Math 61 Spring 2006

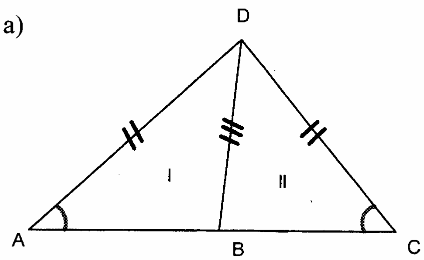
Name: SOLUTIONS

TEST #2@ 130 points

Write in a neat and organized fashion. Use a straightedge and compass for your drawings.

1. i) Write the congruences given by the indicated measures or marks.
   ii) State whether from the given congruences only you may conclude that triangles I and II are congruent.
   iii) If so, write what case of congruency applies.

a)

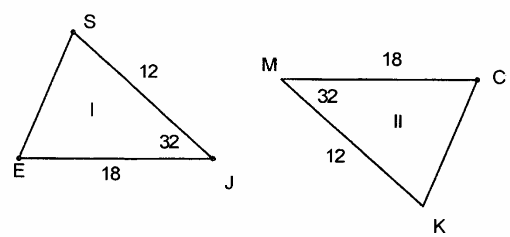


a) $\overline{AD} \cong \overline{CD}$
$\angle A \cong \angle C$
$\overline{BD} \cong \overline{BD}$

b) $\triangle I \ncong \triangle II$ No

c) N/A

b)

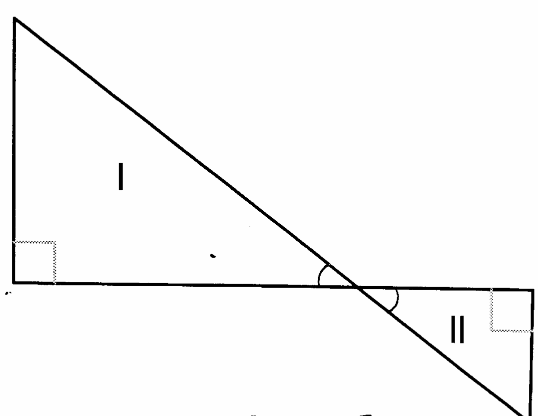


a) $\overline{SJ} \cong \overline{KM}$
$\overline{EJ} \cong \overline{CM}$
$\angle J \cong \angle M$

b) $\triangle SJE \cong \triangle KMC$ yes

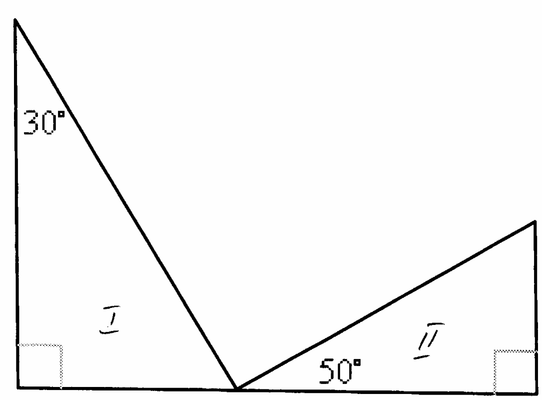
c) SAS

2. a) State whether $\triangle I \sim \triangle II$.
   b) If so, write what case of similarity applies.



a) yes, $\triangle I \sim \triangle II$

b) AA



a) No, $\triangle I \not\sim \triangle II$

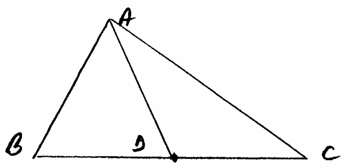
b) N/A

3)
a) Answers the following:

i) Complete the following statement:

A median of a triangle is the segment that joins one vertex with the midpoint of the opposite side

ii) Make a drawing to represent the above statement.



iii) Translate the statement mathematically.

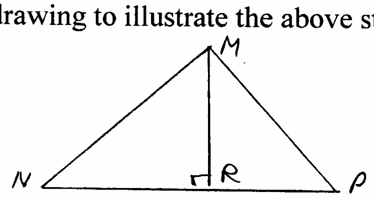
$\overline{AD}$ - median: $D \in \overline{BC}$, $\overline{BD} \cong \overline{DC}$

b) Answers the following:

i) Complete the following statement:

An altitude of a triangle is the line segment from one vertex perpendicular to the opposite side (or its extension)

ii) Make a drawing to illustrate the above statement.



iii) Translate the statement mathematically.

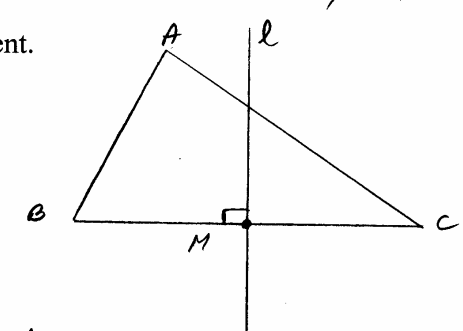
$\overline{MR}$ - altitude: $\overline{MR} \perp \overline{NP}$, $R \in \overleftrightarrow{NP}$

c) Answers the following:

i) Complete the following statement:

A perpendicular bisector of a side of a triangle is the line that is perpendicular to the side at the midpoint.

ii) Make a drawing to illustrate the above statement.



iii) Translate the statement mathematically.

l = perpendicular bisector of side $\overline{BC}$: $l \perp \overline{BC}$
$l \cap \overline{BC} = \{M\}$ with $\overline{BM} \cong \overline{MC}$

d) Answer the following:

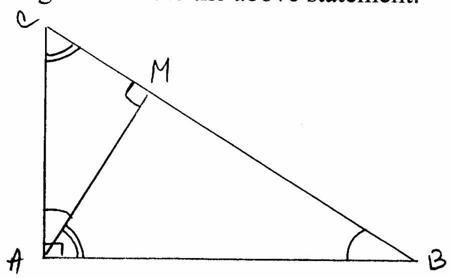
What do you know about the **altitude to the hypotenuse** in a right triangle?

i) Complete the following statements:

The altitude divides the right triangle into two ___similar___ triangles.

Each of these two triangles is also similar to ___the given triangle___

ii) Make a drawing to illustrate the above statement.



iii) Translate each of the above statements mathematically.

$\triangle ABC$ right at $\angle A$

$\overline{AM} \perp \overline{BC}$

Then, $\triangle AMC \sim \triangle BMA$ and $\triangle AMC \sim \triangle BAC$

$\triangle BMA \sim \triangle BAC$

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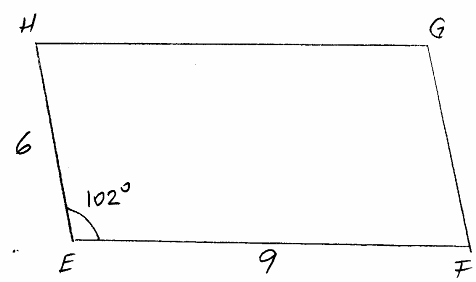
4) EFGH is a parallelogram. Suppose that EH = 6, EF = 9, and $m\angle E = 102^\circ$.
Find HG, FG, $m\angle H$, and $m\angle G$.



Given: EFGH - parallelogram
$EH = 6$
$EF = 9$
$m\angle E = 102^\circ$

Find: $HG = ?$
$FG = ?$
$m\angle G = ?$

Solution

EFGH - parallelogram $\Rightarrow \overline{HG} \cong \overline{EF}$, $\overline{HE} \cong \overline{GF}$ (opposite sides)

Therefore, $HG = EF = 9$
$FG = EH = 6$

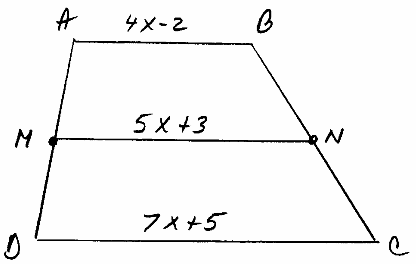
EFGH - parallelogram $\Rightarrow \angle E \cong \angle G$ (opposite angles)

Therefore, $m\angle G = m\angle E = 102^\circ$

EFGH - parallelogram $\Rightarrow \angle E$ and $\angle H$ are supplementary

$m\angle E + m\angle H = 180^\circ$
$m\angle H = 180^\circ - 102^\circ$ $m\angle H = 78^\circ$

5) In a trapezoid, the length of one base is $7x+5$, while the length of the other base is $4x-2$. The length of the median is given by $5x+3$. Find x.



Given: ABCD - trapezoid
$AB = 4x-2$
$DC = 7x+5$
$\overline{MN}$ - median
$MN = 5x+3$

Find $x = ?$

Solution

$\overline{MN}$ - median $\Rightarrow MN = \dfrac{AB+DC}{2}$

$5x+3 = \dfrac{(4x-2)+(7x+5)}{2}$

$2(5x+3) = 4x-2+7x+5$

$10x+6 = 11x+3$

$6-3 = 11x-10x$

$x = 3$

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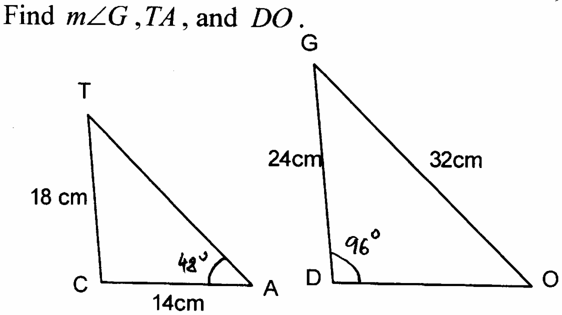
6) Given $\triangle TCA \sim \triangle GDO$, $m\angle D = 96^\circ$, $m\angle A = 48^\circ$, $GD = 24$ cm, $TC = 18$, $CA = 14$ cm, $GO = 32$ cm
Find $m\angle G$, TA, and DO.



Given: $\triangle TCA \sim \triangle GDO$
$m\angle D = 96^\circ$
$m\angle A = 48^\circ$
$GD = 24$ cm
$TC = 18$ cm
$CA = 14$ cm
$GO = 32$ cm

Find $m\angle G = ?$
$TA = ?$
$DO = ?$

Solution

$\triangle TCA \sim \triangle GDO \Rightarrow$

$\dfrac{TC}{GD} = \dfrac{TA}{GO} = \dfrac{CA}{DO}$

and $\begin{cases} \angle G \cong \angle T & (3) \\ \angle A \cong \angle O & (4) \\ \angle C \cong \angle D & (5) \end{cases}$

$\underbrace{\dfrac{18}{24} = \dfrac{TA}{32}}_{(1)} = \dfrac{14}{DO}$, (2): $\dfrac{18}{24} = \dfrac{14}{DO}$

(1) $\dfrac{18}{24} = \dfrac{TA}{32}$

$\dfrac{3}{4} = \dfrac{TA}{32}$

$TA = \dfrac{3 \cdot \cancel{32}^{8}}{\cancel{4}_{1}}$

$TA = 24$ cm

(2) $\dfrac{18}{24} = \dfrac{14}{DO}$

$\dfrac{3}{4} = \dfrac{14}{DO}$

$DO = \dfrac{4 \cdot 14}{3}$

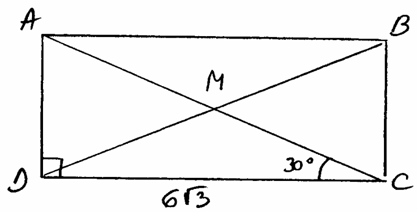
$DO = \dfrac{56}{3}$

(4) $\Rightarrow m\angle O = 48^\circ$

In $\triangle GDO$,
$m\angle G + m\angle D + m\angle O = 180^\circ$
$m\angle G + 96^\circ + 48^\circ = 180^\circ$
$m\angle G = 180^\circ - 144^\circ$
$m\angle G = 36^\circ$

$CD = 6\sqrt{3}$

7) Given a rectangle ABCD with diagonals $\overline{AC}$ and $\overline{BD}$, $m\angle ACD = 30°$, let M be the intersection of the diagonals. Find *AD*, *AC*, and *MC*.



Given: ABCD- rectangle
$\overline{AC}$, $\overline{BD}$ - diagonals
$\overline{AC} \cap \overline{BD} = \{M\}$
$m\angle ACD = 30°$
$CD = 6\sqrt{3}$

Find: $AD = ?$
$AC = ?$
$MC = ?$

Solution

$\triangle ADC$ · right triangle with $m\angle ACD = 30°$
Let $AD = a$
Then $AD = \frac{1}{2} AC$ $\Big\} \Rightarrow AC = 2a$

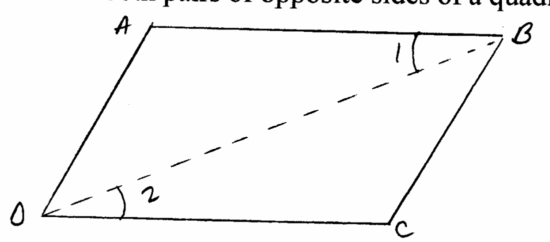
$\triangle ADC$ - right triangle with $AD = a$, $DC = 6\sqrt{3}$, $AC = 2a$

Pythagorean theorem: $AD^2 + DC^2 = AC^2$
$a^2 + (6\sqrt{3})^2 = (2a)^2$
$a^2 + 108 = 4a^2$
$3a^2 = 108$
$a^2 = 36$
$a = 6$

$\rightarrow$ Therefore, $AD = 6$
$AC = 2(6) = 12$
and
$MC = \frac{1}{2} AC = \frac{1}{2} \cdot 12 = 6$
(the diagonals bisect each other)

8) Draw a figure and write the hypothesis and conclusion. Mark the figure and write a formal proof.

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram



Given: ABCD= quadrilateral
$\overline{AB} \cong \overline{DC}$
$\overline{AD} \cong \overline{BC}$

Prove: ABCD = parallelogram

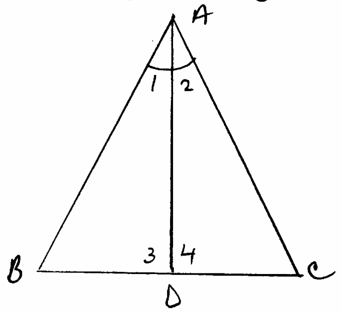
(Condition: $\overline{AB} \parallel \overline{DC}$)

Proof

Statements	Reasons
1. ABCD - quadrilateral	1. given
2. Draw $\overline{BD}$	2. Two points determine a line
3. $\triangle ABD$, $\triangle CDB$ $\left\{ \overline{BD} \cong \overline{BD}, \overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC} \right.$	3. reflexive property $\cong$; given; given
4. $\triangle ABD \cong \triangle CDB$	4. SSS
5. $\angle 1 \cong \angle 2$	5. CPCTC
6. $\overline{AB} \parallel \overline{DC}$	6. $\parallel$ iff alternate interior $\angle$'s $\cong$ ($\overline{AB}$ and $\overline{DC}$ with transversal $\overline{BD}$)
7. ABCD= parallelogram	7. $\square$ if opp. sides $\parallel$ and $\cong$

9) Draw a figure and write the hypothesis and conclusion. Mark the figure and write a formal proof.

In a triangle, if an angle bisector is an altitude, then it is also a median.



Given: $\triangle ABC$
$\overline{AD}$ - bisector of $\angle A$
$\overline{AD}$ - altitude

Prove: $\overline{AD}$ - median

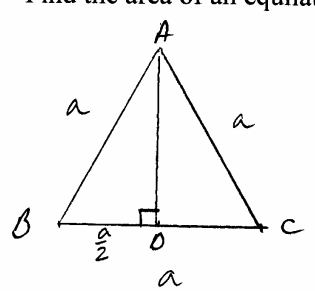
(Condition: D - midpoint, we need to show $\overline{BD} \cong \overline{DC}$)

Proof

Statements	Reasons
1. $\triangle ABC$	1. given
2. $\overline{AD}$ - bisector of $\angle A$	2. given
3. $\angle 1 \cong \angle 2$	3. definition of angle bisector
4. $\overline{AD}$ - altitude	4. given
5. $\overline{AD} \perp \overline{BC}$	5. definition of altitude
6. $\angle 3 \cong \angle 4$	6. $\perp$ iff $\cong$ adj $\angle$'s
7. $\triangle ADB$, $\triangle ADC$: $\angle 1 \cong \angle 2$, $\overline{AD} \cong \overline{AD}$, $\angle 3 \cong \angle 4$	7. (3), reflexive prop. $\cong$, (6)
8. $\triangle ADB \sim \triangle ADC$	8. ASA
9. $\overline{BD} \cong \overline{DC}$	9. CPCTC
10. D - midpoint of $\overline{BC}$	10. definition of midpoint
11. $\overline{AD}$ = median	11. definition of median

Extra Credit

Find the area of an equilateral triangle of side *a*.



Given $\triangle ABC$ - equilateral
$AB = AC = BC = a$

Find Area$(\triangle ABC) = ?$

Solution

Area$_\triangle$ = $\frac{1}{2}$ base$\cdot$height $\overline{BC}$ = base
Let $\overline{AD} \perp \overline{BC}$, $D \in \overline{BC}$ $\overline{AD}$ = height

Then $\triangle ABD \cong \triangle ACD$ (HL)
$\Rightarrow \overline{BD} \cong \overline{DC}$
so, $BD = \frac{1}{2}a$

$\triangle ABD$: $AB^2 = BD^2 + AD^2$ (Pythagorean th)
$a^2 = \frac{a^2}{4} + AD^2$
$AD^2 = \frac{3a^2}{4} \Rightarrow AD = \frac{a\sqrt{3}}{2}$

Then Area $= \frac{1}{2} \cdot a \cdot \frac{a\sqrt{3}}{2}$

Area $= \frac{a^2\sqrt{3}}{4}$