Highlights **Factoring Polynomials**

The Greatest Common Factor

Factoring = writing an expression as a product	Factor 12: $12 = 6 \cdot 2$
Prime factorization = writing an expression as a product of prime factors	The prime factorization of 12: $12 = 2 \cdot 2 \cdot 3$
The Greatest Common Factor (GCF) of a list of terms = The GCF of numerical coefficients (use prime factorization) • The GCF of the variable factors (contains the smallest exponent on each common variable)	Find the GCF of $8x^2y$, $12x^3y^2$, and $60x^2y^3$ $8x^2y = 2 \cdot 2 \cdot 2 \cdot x^2 y$ $12x^3y^2 = 2 \cdot 2 \cdot 3 \cdot x^3 y^2$ $60x^2y^3 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot x^2 y^3$ GCF $= 2 \cdot 2 \cdot x^2 y = 4x^2 y$
To factor out the GCF: • Find the GCF of the terms • Use the distributive property	Factor: 8x + 20 = 4(2x + 5) $8x = 2 \cdot 2 \cdot 2 \cdot x$ $20 = 2 \cdot 2 \cdot 5$ $GCF = 2 \cdot 2 = 4$ Factor: 7(x+2) + y(x+2)= (x+2)(7+y) GCF = x+2
To factor by grouping: • Step 1: Group the terms into two groups of two terms. • Step 2: Factor out the GCF from each group. • Step 3: If there is a common factor, factor it out. • Step 4: If not, rearrange the terms and try Steps 1-3 again.	Factor: $10x^2 + 15x - 6xy - 9y =$ Step 1: $(10x^2 + 15x) - (6xy + 9y) =$ Step 2: $5x(2x+3) - 3y(2x+3) =$ Step 3: $(2x+3)(5x-3y)$

Special products. Factoring Square Trinomials and the Difference of Two Squares

A perfect square = a positive integer that is the square of a natural number.
This concept of perfect squares extends to algebraic expressions.

A perfect square trinomial = a trinomial that is the square of some binomial .

Squaring a binomial:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

Reverse the concept: **Factoring Perfect Square Trinomials**

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

Difference of squares and cubes; Sum of cubes

$$(a-b)(a+b) = a^2 - b^2$$

$$(a-b)\left(a^2+ab+b^2\right) = a^3 - b^3$$

$$(a+b)\left(a^2-ab+b^2\right) = a^3 + b^3$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^3 - b^3 = (a-b)\left(a^2+ab+b^2\right)$$

$$a^3 + b^3 = (a+b)\left(a^2-ab+b^2\right)$$

To recognize a perfect square trinomial:

- Step1: See if there are two terms that are perfect squares: a^2, b^2
- If no perfect squares, then the trinomial is not a perfect square.
- Step 2: See if the third term can be written as twice the product of a and b: 2ab

Perfect squares: $1 = 1^2, 4 = 2^2, 9 = 3^2, 16 = 4^2, ..$

Square each binomial:

$$(x+6)^2 = x^2 + 2 \cdot x \cdot 6 + 6^2 = x^2 + 12x + 36$$

$$(2x-3)^2 = (2x)^2 - 2 \cdot (2x) \cdot 3 + 3^2 =$$

$$= 4x^2 - 12x + 9$$

Multiply:

$$(7-x)(7+x) = 7^2 - x^2 = 49 - x^2$$

$$(2x-1)(2x+1) = (2x)^2 - 1^2 = 4x^2 - 1$$

Factor:

$$x^2 + 6x + 9 = x^2 + 2 \cdot x \cdot 3 + 3^2 = (x+3)^2$$

Step 1: x^2 and 9 are perfect squares.
Step 2: 6x = $2 \cdot x \cdot 3$

Factor:

$$4x^2 - 12x + 9 = (2x)^2 - 2 \cdot (2x) \cdot 3 + 3^2$$

$$= (2x-3)^2$$

Factor: $x^2 - 25 = x^2 - 5^2 = (x-5)(x+5)$

Factoring Trinomials of the Form $Ax^2 + Bx + C$

- **Step1:** Are there any **common factors**? If so, **factor out the GCF.**
- **Step2:** Can you recognize any **special products?**

Two terms: Is it the **difference of two squares?**

$$a^2 - b^2 = (a-b)(a+b)$$

Three terms: Is it a **perfect square trinomial?**

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

- **Step3:** Factor using the method that applies, according to the coefficient of x^2

When the coefficient of x^2 is 1 $x^2 + Bx + C$

$x^2 + Bx + C$ = (x + ?) (x + ?)

*Look after two numbers whose
product = C (the constant term)
sum = B (the coefficient of x)

When the coefficient of x^2 is not 1. $Ax^2 + Bx + C$

$Ax^2 + Bx + C = Ax^2 + ?\,x + ?\,x + C$
p q

*Look after two numbers p and q to rewrite Bx = px + qx whose

product= $A \cdot C$ where A is the coefficient of x^2
C is the constant term

sum= B where B is the coefficient of x

*Then factor by grouping.

Factor: $x^2 + 12x + 20$

Step1 – there are no common factors.
Step2 – there is no perfect square trinomial.

Step3 – the coefficient of x^2 is 1.

$x^2 + 12x + 20 = (x + ?)(x + ?)$ =(x+2)(x+10)

a b +2
ab=20
+10

a+b = 12
20 = 2 · 10

Factor: $6x^2 + 28x - 10$

Step1 – the GCF for all terms is 2. Factor 2 out:

$$6x^2 + 28x - 10 = 2(3x^2 + 14x - 5)$$

Step 2 – there is no perfect square trinomial.

Step 3 – the coefficient of x^2 is 3 (not 1).

Look to rewrite $14x = ax + bx = 15x - x$ +15

ab= $3 \cdot (-5) = -15$
-1
a+b=14
15 =1 · 15

$$3x^2 + 14x - 5 = 3x^2 + 15x - x - 5$$

Factor by grouping.

$$= 3x(x+5) - (x+5) = (x+5)(3x-1)$$

So,

$$6x^2 + 28x - 10 = 2(x+5)(3x-1)$$

Solving Quadratic Equations by Factoring

A quadratic equation is an equation that can be written as

$$Ax^2 + Bx + C = 0, \text{ where } A, B, C \in R, A \neq 0$$

The form $Ax^2 + Bx + C = 0$ is called the **standard form.**

Quadratic Equations	**Same equations in standard form**
$x^2 = 36$	$x^2 - 36 = 0$
$y = -2y^2 + 5$	$2y^2 + y - 5 = 0$

Zero Factor Property If $a, b \in R$ and $a \cdot b = 0$, then $a = 0$ or $b = 0$.

If (x+5)(2x-1)= 0, then x+5 = 0 or 2x-1 = 0

To Solve Quadratic Equations by Factoring:

- **Step 1**: Write the equation in standard form (one side is zero).
- **Step 2**: Factor completely.
- **Step 3**: Set each factor containing a variable equal to zero. (according to the Zero Property)
- **Step 4:** Solve the resulting equations.
- **Step 5:** Check solutions in the original equation.

Solve: $3x^2 = 13x - 4$

Step 1 – standard form: $3x^2 - 13x + 4 = 0$
Step 2 – factor (see 4.4) $(3x-1)(x-4) = 0$
Step 3 – zero property: $3x - 1 = 0$ or $x - 4 = 0$
Step 4 – solve each linear equation:

$$x = \frac{1}{3} \qquad x = 4$$

Step 5 – check both solutions in the original equation by replacing x with $\frac{1}{3}$ and then x with 4

Quadratic Equations and Problem Solving

1. Understand the problem.
   - Read and reread it.
   - Draw a diagram.
   - Choose a variable to represent the unknown.
2. Translate the problem into an equation.
3. Solve the equation.
4. Interpret the results: discard the solutions that do not make sense as solutions of the problem. Check your solution in the stated problem and state your conclusion.

Helpful hints:

Perimeter (P) = the sum of the lengths (l) of all sides.

Triangle Δ P = $l_1 + l_2 + l_3$ $A = \frac{base \cdot height}{2}$

Square □ $P = 4l$ $A = l^2$

Rectangle ▭ $P = 2l + 2w$ $A = l \cdot w$

Right Triangle - **Pythagorean Theorem:**

$$(leg)^2 + (leg)^2 = (hypotenuse)^2$$