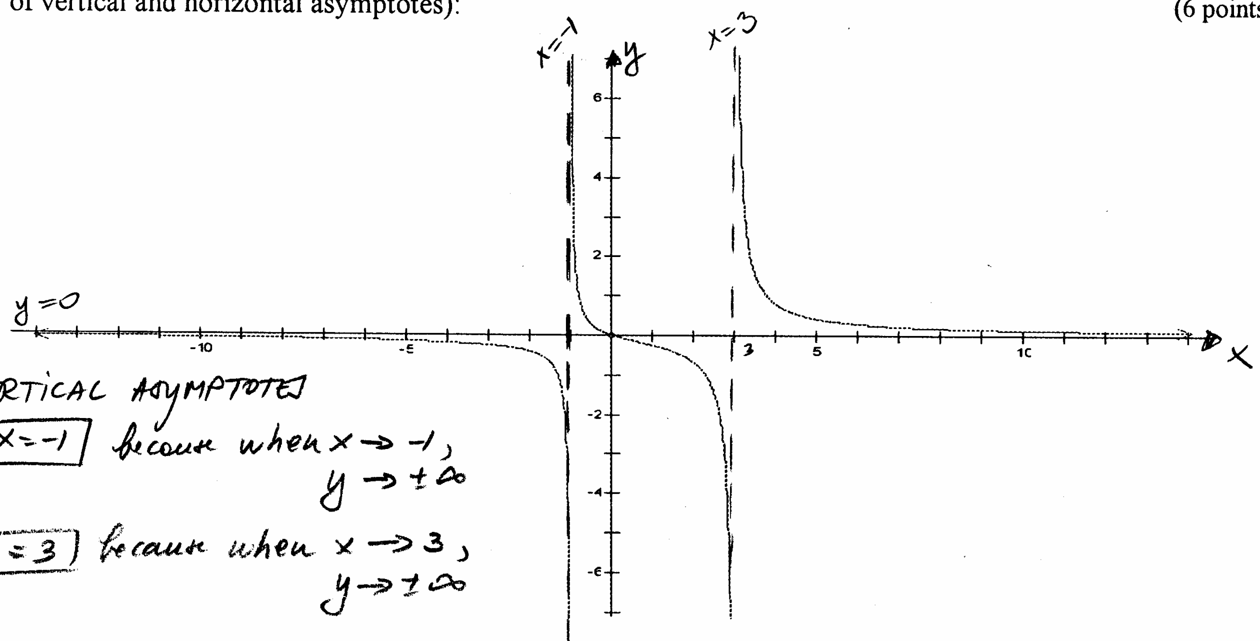
$h(x) = 3x^2 - 18x + 27$
when $x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to \infty$

$p(x) = -x^3 - x^2 + 5x - 3$
when $x \to \infty$, $y \to -\infty$
$x \to -\infty$, $y \to \infty$




5. . Identify all the asymptotes of the following graph. Clearly explain your reasoning (using the definitions of vertical and horizontal asymptotes): (6 points)




VERTICAL ASYMPTOTES

$\boxed{x=-1}$ because when $x \to -1$, $y \to \pm\infty$

$\boxed{x=3}$ because when $x \to 3$, $y \to \pm\infty$

HORIZONTAL ASYMPTOTE

$\boxed{y=0}$ because when $x \to \pm\infty$, $y \to 0$

6. Let $f(x)=6x-3$. Find $\dfrac{f(a+h)-f(a)}{h}$ and simplify. (5 points)

$$\frac{f(a+h)-f(a)}{h}=\frac{\big(6(a+h)-3\big)-(6a-3)}{h}$$
$$=\frac{\cancel{6a}+6h-\cancel{3}-\cancel{6a}+\cancel{3}}{h}$$
$$=\frac{6h}{h}$$
$$=\boxed{6}$$

7. Let $f(x)=\dfrac{x+1}{x-1}$. Find $f\left(\dfrac{1}{x+2}\right)$ and simplify. (6 points)

$$f\left(\frac{1}{x+2}\right)=\frac{\frac{1}{x+2}+1}{\frac{1}{x+2}-1}=\frac{\frac{1+x+2}{x+2}}{\frac{1-(x+2)}{x+2}}$$
$$=\frac{\frac{x+3}{\cancel{x+2}}}{\frac{-x-1}{\cancel{x+2}}}$$
$$=\boxed{\frac{x+3}{-x-1}}$$

8. Find the domain of the following functions: (9 points)

a) $f(x)=\dfrac{3}{x(2x-1)(2-x)\left(x+\frac{1}{2}\right)}$

Condition: $x(2x-1)(2-x)(x+\frac{1}{2})\neq 0$

$x\neq 0$

$x\neq \frac{1}{2}$

$x\neq 2$

$x\neq \frac{-1}{2}$

$\boxed{\text{Domain}=\mathbb{R}\setminus\{0,\frac{1}{2},2,\frac{-1}{2}\}}$

b) $g(x)=\dfrac{2x+3}{x^2+\frac{1}{5}x-\frac{6}{25}}$

Condition: $x^2+\frac{1}{5}x-\frac{6}{25}\neq 0$

$x^2+\frac{1}{5}x-\frac{6}{25}=0$

$(x+\frac{3}{5})(x-\frac{2}{5})=0$

$x=\frac{-3}{5}$ or $x=\frac{2}{5}$

Therefore, $x\neq\frac{-3}{5}$, $x\neq\frac{2}{5}$

$\boxed{\text{Domain}=\mathbb{R}\setminus\{\frac{-3}{5},\frac{2}{5}\}}$

9. Find all values of $a$ for which $f(a) = g(a) + 1$ if $f(x) = \frac{x+2}{x+3}$ and $g(x) = \frac{x+1}{x^2+2x-3}$.

(6 points)

$$f(a) = g(a) + 1$$

$$\frac{a+2}{a+3} = \frac{a+1}{a^2+2a-3} + 1$$

$a^2+2a-3$

$$\frac{a+2}{a+3} = \frac{a+1}{(a+3)(a-1)} + \frac{1}{1}$$

$a-1$

Conditions: $\begin{cases} a \neq -3 \\ a \neq 1 \end{cases}$

$LCD = (a+3)(a-1)$

$$(a-1)(a+2) = a+1 + a^2+2a-3$$

$$\cancel{a^2} + \cancel{2a} - a - 2 = a + 1 + \cancel{a^2} + \cancel{2a} - 3$$

$$-a - \cancel{2} = a - \cancel{2}$$

$$-a = a$$

$$2a = 0$$

$$\boxed{a = 0}$$

The solution set is $\{0\}$.

---

10. In the following, solve or simplify, whichever is appropriate

a) $x^3 + 2x^2 = 16x + 32$ (equation) (5 points)

$$\underbrace{x^3+2x^2} \underbrace{- 16x - 32} = 0$$

$$x^2(x+2) - 16(x+2) = 0$$

$$(x+2)(x^2-16) = 0$$

$$(x+2)(x+4)(x-4) = 0$$

$x+2=0 \Rightarrow \boxed{x=-2}$

OR

$x+4=0 \Rightarrow \boxed{x=-4}$

OR

$x-4=0 \Rightarrow \boxed{x=4}$

The solution set is $\{-2, 4, -4\}$

b) $\left(2a^{-1} + b^{-1}\right)^{-1}$ (expression) (5points)

$$\left(2a^{-1}+b^{-1}\right)^{-1} = \frac{1}{2a^{-1}+b^{-1}}$$

$$= \frac{1}{\frac{2}{a} + \frac{1}{b}}$$

$LCD = ab$

$$= \frac{1}{\frac{2b+a}{ab}}$$

$$= \boxed{\frac{ab}{2b+a}}$$

c) $\frac{1}{x^3-8}+\frac{3}{(x-2)(x^2+2x+4)}-\frac{2}{x^2+2x+4} =$

(expression)

(6 points)

$$= \frac{1}{(x-2)(x^2+2x+4)} + \frac{3}{(x-2)(x^2+2x+4)} - \frac{2}{x^2+2x+4}$$

$x-2$

$$\left( LCD = (x-2)(x^2+2x+4) = x^3-8 \right)$$

$$= \frac{1+3-2(x-2)}{x^3-8}$$

$$= \frac{4-2x+4}{x^3-8}$$

$$= \boxed{\frac{8-2x}{x^3-8}}$$

d) $\frac{x+3}{x^2-x}-\frac{8}{x^2-1}=0$ (equation) (6 points)

$$\frac{x+3}{x(x-1)} - \frac{8}{(x-1)(x+1)} = 0$$

Conditions $\begin{cases} x \neq 0 \\ x \neq 1 \\ x \neq -1 \end{cases}$

$$\frac{x+3}{x\cancel{(x-1)}} = \frac{8}{\cancel{(x-1)}(x+1)} \quad \Big| \cdot (x-1) \neq 0$$

$$\frac{x+3}{x} = \frac{8}{x+1}$$

$$(x+3)(x+1) = 8x$$

$$x^2+4x+3 = 8x$$

$$x^2-4x+3 = 0$$

$$(x-3)(x-1) = 0$$

$x-3=0$ OR $x-1=0$

$\boxed{x=3}$ $\quad$ $\cancel{x=1}$ not possible

The solution set is $\{3\}$

**Choose TWO of the following three problems @ 7 points each**

**1**. A city commission has proposed two tax bills. The first bill requires that a homeowner pay \$1800 plus 3% of the assessed home value in taxes. The second bill requires taxes of \$200 plus 8% of the assessed home value. What price range of home assessment would make the first bill a better deal?

**2**. The function $f(x) = -\frac{1}{4}x^2 + 3x + 17$ models the number of people, f(x), in millions, receiving food stamps $x$ years after 1990.

a) In which year did 25 million people receive food stamps?

b) How many people received food stamps in 1996?

**3**. James Bond stands on top of a 240-foot building and throws a film canister upward to a fellow agent in a helicopter 16 feet above the building. The height of the film above the ground $t$ seconds later is given by the formula $h = -16t^2 + 32t + 240$ where $h$ is in feet.

a) Calculate $h(0)$. What is its meaning in this context?

b) How long will it take the canister to hit the ground?

1 Let $x$ = assessed home value

1st bill : Taxes = $1800 + 3\% x$

2nd bill : Taxes = $200 + 8\% x$

in order for the 1st bill to be a better deal,

$$1800 + 3\% x < 200 + 8\% x$$

$$1800 + \frac{3}{100} x < 200 + \frac{8}{100} x \quad /\cdot 100$$

$$180000 + 3x < 20000 + 8x$$

$$180{,}000 - 20{,}000 < 8x - 3x$$

$$160{,}000 < 5x$$

$$\frac{160{,}000}{5} < x$$

Therefor, the home value $x > 32{,}000$ \$

(2) $f(x) = \frac{-1}{4}x^2 + 3x + 17$

where $x$ = the number of years after 1990

$f(x)$ = the number of people (in million)

a) $x = ?$ if $f(x) = 25$

$25 = \frac{-1}{4}x^2 + 3x + 17$

$100 = -x^2 + 12x + 68$

$x^2 - 12x + 32 = 0$

$(x-8)(x-4) = 0$ $\left\langle \begin{array}{l} x = 4 \\ \text{OR} \\ x = 8 \end{array} \right.$

Therefore, in 1994 ($x=4$) and 1998 ($x=8$), 25 million people received food stamps.

b) $x = 6$, find $f(6)$

$f(6) = \frac{-1}{4}(6)^2 + 3(6) + 17$

$= 26$

Therefore, 26 million people received food stamps in 1996

(3) $h = -16t^2 + 32t + 240$

when $t$ = time (# of seconds after the film has been thrown up in the air)

$h = h(t)$ = the height of the film above the ground

a) $h(0) = -16(0) + 32(0) + 240$

$h(0) = 240$ feet the initial height (where the film was right before being thrown upward)

b) $t = ?$ if $h = 0$

$0 = -16t^2 + 32t + 240$ $/ \div 16$

$16t^2 - 32t - 240 = 0$

$t^2 - 2t - 15 = 0$

$(t-5)(t+3) = 0$ $\left\langle \begin{array}{l} t = 5 \\ \text{OR} \\ t = \cancel{-3} \end{array} \right.$

It took the film 5 seconds to hit the ground.

**EXTRA CREDIT - You may solve TWO of the following:**

**1.** Simplify $x+\cfrac{1}{x+\cfrac{1}{x+\cfrac{1}{x}}}$ (5 points)

on next page

**2.** Solve the equation: $\left(\frac{4}{x-1}\right)^2+2\left(\frac{4}{x-1}\right)+1=0$ (5 points)

on next page

**3.** Suppose 12 kg of a certain alloy initially contains 3 kg copper and 9kg tin. Suppose you add $x$ kg of copper to this 12 kg of alloy. Define the concentration of copper in the alloy as a function of $x$, so that (6 points)

$$f(x) = Concentration\ of\ copper = \frac{Total\ amount\ of\ copper}{Total\ amount\ of\ alloy}.$$

a) Find a formula for $f$ in terms of $x$, the amount of copper added.

$$\boxed{f(x)=\frac{3+x}{12+x}}$$

b) Evaluate the following expressions and explain their significance in the context of the alloy:

i) $f\left(\frac{1}{2}\right) = \frac{3+\frac{1}{2}}{12+\frac{1}{2}} = \frac{\frac{7}{2}}{\frac{25}{2}}$

$\boxed{f(\tfrac{1}{2})=\frac{7}{25}}$ it represents the concentration of copper if we add $\frac{1}{2}$ kg copper to the initial alloy

ii) $f(-1) = \frac{3-1}{12-1}$

$\boxed{f(-1)=\frac{2}{11}}$ it represents the concentration of copper if we take 1kg of copper out of the original alloy

Extra credit

(1) $$x+\cfrac{1}{x+\cfrac{1}{x+\frac{1}{x}}} = x+\cfrac{1}{x+\cfrac{1}{\frac{x^2+1}{x}}} =$$

$$= x+\cfrac{1}{x+\frac{x}{x^2+1}} = x+\cfrac{1}{\frac{x(x^2+1)+x}{x^2+1}}$$

$$= x+\frac{x^2+1}{x^3+x+x} = x+\frac{x^2+1}{x^3+2x}$$

$$= \frac{x(x^3+2x)+x^2+1}{x^3+2x} = \frac{x^4+2x^2+x^2+1}{x^3+2x} = \boxed{\frac{x^4+3x^2+1}{x^3+2x}}$$

(2) $$\left(\frac{4}{x-1}\right)^2 + 2\left(\frac{4}{x-1}\right)+1=0$$

Condition: $\{x\neq 1$

Let $\frac{4}{x-1}=t$

Then $t^2+2t+1=0$

$(t+1)^2=0$

$t=-1$

So, $\frac{4}{x-1}=-1$

$4=-(x-1)$

$4=-x+1$

$\boxed{x=-3}$

The solution set is $\{-3\}$.