Math 71 Spring 06
January 10 & 12, 2006

REVIEW
Chapter 1 – The Real Number System

In class work: Solve all exercises.

(Sections 1.1 & 1.2)

Definition **A set** is a collection of objects (elements).

The Set of Natural Numbers $\mathbb{N}$

$$\mathbb{N} = \{ 1, 2, 3, 4, 5, \ldots \}$$

The Set of Whole Numbers W — $\mathbb{N} \subset W$

$$W = \{ 0, 1, 2, 3, 4, 5, \ldots \}$$

The Set of Integers $\mathbb{Z}$ — $\mathbb{N} \subset W \subset \mathbb{Z}$

$$\mathbb{Z} = \{ \ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots \}$$

The Set of Rational Numbers $\mathbb{Q}$ — $\mathbb{N} \subset W \subset \mathbb{Z} \subset \mathbb{Q}$

$$\mathbb{Q} = \left\{ \frac{a}{b} \middle| a, b \in \mathbb{Z}, b \neq 0 \right\}$$

The Set of Irrational Numbers

Examples: $\sqrt{2}, -\sqrt{5}, p$

The Set of Real Numbers $\mathbb{R}$ — $\mathbb{N} \subset W \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

$$\mathbb{R} = \{ x \mid x \text{ isrational or } x \text{ is irrational} \}$$

Mathematical Symbols

SYMBOL	MEANING	EXAMPLES
$=$	is equal to	
$\neq$	is not equal to	
$\in$	belongs to (about an element)	
$\notin$	it doesn't belong to	
$<$	is less than	
$\leq$	is less than or equal to	
$>$	is greater than	
$\geq$	is greater than or equal to	
$\forall$	any	

Properties of Real Numbers

PROPERTIES	ADDITION +	MULTIPLICATION •
COMMUTATIVITY	$a+b=b+a, \quad \forall a,b\in\mathbb{R}$	$ab=ba \quad \forall a,b\in\mathbb{R}$
ASSOCIATIVITY	$(a+b)+c=a+(b+c), \forall a,b,c\in\mathbb{R}$	$(ab)c=a(bc), \quad \forall a,b,c\in\mathbb{R}$
IDENTITY ELEMENT	0 $a+0=0+a=a, \forall a\in\mathbb{R}$	1 $a\cdot 1=1\cdot a=a, \forall a\in\mathbb{R}$
INVERSE ELEMENT	$\forall a\in\mathbb{R}$, there is $-a\in\mathbb{R}$ such that $a+(-a)=(-a)+a=0$	$\forall a\in\mathbb{R}, a\neq 0$, there is $\frac{1}{a}\in\mathbb{R}$ such that $a\cdot\frac{1}{a}=\frac{1}{a}\cdot a=1$
DISTRIBUTIVITY	$a(b+c)=ab+ac$ $\xrightarrow{\hspace{3em}}$ multiply out (remove parentheses) $\xleftarrow{\hspace{3em}}$ factor out the common factor	

Exercise #1 Find the opposite and the reciprocal (if any) of each number:

The Number	Its Opposite	Its Reciprocal

The Double Negative Rule

$$-(-a)=a$$

(Section 1.2)

The Absolute Value of a Number

<u>Definition (1)</u> **The absolute value of a number** is the distance between the number and 0 (the origin) on the number line.

$$|a| = dist(a,0)$$

<u>Property</u> $|a| \geq 0, \quad \forall a \in R$

<u>Definition (2)</u>
$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

<u>Properties</u> (1) $|ab| = |a| \cdot |b|, \ \forall a, b \in \mathbb{R}$

(2) $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}, \forall a, b \in \mathbb{R}, b \neq 0$

Note: $|a+b| \neq |a| + |b|$ Example: ________________________________

$|a-b| \neq |a| - |b|$ Example: ________________________________

<u>Exercise #2</u> Simplify the following:

a) $|-7| =$

c) $-|-7| =$

b) $-(-7) =$

d) $-|-(-7)| =$

<u>Exercise #3</u> Simplify the following:

a) $(-5)^2 - 3^2 + |10 - 2 \cdot 3|$ (A: 20)

b) $-18 \div (-3)^2 + |-8| - |-4|$ (A: 2)

c) $\dfrac{(-4)^2 - |1 - 2^3|}{-(-2)^3 + (-1)^{125}}$ (A: $\dfrac{9}{7}$)

d) $\dfrac{9[4-(1+6)]-(3-9)^2}{5+\dfrac{12}{5-\dfrac{6}{2+1}}}$ (A: -7)

<u>Exercise #4</u> Evaluate the following expressions if $x = 2, y = -3, z = -1$:

a) $\dfrac{|xy|}{3z}$ (A: -2)

b) $\dfrac{3y^2 - x^2 + 1}{y|z|}$ (A: -8)

c) $yz^3 - (xy)^3$ (A: 219)

(Section 1.6)

Properties of Integral Exponents

Definition If $n \in \mathbb{N}$, then $a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ times}}$

a is called **base** and *n* is called **power (exponent).**

PROPERTY		EXAMPLES
The Product Rule	$a^m \cdot a^n = a^{m+n}$	
The Quotient Rule	$\dfrac{a^m}{a^n} = a^{m-n}$	
The Zero-Exponent Rule	$a^0 = 1, \forall a \neq 0$	
The Negative-Exponent Rule	$a^{-n} = \dfrac{1}{a^n}$	
The Power Rule	$\left(a^m\right)^n = a^{m \cdot n}$	
Products to Power	$(ab)^n = a^n \cdot b^n$	
Quotients to Power	$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	

Exercise #5 Simplify the following expressions:

a) $x-5[x-5(x-5)]$ $(A\colon 21x-125)$

b) $5(a-1)-4[2a-4(a-3)]$ $\left(A\colon 13a-53\right)$

c) $x^2y(xy-x)-7xy(x^2y-x^2)$ $\left(A\colon -6x^3y^2+6x^3y\right)$

d) $(-8xy)(x^5y^4)(-4xy)$ $\left(A\colon 32x^7y^6\right)$

e) $x\left[2x^2+x\left(x-3\left(x-1\right)\right)\right]$ $\left(A\colon 3x^2\right)$

Exercise #6 Simplify each expression .
Write answers without using parentheses or negative exponents.

a) $\dfrac{y^2}{yy^{-2}}$ $\left(A\colon y^3\right)$

b) $\left(\dfrac{a^2b^{-1}}{4a^3b^{-2}}\right)^{-3}$ $\left(A\colon \dfrac{64a^3}{b^3}\right)$

c) $\dfrac{a^0+b^0}{2\left(a+b\right)^0}$ $\left(A\colon 1\right)$

d) $\left(\dfrac{-2a^{-4}b^{3}c^{-1}}{3a^{-2}b^{-5}c^{-2}}\right)^{-4}$ $\left(A\colon \dfrac{81a^8}{16b^{32}c^4}\right)$

e) $\left(\dfrac{2x^{-4}y}{x^5y^5}\right)^{-3}\left(\dfrac{4x^{-2}y^0}{x^7y^2}\right)^2$ $\left(A\colon 2x^9y^8\right)$

f) $\dfrac{24x^2y^{13}}{-2x^5y^{-2}}$ $\left(A\colon -\dfrac{12y^{15}}{x^3}\right)$

g) $\left(-4x^{-4}y^5\right)^{-2}\left(-2x^5y^{-6}\right)$ $\left(A\colon -\dfrac{x^{13}}{y^{16}}\right)$

Sets. Operations with Sets

<u>Example#1</u> Let A and B be two sets of elements: $A = \{a, b, c\}$, $B = \{a, b, c, d\}$

$a \in A$ because a is an element of A

$d \notin A$ because d is not an element of A.

$\{a, b, c\} = \{b, a, c\}$

<u>Definition</u> $A \subset B$ **A is included in B** if any element of A is also in B.

<u>Example #2</u> $\{a, b, c\} \subset \{a, b, c, d\}$

$\{1,2,3\} \not\subset \{1,2\}$

<u>Operations with sets</u>

$\bigcup$ **- "union"** $A \bigcup B = \{x \mid x \in A \text{ or } x \in B\}$

<u>Examples:</u>

$\bigcap$ **- "intersection"** $A \bigcap B = \{x \mid x \in A \text{ and } x \in B\}$

<u>Examples:</u>

The Empty Set $\varnothing$ **-** the set with no elements

<u>Definition</u> A number ***a* is less than a number *b*** ($a < b$) if a is to the left of b on the number line.

<u>Exercise #7</u> Write equivalent statements:

a) $2 \le 3$ ______________

b) $2 > y$ ______________

c) $5 > x \ge -2$ ______________

d) $-4 < -2$ ______________

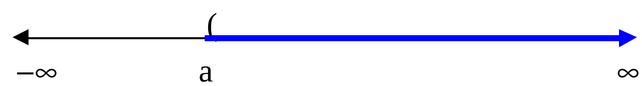
Intervals of real numbers

$[a,b] = \{x \mid a \le x \le b\}$

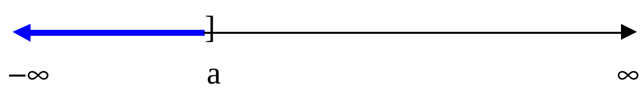
$(a,b) = \{x \mid a < x < b\}$

$[a,\infty) = \{x \mid x \ge a\}$

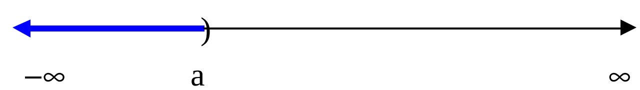
$(a,\infty) = \{x \mid x > a\}$



$(-\infty,a] = \{x \mid x \le a\}$



$(-\infty,a) = \{x \mid x < a\}$



<u>Exercise #8</u> Do the following operations and graph the solution set:

a) $[-2,5] \cup [-3,1] =$

b) $[-2,5] \cap [-3,1] =$

c) $(1,\infty) \cap (-3,4) =$

d) $(-\infty,2) \cup [0,\infty) =$

e) $(-4,-1) \cap (-1,2) =$

<u>Exercise #9</u> Graph the following sets and express them using interval notation:

a) $\{x \mid x \le -2\} =$

b) $\{x \mid 2 < x \le 3\} =$

c) $\{x \mid -3 \ge x \ge -7\} =$

(Sections 1.4 & 1.5)

Linear Equations

Definition An **equation** is a mathematical statement that two algebraic expressions are equal.

Examples:

Types of Equations

(1) **IDENTITY** = an equation which is always **true** regardless of the value of the variable.

Examples: $3 = 3$

$x + 1 = x + 1$

(2) **CONTRADICTION** = an equation which is always **false** regardless of the value of the variable.
(INCONSISTENT)

Examples: $5 = 7$

$x + 2 = x + 4$

(3) **CONDITIONAL** = an equation whose truth or falsehood depends on the value of the variable.

Examples: $x + 2 = 5$

Exercise #10 Determine the type of each of the following equations:

a) $2(x - 3) = 2x - 3$

b) $5u - 4(u - 1) - u = u - 4 - (u - 2)$

c) $5(x + 2) = 5x + 10$

d) $3(w + 1) = w + 3$

<u>Definition</u> A **solution** of an equation is the value of the variable that **satisfies** the equation.

<u>Definition</u> The process of finding the values that satisfy an equation is called **solving the equation.**

<u>Exercise #11</u> Determine which of the listed values satisfies the given equation:

a) $2x+3=6$, $x=0, x=\dfrac{3}{2}$

b) $6-2w=10-3w$, $w=-4, w=1$

<u>Properties of Equality</u>

$$\text{If } a=b\text{, then } \begin{cases} a+c=b+c, \forall c\in \mathbb{R} \\ a-c=b-c, \forall c\in \mathbb{R} \\ ac=bc, \forall c\in \mathbb{R} \\ \dfrac{a}{c}=\dfrac{b}{c}, \forall c\neq 0 \end{cases}$$

<u>Exercise #12</u> Solve the following equations .

a) $9(6+x)=-7(2+x)$ $\left(A: x=-\dfrac{17}{4}\right)$

b) $\dfrac{3}{5}x+2=\dfrac{10}{3}$ $\left(A: x=\dfrac{20}{9}\right)$

c) $\dfrac{3(n-2)}{5}=\dfrac{3n+6}{6}$ $(A: n=22)$

d) $9(a+5)-10(1-a)=14$ $\left(A: a=-\dfrac{21}{19}\right)$

e) $2[3(y+1)-(2y+3)]=-2$ $(A: y=-1)$

f) $\dfrac{x+1}{3}=5-\dfrac{x+2}{7}$ $\left(A: x=\dfrac{46}{5}\right)$

i) $0.8q-3.2=1.6$ $(A: q=6)$

j) $\dfrac{x+1}{4}=\dfrac{1}{6}+\dfrac{2-x}{3}$ $(A: x=1)$

k) $\dfrac{1}{4}m+\dfrac{2}{3}m=\dfrac{1}{6}$ $\left(m=\dfrac{2}{11}\right)$

l) $\dfrac{5}{6}=\dfrac{2u-3}{5}$ $\left(A: u=\dfrac{43}{12}\right)$

m) $\dfrac{2m-1}{2}-\dfrac{3m-1}{3}=\dfrac{4m-1}{4}$ $\left(A: m=\dfrac{1}{12}\right)$

g) $\frac{x+4}{2}+\frac{x+1}{4}=3$ $(A: x=1)$

o) $\frac{2}{5}=\frac{2t+3}{6}$ $\left(A: t=-\frac{3}{10}\right)$

h) $\frac{2}{3}(v-4)=2$ $(A: v=7)$

p) $0.3r+1.2(20)=0.8(r+20)$ $(A: r=16)$

q) Evaluate $x^2-(xy-y)$ for x satisfying $\frac{3(x+3)}{5}=2x+6$ and y satisfying $-2y-10=5y+18$. $(A: -7)$

Exercise #13 Solve each formula for the specified variable:

a) $v=k+gt$, for t $\left(A: t=\frac{v-k}{g}\right)$

b) S=3pd+pa, for d $\left(A: d=\frac{S-pa}{3p}\right)$

c) $A=P(1+rt)$, for r $\left(A: r=\frac{A-p}{pt}\right)$

d) $A=2w^2+4lw$, for l $\left(A: l=\frac{A-2w^2}{4w}\right)$

e) $A=\frac{1}{2}h(a+b)$ for a $\left(A: a=\frac{2A}{h}-b\right)$

f) $A=2lw+2lh+2wh$ for l $\left(A: l=\frac{A-2wh}{2(w+h)}\right)$

Exercise #14 In a triangle, the measures of the three angles are consecutive integers. What is the measure of each angle? (A: 59, 60, 61)

Exercise #15 According to one mathematical model, the average life expectancy for American men born in 1900 was 55 years. Life expectancy has increased by about 0.2 year for each birth year after 1900. If this trend continues, for which birth year will the average life expectancy be 85 years? (A: 2050)

Exercise #16 In 2003, the price of a BMW 7 Series was approximately \$80,500 with a depreciation of \$8705 per year. After how many years will the car's value be \$19,565? (A: 7 years)

Exercise #17 Suppose that we agree to pay you 8 cents for every problem in this chapter that you solve correctly and fine you 5 cents for every problem done incorrectly. If at the end of 26 problems we do not owe each other any money, how many problems did you solve correctly? (A:10)

(Section 1.3)

A Review of the Rectangular Coordinate System
Graphing Equations

Definition — The **Rectangular Coordinate System** (Cartesian coordinate system) is a system of two perpendicular number lines:

- the horizontal number line **(x-axis)**
- the vertical number line **(y-axis)**
- the point of intersection of the coordinate axes is called the **origin.**

Definition — The **general form of a linear equation in two variables** is

$ax + by = c$, where a and b are not both zero.

Definition — A **solution** of an equation in two variable is an **ordered pair** (x, y)

that satisfies the equation.

Definition — The **graph** of an equation is the set of all points that **satisfy** the equation.

Theorem — The graph of an equation of the form $ax + by = c$ is a **line** provided that a and b are not both zero.

Property — i) The ordered pair (x, y) is a solution of an equation if and only if (x, y)

belongs to the **graph of the given equation.**

ii) The ordered pair (x, y) is **not a solution** of an equation if and

only if (x, y) doesn't belong to the **graph of the given equation.**

Definition — The x- intercept of a line is the point where the line intersects the ***x*-axis.**

The y-intercept of a line is the point where the line intersects the ***y*-axis.**

Exercise #19 Graph the following equations:

a) $y = x^3$

b) $y = |x|$

c) $y = \frac{1}{x}$

d) $y = 2x + 4$

e) $2x + 3y = 6$

Exercise #20 Match the story with the correct figure.

a) As the blizzard got worse, the snow fell harder and harder.

b) The snow fell more and more softly.

c) It snowed hard, but then it stopped. After a short time, the snow started falling softly.

d) It snowed softly, and then it stopped. After a short time, the snow started falling hard.