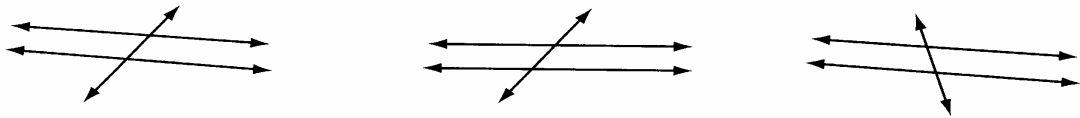# Exploring Angles Formed by Parallel Lines and a Transversal

When parallel lines are intersected by a third line, called a *transversal,* there are certain relationships among the angles formed. Use Sketchpad to explore these relationships.

1. Open the sketch **Parallel.gsp.**

2. Find ∠**HEX** and ∠**EOY**. They are a pair of *corresponding angles.* How do their measures compare? _____

3. For each angle given below, list its corresponding angle and tell how the measures compare.

   a. ∠**HEM** _____

   b. ∠**MEO** _____

   c. ∠**XEO** _____

4. Drag point **N** slowly and notice the measures of each pair of corresponding angles. How do their measures always compare? _____

   _____

   Complete the following statement: If two parallel lines are cut by a transversal, then the corresponding angles are _____.

5. Find ∠**MEO** and ∠**EOY**. They are a pair of *alternate interior angles.* How do their measures compare? _____

6. Name the other pair of alternate interior angles in the sketch: _____ and _____. How do their measures compare? _____

   _____

7. Drag point **N** slowly and notice the measures of both pairs of alternate interior angles. How do their measures compare? _____
Complete the following statement: If two parallel lines are cut by a transversal, then the alternate interior angles are _____.

8. Find ∠**HEM** and ∠**YON**. They are a pair of alternate exterior angles. How do their measures compare? _____

9. Name the other pair of alternate exterior angles in the sketch:
_____ and _____. How do their measures compare?

_____

10. Drag point **N** slowly and notice the measures of both pairs of alternate exterior angles. How do their measures always compare? _____

_____

Complete the following statement: If two parallel lines are cut by a transversal, then the alternate exterior angles are _____

_____.

Properties of Parallel Lines

Name(s): _____

In this investigation, you'll discover relationships among the angles formed when parallel lines are intersected by a third line, called a *transversal*.
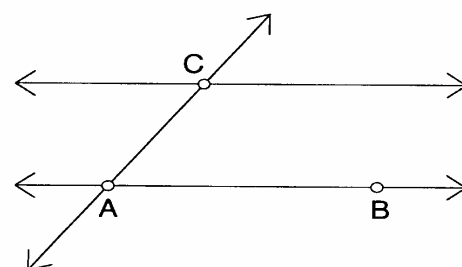
Sketch and Investigate

1. Construct $\overleftrightarrow{AB}$ and point C, not on $\overleftrightarrow{AB}$.

2. Construct a line parallel to $\overleftrightarrow{AB}$ through point C.

3. Construct $\overleftrightarrow{CA}$. Drag points C and A to make sure the three lines are attached at those points.

Step 1
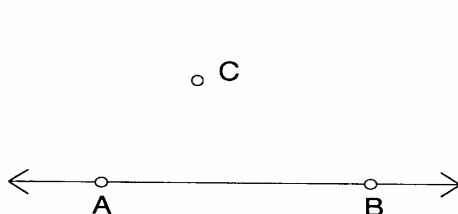
Steps 2 and 3

4. Construct points D, E, F, G, and H as shown at right.
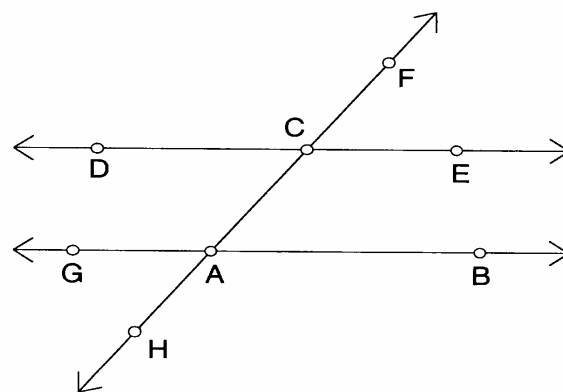
5. Measure the eight angles in your figure. Be systematic about your measuring to be sure you don't measure the same angle twice.

6. Drag point A or B and see which angles stay congruent. Also drag the transversal $\overleftrightarrow{CA}$. (Be careful not to change the point order on your lines. That would change some angles into other angles.) Observe how many of the eight angles you measured appear to be always congruent.

Step 4

Q1 When two parallel lines are crossed by a transversal, the pairs of angles formed have specific names and properties. The chart on the next page shows one example of each type of angle pair. Fill in the chart with a second angle pair of each type, then state what relationship, if any, you observe between the angles in a pair type.

Properties of Parallel Lines (continued)

Angle Type	Pair 1	Pair 2	Relationship
Corresponding	∠FCE and ∠CAB		
Alternate interior	∠ECA and ∠CAG		
Alternate exterior	∠FCE and ∠HAG		
Same-side interior	∠ECA and ∠BAC		
Same-side exterior	∠FCD and ∠HAG		

Q2 One of the angle types has more than one pair. Name that angle type in the chart below, and name the third and fourth pairs of angles of that type.

Angle Type	Pair 3	Pair 4	Relationship

7. Next, you'll investigate the converses of your conjectures. In a new sketch, draw two lines that are not quite parallel. Construct a transversal.

8. Add points as needed, then measure all eight angles formed by the three lines.

9. Move the lines until you have two sets of four congruent angles.

Q3 If two lines are crossed by a transversal so that corresponding angles, alternate interior angles, and alternate exterior angles are congruent, what can you say about the lines?
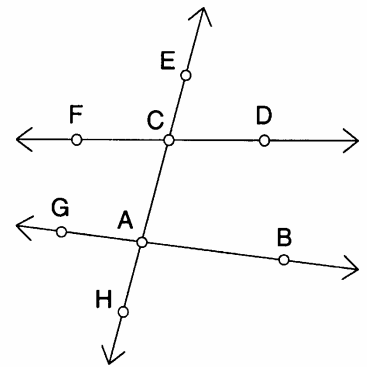
Explore More

1. You can use the converse of the parallel-lines conjecture to construct parallel lines. Construct a pair of intersecting lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ as shown. Select, in order, points C, A, and B. Then, in the Transform menu, choose **Mark Angle**. Double-click point C to mark it as a center for rotation. You figure out the rest. Explain why this method works.
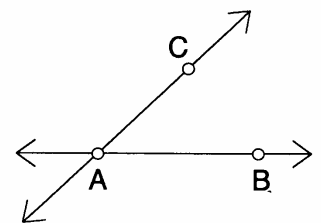
Angle Puzzle

1. Using only the information you are given in the drawing below, work with the students in your group to find the measure of each numbered angle.

2. Use what you know about the measures of special pairs of angles and angles formed by parallel lines and a transversal to help you. Also, remember that the sum of the angles of any triangle is 180°.

a ‖ *b*
c ‖ *d*
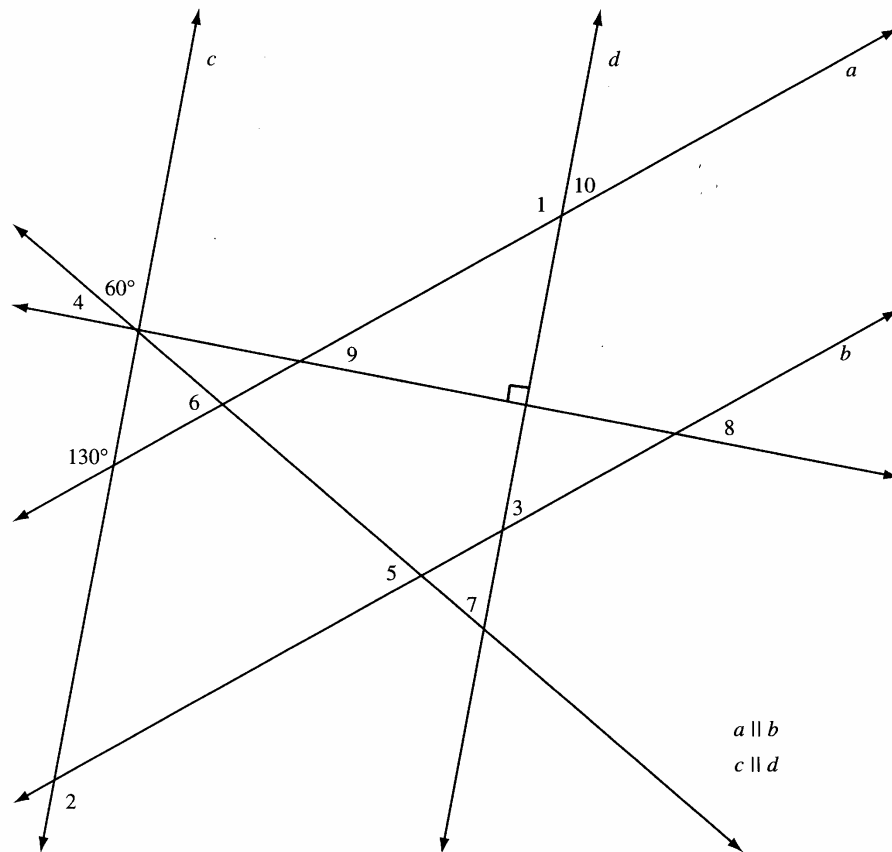
m∠1 = _____   m∠2 = _____   m∠3 = _____   m∠4 = _____

m∠5 = _____   m∠6 = _____   m∠7 = _____   m∠8 = _____

m∠9 = _____   m∠10 = _____

Challenge: Use Sketchpad to design your own Angle Puzzle. You must be careful to construct parallel lines that will stay parallel. Also, try only a few angles at first! Have another student try your first draft, and don't be surprised if you have to revise.