Math 61 Spring 2006 Name:

TEST #1 @ 130 points SOLUTIONS

Write in a neat and organized fashion. Use a straightedge and compass for your drawings.

1)

a) Write the inverse, converse, and contrapositive of the following statement:

if a baby gets pepper in its nose, then it sneezes.

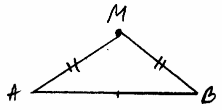
"A baby sneezes when it gets pepper in its nose."

Inverse: If a baby doesn't get pepper in its nose, then it doesn't sneeze.

Converse: If a baby sneezes, then it gets pepper in its nose.

Contrapositive: If a baby doesn't sneeze, then it doesn't get pepper in its nose.

b) Write the inverse, converse, and contrapositive of the following statement and classify the statements as true or false. If true, state the definition, postulate, or theorem your conclusion is based on. If false, say why or draw a counterexample.

	Circle one	Justify your choice
" *If M is the midpoint of* $\overline{AB}$, *then* $\overline{AM} \cong \overline{MB}$."	[✓ True] False	definition of midpoint.
Inverse: If M is not the midpoint of $\overline{AB}$, then $\overline{AM} \ncong \overline{MB}$	True [✓ False]	$M \neq$ midpoint but $\overline{AM} \cong \overline{MB}$ 
Converse: If $\overline{AM} \cong \overline{MB}$, then M is the midpoint of $\overline{AB}$.	True [✓ False]	Not necessarily. In order for M to be the midpoint, we also need $M \in \overline{AB}$, A-M-B (see above)
Contrapositive: If $\overline{AM} \ncong \overline{MB}$, then M is not the midpoint of $\overline{AB}$	[✓ True] False	definition of midpoint.

2) Study each argument carefully to decide whether or not it is valid.

a) ②If you walk under a coconut tree, you will probably be hit on the head.
①If you visit Hawaii, then you will walk under coconut trees.
Ⓒ Therefore, if you visit Hawaii, you will probably be hit on the head.

(Law of Syllogism)

VALID: [✓ YES] NO

b) If I go to the football game, I'll cheer for the Cowboys.
I cheered for the Cowboys.
Therefore, I went to the football game.

VALID: YES [✓ NO]

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3) Given the premises
$P \to Q$ Premise 1
$\sim Q$ Premise 2

a) State the conclusion: $\sim P$

b) Give the symbolic form of the law. $[(P \to Q) \wedge \sim Q] \to \sim P$

c) Prove the law using a truth table: Must show the above statement is a tautologie

P	Q	$P \to Q$	$\sim Q$	$(P \to Q) \wedge \sim Q$	$\sim P$	$[(P \to Q) \wedge \sim Q] \to \sim P$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

therefore, it's a tautologie.

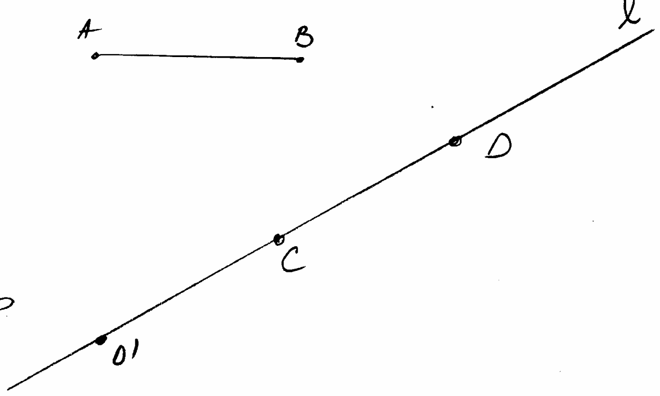
4) **Choose only ONE of the following constructions**. Explain how you are constructing it.

Given a segment $\overline{AB}$, construct using only a compass and a straightedge, a segment $\overline{CD}$ congruent with $\overline{AB}$.

Given: $\overline{AB}$

Construct: $\overline{CD}$ such that $\overline{CD} \cong \overline{AB}$

(Condition: $CD = AB$)



1. draw l - a line
2. let C a point on the line
3. construct circle with center at C and radius AB — $\mathcal{C}$
4. $\mathcal{C} \cap l = \{D, D'\}$
5. $CD = AB$ - radius
6. $\overline{CD} \cong \overline{AB}$ Therefore, $\overline{CD}$ is a segment such that $\overline{CD} \cong \overline{AB}$

(Note that also $\overline{CD'} \cong \overline{AB}$)

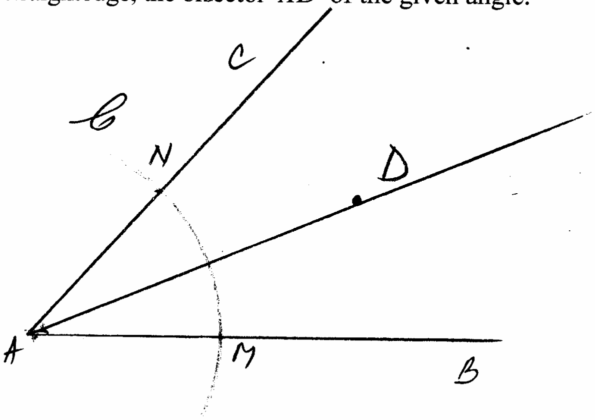
OR

Given an angle $\angle BAC$, construct using only a compass and a straightedge, the bisector $\overrightarrow{AD}$ of the given angle.

Given: $\angle BAC$

Construct: $\overrightarrow{AD}$ such that $\angle BAD \cong \angle DAC$

(Condition: $D \in \text{int} \angle BAC$
$m\angle BAD = m\angle DAC$)



1. Construct a circle with center A and a radius $r > 0$. $\mathcal{C}$
2. $\mathcal{C} \cap \overrightarrow{AB} = \{M\}$
   $\mathcal{C} \cap \overrightarrow{AC} = \{N\}$
3. Construct two congruent circles ⟨ one with center M and radius R
   another with center N and radius R
   (Note that R can be equal or not to r)
4. Let D = the intersection of the two circles from (3).
5. $\overrightarrow{AD}$ = bisector of $\angle BAC$

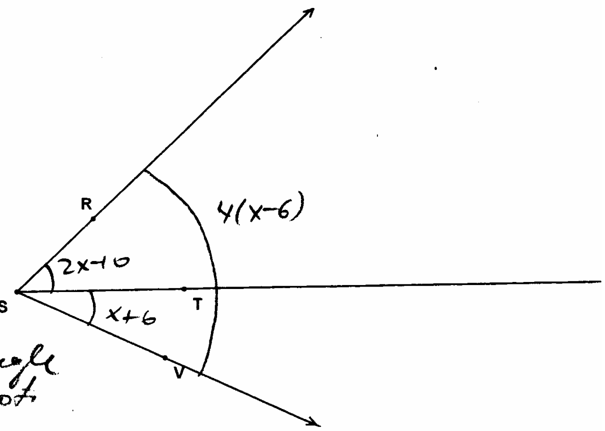
5) Given: $m\angle RST = 2x - 10$

$m\angle TSV = x + 6$

$m\angle RSV = 4(x - 6)$

Find: x and $m\angle RSV$.

(Note: Your proof could be formal or informal. Do not just write down an answer!)

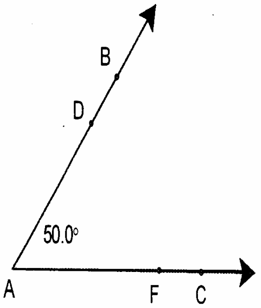


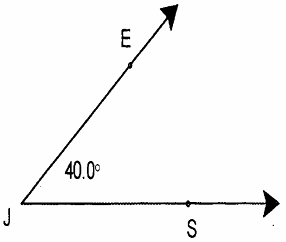
Proof

1. $m< RST + m< TSV = m< RSV$	1. Addition Angle Postulate
2. $(2x-10)+(x+6) = 4(x-6)$	2. Substitution
3. $3x - 4 = 4x - 24$	3. combining like terms & distributive prop
4. $4x - 3x = -4 + 24$	4. Addition property of equality
5. $\boxed{x = 20}$	5. Combining like terms
6. $m< RSV = 4(x-6) = 4(20-6) = 4(14) = 56°$	6. Substitution

$\boxed{m< RSV = 56°}$

6)





Refer to the figure to answer true or false.

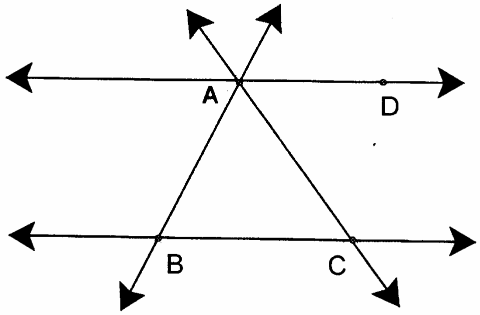
a) $\angle BAC$ is the same angle as $\angle BAF$ TRUE

b) $\angle SJE$ is complementary to $\angle CAD$ TRUE

c) $\angle EJS$ is an obtuse angle FALSE

d) $m\angle DAC + m\angle EJS = 90°$ TRUE

7)



Use the figure to name the geometric figures requested:

a) four lines $\overleftrightarrow{AD}, \overleftrightarrow{AB}, \overleftrightarrow{AC}, \overleftrightarrow{BC}$

b) four line segments $\overline{AB}, \overline{BC}, \overline{AC}, \overline{AD}$

c) eight rays $\overrightarrow{AD}$ $\overrightarrow{BA}$ $\overrightarrow{CB}$ $\overrightarrow{DA}$ $\overrightarrow{AC}$ $\overrightarrow{BC}$ $\overrightarrow{CA}$ $\overrightarrow{AB}$

d) two segments whose intersection is empty.

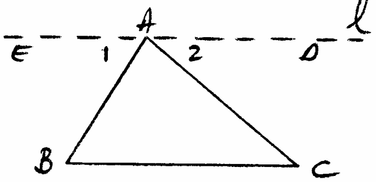
$\overline{AD}$ and $\overline{BC}$

8) **Choose only ONE of the following** . Do not prove both.

First, **complete the theorem**:

Prove that in any triangle, the sum of the measures of the interior angles is $180°$

Then, **prove the theorem**. (Hint: An auxiliary construction (line) is needed to complete the proof.) Make sure you state the hypothesis and the conclusion of the theorem.



Given: $\triangle ABC$

Prove: $m\angle A + m\angle B + m\angle C = 180°$

Proof

1. Construct $l \parallel \overleftrightarrow{BC}$, $l \ni A$	1 Through a point not on a line, there is only one line parallel to the given line
2 $m\angle EAD = 180°$	2. Definition of straight angle
3. $m\angle 1 + m\angle A + m\angle 2 = m\angle EAD$	3 Angle-Addition Postulate
4 $m\angle 1 + m\angle A + m\angle 2 = 180°$	4. Substitution ((2) in (3))
5 $m\angle 1 = m\angle B$	5. Alternate interior angles ($\overleftrightarrow{l} \parallel BC$, $\overline{AB}$ transversal)
6. $m\angle 2 = m\angle C$	6 Alternate int angles ($l \parallel \overline{BC}$, $\overline{AC}$ transv)
7 $m\angle B + m\angle A + m\angle C = 180°$	7. Substitution (5, 6 in 4)

Therefore, $m\angle A + m\angle B + m\angle C = 180°$

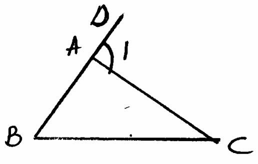
q.e.d.

OR

First, **complete the theorem:**

The measure of an exterior angle of a triangle equals the sum of the measures of the two nonadjacent interior angles of the triangle.

Then, **prove the theorem**. Make sure you state the hypothesis and conclusion of the theorem.



Given: $\triangle ABC$
$D \in \overrightarrow{BA}$, $B\text{-}A\text{-}D$

Prove: $m\angle DAC = m\angle B + m\angle C$

Proof

$$\left.\begin{array}{ll}\text{We know that} & m\angle A + m\angle B + m\angle C = 180° \text{ (Theorem)} \\ \text{Also,} & m\angle A + m\angle 1 = 180° \text{ (straight angle)}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \quad \cancel{m\angle A} + m\angle B + m\angle C = \cancel{m\angle A} + m\angle 1$$

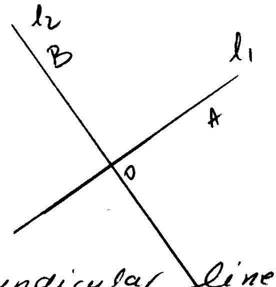
$$m\angle B + m\angle C = m\angle 1$$

Therefore, $m\angle B + m\angle C = m\angle DAC$

q.e.d.

9) Show the **formal proof** of the following theorem:

If two lines are perpendicular, then they meet to form right angles.



Given: $l_1 \perp l_2$

$A \in l_1$, $B \in l_2$

Prove: $m\angle AOB = 90°$

Proof

1.	$l_1 \perp l_2$	1. given
2.	$\angle AOB \cong \angle BOC$	2 definition of perpendicular lines
3.	$m\angle AOB = m\angle BOC$	3 definition of congruent angles
4.	$\angle AOC$ - straight angle $m\angle AOC = 180°$	4. definition of straight angle
5.	$m\angle AOB + m\angle BOC = m\angle AOC$	5. Angle-Addition Postulate
6.	$m\angle AOB + m\angle AOB = 180°$	6. Substitution (4 & 3 in 5)
7.	$2\, m\angle AOB = 180°$	7. combining like terms
8.	$m\angle AOB = 90°$	8. Multiplication property of equality

q.e.d.

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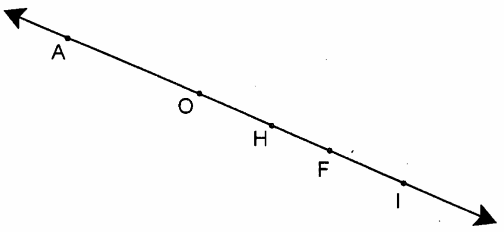
10)



Give the indirect proof of the following problem.

Given: $AO > HF$

$OH = FI$

Prove: H is not the midpoint of $\overline{AI}$

Proof

Assume H = midpoint of $\overline{AI}$

① $AH = HI$ (definition of midpoint)

but $\left\{ \begin{array}{l} AH = AO + OH \\ HI = HF + FI \end{array} \right.$ ② (Segment-Addition Postulate)

Substitute ② in ① ⇒

$AO + OH = HF + FI$) ⇒

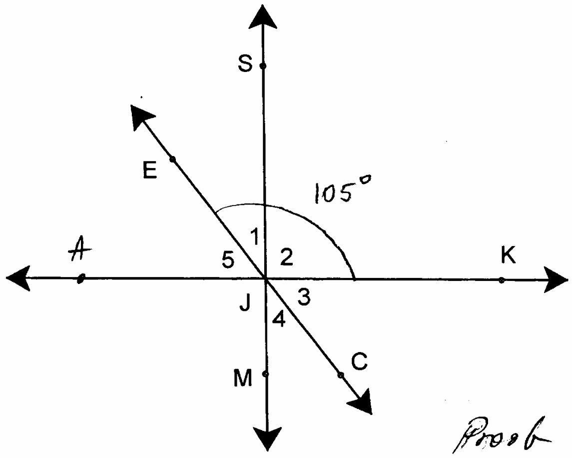
But $OH = FI$

⇒ $AO + \cancel{OH} = HF + \cancel{OH}$

$AO = HF$ Contradiction with the given $AO > HF$

Therefore, our assumption is false ⇒ H is not the midpoint of $\overline{AI}$

11)



Given $\overleftrightarrow{JK} \perp \overleftrightarrow{SM}$

$m\angle EJK = 105^\circ$

Find angles 1 through 5
(justify your steps)

<u>Proof</u> (informal)

$\overleftrightarrow{JK} \perp \overleftrightarrow{SM} \Rightarrow \boxed{m\angle 2 = 90^\circ}$

$m\angle 2 + m\angle 1 = 105^\circ$

$90^\circ + m\angle 1 = 105^\circ \Rightarrow \boxed{m\angle 1 = 15^\circ}$

$m\angle 1 = m\angle 4$ (vertical angles) $\Rightarrow \boxed{m\angle 4 = 15^\circ}$

$\overleftrightarrow{JK} \perp \overleftrightarrow{SM} \Rightarrow m\angle AJS = 90^\circ$

$m\angle 5 + m\angle 1 = 90^\circ$

$m\angle 5 + 15^\circ = 90^\circ \Rightarrow \boxed{m\angle 5 = 75^\circ}$

$m\angle 5 = m\angle 3$ (vertical angles) $\Rightarrow \boxed{m\angle 3 = 75^\circ}$

<u>OR</u>

Proof (formal)

1. $\overleftrightarrow{JK} \perp \overleftrightarrow{SM}$	1. given
2. $\boxed{m\angle 2 = 90^\circ}$	2. Perpendicular lines form right $<$'s.
3. $m\angle 2 + m\angle 1 = m\angle KJE$	3. Angle-Addition Postulate
4. $m\angle KJE = 105^\circ$	4. given
5 $90^\circ + m\angle 1 = 105^\circ$	5. Substitution (2 and 4 in 3)
6. $m\angle 1 = 105^\circ - 90^\circ$ $\boxed{m\angle 1 = 15^\circ}$	6. Addition Property of equality
7. $\angle 1 \cong \angle 4$	7. Vertical angles
8. $m\angle 1 = m\angle 4$	8. Definition of congruent angles
9. $\boxed{m\angle 4 = 15^\circ}$	9. Substitution (6 in 8)
10. $m\angle AJK = 180^\circ$	10 Straight angle
11. $m\angle KJE + m\angle 5 = m\angle AJK$	11. Angle-Addition Postulate
12. $105^\circ + m\angle 5 = 180^\circ$	12. Substitution (4 & 10 in 11)
13. $m\angle 5 = 180^\circ - 105^\circ$ $\boxed{m\angle 5 = 75^\circ}$	13 Addition Property of equality.
14. $\angle 5 \cong \angle 3$	14. Vertical angles
15 $m\angle 5 = m\angle 3$	15 definition of congruent angles
16. $\boxed{m\angle 3 = 75^\circ}$	16. Substitution. (13 in 15)