Math 71 Spring 06 Name:

# TEST 1 @ 120 points

Write in a neat and organized fashion. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given!

1. If $f(x)=x-3$ and $g(x)=2x+5$, find each of the following: (10 points)

a) What is the domain of $g$?

$x \in \mathbb{R}$

b) What is the range of g ?

$y \in \mathbb{R}$

c) Does $g$ have an inverse? Explain.

Yes. $y=g(x)$ is an ascending line, thus it passes the horizontal line test, thus for it is a one-to-one function

d) Find $g^{-1}(x)$.

$y = 2x+5$

$2x = y-5$

$x = \frac{y-5}{2}$

$x \leftrightarrow y$

$y = \frac{x-5}{2}$

$g^{-1}(x) = \frac{x-5}{2}$

e) Find $(f \circ g)(x)$.

$(f \circ g)(x) = f(g(x))$

$= f(2x+5)$

$= 2x+5-3$

$= 2x+2$

$(f \circ g)(x) = 2x+2$

f) Find $f(g(0))$;

We know $(f \circ g)(x) = 2x+2$

then $(f \circ g)(0) = f(g(0)) = 2 \cdot 0 + 2$

$= 2$

$f(g(0)) = 2$

2. Let $f(x)=\frac{3x-1}{x-5}$. (10 points)

a) Find $f(1)$.

$f(1) = \frac{3 \cdot 1 - 1}{1-5}$

$= \frac{2}{-4}$

$= \frac{-1}{2}$

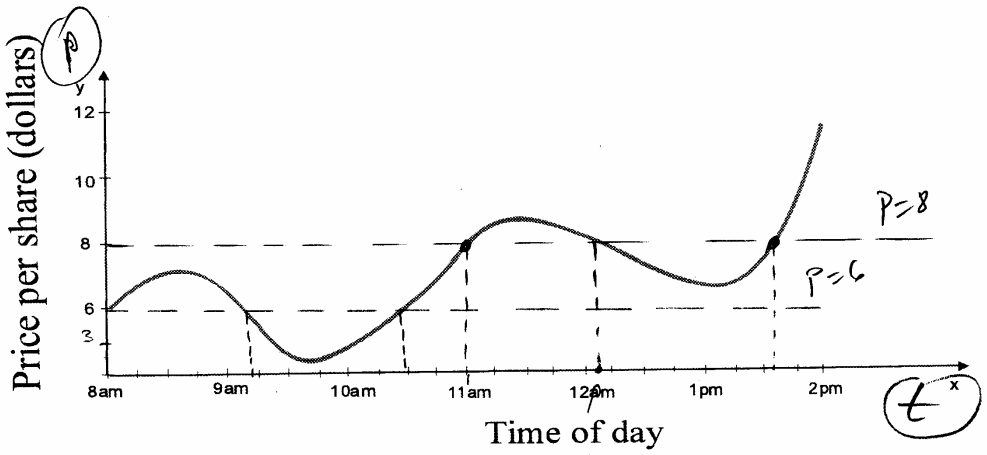
$f(1) = \frac{-1}{2}$

b) Find $f(a+h)$

$f(a+h) = \frac{3(a+h)-1}{a+h-5}$

$f(a+h) = \frac{3a+3h-1}{a+h-5}$

. The value of a stock varies during the course of any trading day. The price per share "P" of a certain stock is shown on the graph below for a particular trading day. Note "t" represents any time between 8 am and 2 pm. (10 points)




a) Is "t" ( the time of the day) a function of "P" ( the price per share)?
Is "P" a function of "t"? Explain using the definition of function.

- t is not a function of P because there are P values (input) for which there is more than one t value (output) if P = 6, t ≈ 9:15 and 10:30
- P is a function of t because for every t, there is only one P
  P = f(t)

Using the graph, estimate the answers to the following questions (Use the correct units).

b) What is the domain?

$t \in [8am, 2pm]$

c) What is the range?

$P \in [1.5\$, 12\$]$

c) For what value(s) of "t" does P(t)=8 and what does it mean in practical terms?

P(t) = 8 when t ≈ 11am, t ≈ 1:35 pm and t ≈ 12:00 pm
P(t) = 8 tells us the times when the share was $8.

d) What is P(2) and what does it mean in practical terms?

P(2) = 12 P(2) gives the value of the share at 2:00 pm.
The share was $12 at 2 pm

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4. Let $g(x) = \begin{cases} 1-2x^2, x \le 4 \\ 3x+9, x > 4 \end{cases}$. (10 points)

a) Find $g(10)$

$g(10) = 3 \cdot 10 + 9$
$= 39$
(because $x = 10 > 4$)

b) Find $g(\sqrt{2})$.

$g(\sqrt{2}) = 1 - 2(\sqrt{2})^2$
$= 1 - 2 \cdot 2$
$= -3$
(because $x = \sqrt{2} \approx 1.4 \le 4$)

5. Match the graphs (I) – (VI) with the equations given below. (You shouldn't need to graph each equation to determine which is which!) NOTE: The x and y scales may be unequal. Show all work. (10 points)

a. $y = .005x + .009$

$m = 0.005 > 0$
$b = 0.009 > 0$ $\Rightarrow$
ascending line
positive y-int

(IV)

b. $x = -\pi y$

$y = \frac{-1}{\pi} x$

$m = \frac{-1}{\pi} < 0$
$b = 0$ $\Rightarrow$
descending line
through (0,0)

(III)

c. $y = \frac{5}{2} - \frac{3}{4}x$

$y = \frac{-3}{4}x + \frac{5}{2}$

$m = \frac{-3}{4} < 0$
$b = \frac{5}{2} > 0$ $\Rightarrow$
descending line
positive y-int

(I)

d. $x - \sqrt{1000} = 0$

$x = \sqrt{1000}$
vertical line

(V)

e. $3x + 4y + 10 = 0$

$4y = -3x - 10$

$y = \frac{-3}{4}x - \frac{10}{4}$

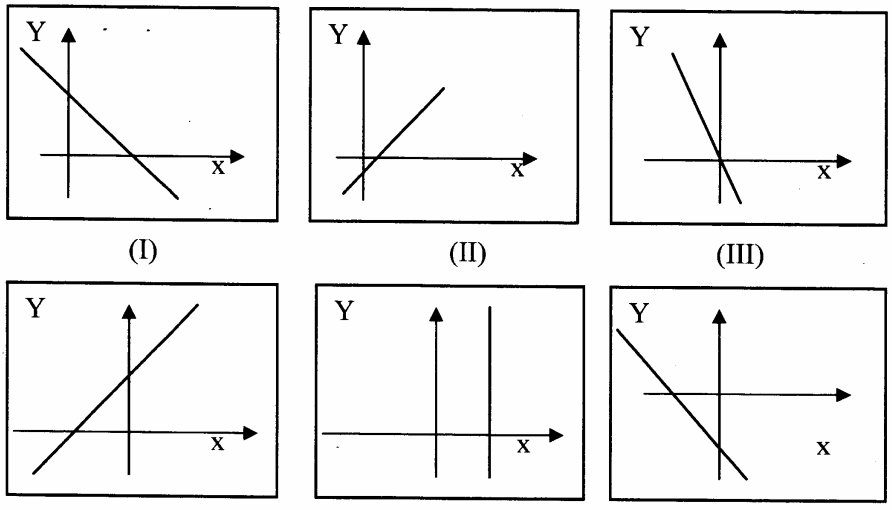
$m = \frac{-3}{4} < 0$
$b = \frac{-5}{2} < 0$ $\Rightarrow$
descending line
negative y-int

(VI)

f. $y = 351x - 140$

$m = 351 > 0$
$b = -140 < 0$ $\Rightarrow$
ascending line
negative y-int

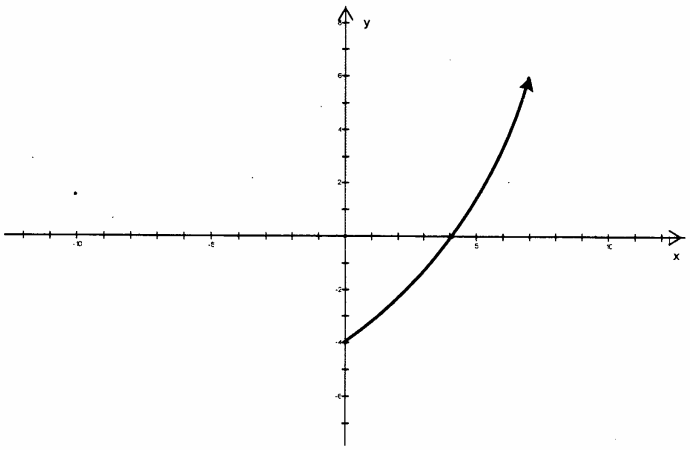
(II)




(I) (II) (III)

(IV) (V) (VI)

6. A graph is given. Answer the following:
(10 points)




a) Is $y$ a function of $x$? Explain.

yes. The graph posses the vertical line test.

b) If $y = f(x)$, find the domain and range of $f$.

$x \in [0, \infty)$

$y \in [-4, \infty)$

d) If $f$ a one-to-one function? Explain.

yes it posses the horizontal line test

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7. Find an equation of the line satisfying each of the conditions: (10 points)

a) Slope 5 and passing through $(-3, -5)$

$m = 5$
$(-3, -5)$
$y - y_1 = m(x - x_1)$
$y - (-5) = 5(x - (-3))$
$\boxed{y + 5 = 5(x + 3)}$

b) passing through $\left(\frac{1}{2}, -3\right)$ and $\left(\frac{3}{2}, -5\right)$.

$m = \frac{\Delta y}{\Delta x} = \frac{-3 - (-5)}{\frac{1}{2} - \frac{3}{2}} = \frac{-3 + 5}{-\frac{2}{2}} = \frac{2}{-1}$

$m = -2$ , $(\frac{1}{2}, -3)$
$y - y_1 = m(x - x_1)$
$y - (-3) = -2\left(x - \frac{1}{2}\right)$
$\boxed{y + 3 = -2\left(x - \frac{1}{2}\right)}$

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8. Find an equation of the line that passes through the point $(2,1)$ and is perpendicular to $3x + 4y = 12$.
(10 points)

line $\begin{cases} \ni (2,1) \\ \perp\ 3x + 4y = 12 \end{cases}$

Let's find the slope of the given line: $3x + 4y = 12$

$4y = -3x + 12$
$y = \frac{-3}{4}x + 3$

$m = \frac{-3}{4}$

$\rightarrow$ then, $m_\perp = \frac{4}{3}$
$(2,1)$

$y - y_1 = m(x - x_1)$

$\boxed{y - 1 = \frac{4}{3}(x - 2)}$

9. Solve the following systems of equations: (10 points each)

a) $\begin{cases} \frac{1}{16}x - \frac{3}{4}y = -1 \\ \frac{3}{4}x + \frac{5}{2}y = 11 \end{cases}$ | LCD = 16, LCD = 4

$\begin{cases} x - 12y = -16 \\ 3x + 10y = 44 \end{cases}$ | $\cdot(-3)$

$\begin{cases} -3x + 36y = 48 \\ 3x + 10y = 44 \end{cases}$

$46y = 92$

$y = \frac{92}{46} = 2$

$y = 2$

$x - 12y = -16$

$x - 12 \cdot 2 = -16$

$x - 24 = -16$

$x = 24 - 16$

$x = 8$

So, the solution is $(8, 2)$

b) $\begin{cases} 2x - y + 1 = 0 \\ -4x + 2y = -6 \end{cases}$ | 2

$\begin{cases} 4x - 2y + 2 = 0 \\ -4x + 2y = -6 \end{cases}$

$\begin{cases} 4x - 2y = -2 \\ -4x + 2y = -6 \end{cases}$

$0 = -8$

Contradiction

Therefore, the system has NO SOLUTIONS

10. **Choose TWO** of the following word problems. Show clearly what your variables represent. Show clearly the equation(s) you use to solve each problem. (10 points each)

(A) One week, a computer store sold a total of 36 computers and hard drivers. The revenue from these sales was $27,710. If computers sold for $1180 per unit and hard drivers for $125per unit, how many of each did the store sell?

(B) You invested $7000 in two accounts paying 6% and 8% annual interest, respectively. If the total interest earned for the year was $520, how much was invested at each rate?

(C) When a plane flies with the wind, it can travel 800 miles in 5 hours. When the plane flies in the opposite direction, against the wind, it takes 8 hours to fly the same distance. Find the rate of the plane in still air and the rate of the wind.

(A) 36 TOTAL < COMPUTERS $1180/unit x ; HARD DRIVERS $125/unit y

REVENUE = $27,710

Let x = the number of computers sold

y = the number of hard drives sold

$$\begin{cases} x+y=36 \\ 1180x+125y=27{,}710 \end{cases} \quad \Big| -1180$$

$$\begin{cases} -1180x-1180y=-42{,}480 \\ 1180x+125y=27{,}710 \end{cases}$$

(+) $-1055y=-14{,}770$

$$y=\frac{14770}{1055}=14$$

$x+y=36$

$x+14=36$

$x=36-14$

$x=22$

Therefore, the computer sold 22 computers and 14 hard drives.

(B) $7000 < I account @ 6% x $ ; II account @ 8% y $

TOTAL INTEREST = $520

Let x = the amount invested into the 1st account at 6% interest

y = the amount invested in the 2nd account at 8% interest

Then, $\begin{cases} x+y = 7000 \\ 6\%x + 8\%y = 520 \end{cases}$

$\begin{cases} x+y = 7000 \\ \frac{6}{100}x + \frac{8}{100}y = 520 \end{cases} \Big| \cdot 100$

$\begin{cases} x+y = 7000 \\ 6x+8y = 52000 \end{cases} \Big| -6$

$\begin{cases} -6x-6y = -42{,}000 \\ 6x+8y = 52{,}000 \end{cases}$

(+) $2y = 10{,}000$

$y = 5000$

$x+y = 7000$

$x + 5000 = 7000$

$x = 2000$

Therefore, 2000 $ were invested in the 1st account
5000 $ were invested in the 2nd account

(C)

| | Distance | Rate | Time |
|---|---|---|---|
| with wind | 800 mi | $p+w$ | 5h |
| against wind | 800 mi | $p-w$ | 8h |

Let $p$ = the speed of the plane in still air
$w$ = the speed of the wind

we know Distance = Rate · Time

$$\begin{cases} 800 = 5(p+w) \\ 800 = 8(p-w) \end{cases} \quad \begin{matrix} \div 5 \\ \div 8 \end{matrix}$$

$$\begin{cases} 160 = p+w \\ 100 = p-w \end{cases}$$

(+) $260 = 2p$

$p = 130$ mi/h the speed of the plane

$p+w = 160$

$130+w = 160$

$w = 30$ mi/h the speed of the wind

**EXTRA CREDIT @ 5 POINTS**

.Let $n = f(A)$ be a function that gives the number of gallons of paint required to cover a house of area $A\ ft^2$.

A = input (independent variable)
n = output (dependent variable)

(a) Using the fact that 1 gallon of paint will cover 250 $ft^2$, evaluate $f(20{,}000)$.

$f(20{,}000) = ?$ If $A = 20{,}000\ ft^2$, find how many gallons of paint are needed

input

$f(20{,}000) = \frac{20{,}000}{250}$

$f(20{,}000) = 80$ gallons

(b) Find a formula for $f(A)$, the amount of paint ( in gallons) required to cover a house of area $A\ ft^2$.

$f(A) = \frac{\text{total area}}{250\ ft^2}$

$f(A) = \frac{A}{250}$

(c) Explain the meaning of the expressions $f(A+10)$ and $f(A)+10$ in the context of painting.

$f(A+10) = n$ = amount of paint needed to cover an area $10\ ft^2$ larger than A

$f(A)+10 = n$ = 10 gallons more paint than the amount needed to cover an area A

**EXTRA CREDIT @ 4 POINTS**

If $f(x) = 3x$ and $g(x) = x+5$, find $(f \circ g)^{-1}(x)$.

First, let's find $(f \circ g)(x)$

$(f \circ g)(x) = f(g(x))$

$= f(x+5)$

$= 3(x+5)$

$(f \circ g)(x) = 3x+15$

Now, we'll find $(f \circ g)^{-1}(x)$

Let $y = 3x+15$

$3x = y-15$

$x = \frac{y-15}{3}$

So, $(f \circ g)^{-1}(x) = \frac{y-15}{3}$