Section 8.2
Quadratic Equations and Their Applications

1) $h = -16t^2 + 20t + 5$
where t = time the ball is in the air (in seconds);
h=height of the ball above the ground (in feet)
a) If nobody hits the ball, how long will it take the ball to hit the ground?
t=? when h=0

$$0 = -16t^2 + 20t + 5$$
$$16t^2 - 20t - 5 = 0$$
$$t_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$$
$$= \frac{-(-20) \pm \sqrt{(-20)^2 - 4(16)(-5)}}{2(16)} = \frac{20 \pm \sqrt{720}}{32}$$
$$t_1 = \frac{20 + \sqrt{720}}{32} \approx 1.46, \quad t_2 = \frac{20 - \sqrt{720}}{32} \approx -0.21$$

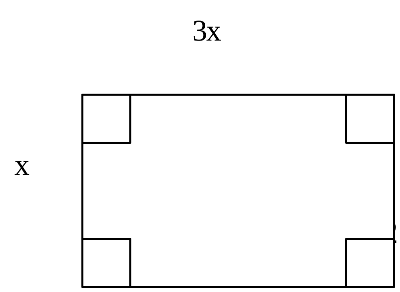
Negative time doesn't make sense, therefore it takes the ball about 1.46 seconds until it hits the ground.

b)If nobody hits the ball, how long will it take the ball to reach its initial height again?
The initail height occurs when t=o, so the initial height is h=0+0+5 = 5 feet.
Find t when h=5.

$$5 = -16t^2 + 20t + 5$$
$$16t^2 - 20t = 0$$
$$4t(4t - 5) = 0$$

$t_1 = 0$ (the initial moment); $t_2 = \frac{5}{4}$.
The ball reaches 5 ft again at 5/4 seconds.

2)



a) Find a formula for the volume of the box V in terms of the width x of the cardboard.
If the width of the original piece of cardboard = x,
Then the length of the cardboard = 3x.
If we cut a square from each corner of side 2 and fold up the sides, the box will have length = 3x-4, width = x-4, and height =2.

$V = l \cdot w \cdot h$.
$V = (3x - 4)(x - 4)2$
$V = 6x^2 - 32x + 32$

b)) If the volume of the box is 1222 cubic inches, find the dimensions of the cardboard.
V=1222

$$6x^2 - 32x + 32 = 1222$$
$$6x^2 - 32x - 1990 = 0$$
$$2\left(3x^2 - 16x - 595\right) = 0$$
$$3x^2 - 16x - 595 = 0$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(3)(-595)}}{2(3)}$$
$$= \frac{16 \pm \sqrt{7396}}{6} = \frac{16 \pm 86}{6}$$

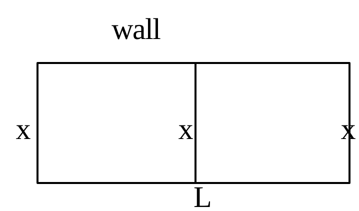
$x_1 = \frac{16+86}{6} = 17$; $x_2 = \frac{16-86}{6} < 0$ - it doesn't make sense. So , the width of the cardboard is 17 inches and the length = 3 (17)=51 inches.

3) $A = P(1+r)^t$, where P=2600, t=2, A=2(2600)=5200, r=?

$5200 = 2600(1+r)^2$
$(1+r)^2 = \frac{5200}{2600}$
$(1+r)^2 = 2$
$\sqrt{(1+r)^2} = \sqrt{2}$
$|1+r| = \sqrt{2}$
$1 + r = \pm\sqrt{2}$
$r = -1 \pm \sqrt{2}$
$r = -1 + \sqrt{2} \approx 0.4142 = 41.42\%$
$r = -1 - \sqrt{2} \approx -2.414$ which doesn't make sense.

Therefore, she needs a rate of 41.42% !!!!!!!

4)



The fence will go only on the three widths and one length (not on the wall). Total fence = 300 feet means that 3x + L = 300, where x = width, L = length.
Solving the above equation for L, we obtain: L=300-3x

(Cont. #4) Area = 6000

Area = length (width) = (300-3x)x

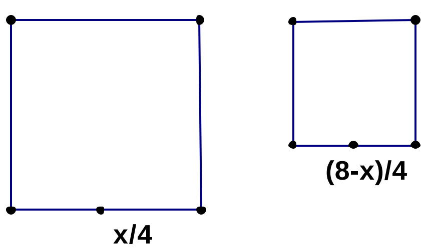
$$6000 = (300 - 3x)x$$
$$6000 = 300x - 3x^2$$
$$3x^2 - 300x + 6000 = 0$$
$$3(x^2 - 100x + 2000) = 0$$
$$x^2 - 100x + 2000 = 0$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-100) \pm \sqrt{(-100)^2 - 4(1)(2000)}}{2(1)}$$

$$= \frac{100 \pm \sqrt{2000}}{2}$$
$$x_1 = \frac{100 + \sqrt{2000}}{2} = 72.36$$
$$x_2 = \frac{100 - \sqrt{2000}}{2} = 27.64$$

So, width = 72.36 ft and length = 300-3(72.36) =82.92 ft or

width = 27.64 ft and length = 300-3(27.64)=217.08 ft.

5)



The total area of the two squares must be 2.

We know that the area of a square is side squared.

Therefore,

$$\left(\frac{x}{4}\right)^2 + \left(\frac{8-x}{4}\right)^2 = 2$$
$$\frac{x^2}{16} + \frac{(8-x)^2}{16} = 2$$
$$x^2 + (8-x)^2 = 2(16)$$
$$2x^2 - 16x + 32 = 0, \; 2(x^2 - 8x + 16) = 0$$
$$(x-4)^2 = 0, \; x = 4$$

6)

	Time to complete the job (in hours)	Part of job done per hour
I person	x	1/x
II person	x+1	1/(x+1)
Together	4	1/4

$$\frac{1}{x} + \frac{1}{x+1} = \frac{1}{4}$$
$$\frac{x+1+x}{x(x+1)} = \frac{1}{4}$$
$$4(2x+1) = x(x+1)$$
$$x^2 - 7x - 4 = 0$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{7 \pm \sqrt{65}}{2}$$
$$x_1 = \frac{7 + \sqrt{65}}{2} \approx 7.5$$

$x_2 = \frac{7 - \sqrt{65}}{2} < 0$ - not possible. Therefore, one of them needs 7.5 hours and the other one needs 8.5 hours.

7) $A = P(1+r)^t$, where P=5500, t=2, A=6474.74, r=?

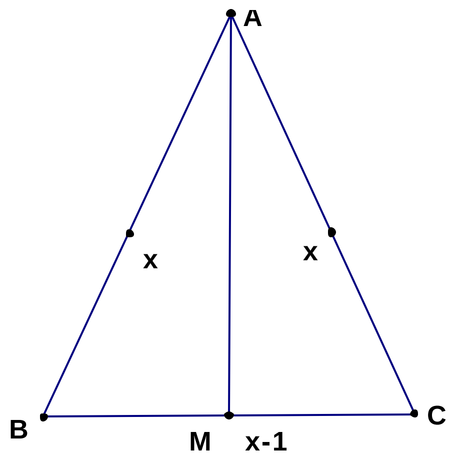
$$6474.74 = 5500(1+r)^2$$
$$(1+r)^2 = \frac{6474.74}{5500}$$
$$\sqrt{(1+r)^2} = \sqrt{\frac{6474.74}{5500}}$$

$$|1+r| = \sqrt{\frac{6474.74}{5500}}$$
$$1 + r = \pm\sqrt{\frac{6474.74}{5500}}$$
$$r = -1 \pm \sqrt{\frac{6474.74}{5500}},$$
$$r = -1 + \sqrt{\frac{6474.74}{5500}} \approx 0.085 = 8.5\%$$

(cont. # 7)

$r = -1 - \sqrt{\dfrac{6474.74}{5500}} < 0$ - it doesn't make sense.
Therefore, the interest rate was 8.5%.

8)



Let x – the length of the equal sides AB and AC.
Then x-1 is the length of the base BC.
Then the height (altitude) AM is x-2.
We'll apply the Pythagorean theorem in
The triangle AMC, where
AC = x, AM = x-2, MC=(x-1)/2 (half of BC).

$$(AM)^2 + (MC)^2 = (AC)^2$$

$$(x-2)^2 + \left(\frac{x-1}{2}\right)^2 = x^2$$

$$4\left(x^2 - 4x + 4\right) + \left(x^2 - 2x + 1\right) = 4x^2$$

$$(x-17)(x-1) = 0$$

$x = 17$ or

$x = 1$ - not possible (x-1 and x-2 become negative).

Therefore, the length of the equal sides is 17 in.