PROJECT #4 - DUE 05/04

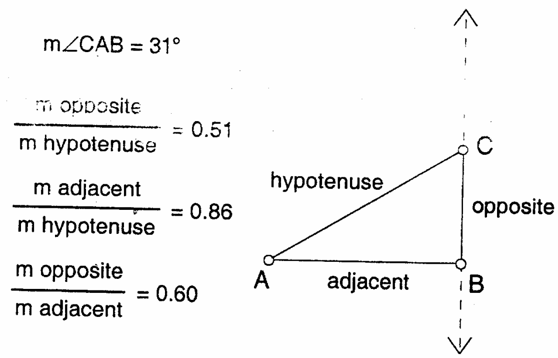
Trigonometric Ratios

Name(s): ____________________

Right-triangle trigonometry builds on similar-triangle concepts to give you more ways to find unknown measures in triangles. In this activity, you'll learn about trigonometric ratios and how you can use them.

Sketch and Investigate

In steps 1–5, you'll construct a right triangle.



1. Construct $\overline{AB}$.

Select point B and $\overline{AB}$; then in the Construct menu, choose **Perpendicular Line**.

2. Construct a line through point B perpendicular to $\overline{AB}$.
3. Construct $\overline{AC}$, where point C is a point on the perpendicular line.
4. Hide the line.
5. Construct $\overline{BC}$ to finish the right triangle.

Using the **Text** tool, click once on a segment to show its label. Double-click the label to change it.

6. Show the three segments' labels and change the labels to match the figure above right.

Select, in order, points *C*, *A*, and *B*. Then, in the Measure menu, choose **Angle**.

7. Measure angle *CAB*.

For each ratio, select the two segments in order. Then, in the Measure menu, choose **Ratio**.

8. Measure the ratios *opposite/hypotenuse*, *adjacent/hypotenuse*, and *opposite/adjacent*.

Q1 Drag point C to change the angles. When the angles change, do the ratios also change?

Q2 Drag point A or point B to scale the triangle. What do you notice about the ratios when the angles don't change? Explain why you think this happens.

Your observations in Q2 give you a useful fact about right triangles. For any right triangle with a given acute angle, each ratio of side lengths has a given value, regardless of the size of the triangle. The three ratios you measured are called *sine*, *cosine*, and *tangent*.

Choose **Calculate** from the Measure menu to open the Calculator. In the Functions pop-up menu, choose **sin**. Click in the sketch on the measure of ∠*CAB*, then click OK. Use the same process to calculate cosine and tangent.

9. The sine, cosine, and tangent functions can be found on all scientific calculators, commonly abbreviated as sin, cos, and tan. Use Sketchpad's Calculator to calculate the sine, cosine, and tangent of ∠*CAB*. Match these calculations with the ratios they are equal to.

Q3 Complete the ratios for cosine and tangent below.

$$\text{sine } \angle A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}}$$

cosine $\angle A$ = ______________________

tangent $\angle A$ = ______________________

Q4 Drag point C so that $\angle A$ measures as close to 30° as you can get it. Write approximate values for the sine, cosine, and tangent of 30° below. Use the definitions in Q3 and refer to the calculations in your sketch to find these values.

sin 30° = ________ cos 30° = ________ tan 30° = ________

Q5 Without measuring, figure out the measure of $\angle C$ and write down that number. Calculate the sine of that angle measure. The sine of $\angle C$ should be close to one of the trigonometric ratios for $\angle A$. Which one? Explain why this is so.

Q6 Drag point C and answer the following questions.

Hint: Make $\overline{AB}$ short so that you can drag point C up farther.

a. What's the smallest possible value for the sine of an angle in a right triangle? What angle has this value? ________

b. What's the greatest possible value for the sine of an angle in a right triangle? What angle has this value? ________

c. Why can't you make m$\angle CAB$ exactly equal to 90°?

d. Even though you can't make m$\angle CAB$ exactly equal to 90°, what do you think is the value of tan 90°? Explain.

e. For what angle is the tangent equal to 1? Why?

f. For what angle are the sine and cosine equal? Why?

g. Suppose an angle has measure x. Complete this equation:

$\sin x = \cos$ ____________.

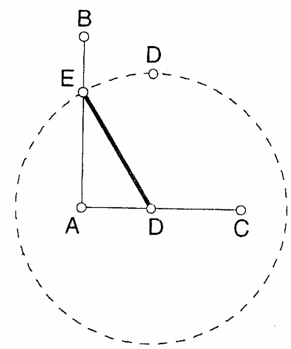
Modeling a Ladder Problem

Name(s): ____________________

Drawing diagrams is a useful method to help solve many types of realistic problems. Dynamic diagrams can be even more useful. Here's a problem that can be solved with a Sketchpad sketch.

The Occupational Safety and Health Administration (OSHA) recommends that when you use a ladder, you should lean it against a wall so that the height at which it touches the wall is four times the distance from the wall to the foot of the ladder. Any more and you risk tipping the ladder backward. Any less and you risk having the bottom slide out from under the ladder. What's the height from the floor that you can reach with a 20-foot ladder? What angle will the ladder make with the floor?

Sketch and Investigate



Choose **Preferences** from the Edit menu and go to the Units panel.

1. Set Preferences to display the Distance Units in **inches**.

Holding down the Shift key while you draw makes it easier to draw vertical and horizontal segments.

2. Construct vertical segment AB and horizontal segment AC. These segments represent the wall and the floor.
3. Construct point D on the floor. This point will be the foot of your ladder.

Select point D; then, in the Transform menu, choose **Translate**.

4. Translate point D vertically by 2 inches. The 2 inches will represent the length of your ladder, so the scale of your drawing will be 1 in. = 10 ft.
5. Construct circle DD'.
6. Construct point E where the circle intersects the wall. You may have to move point D first so that the circle and the wall intersect.
7. Construct $\overline{DE}$. This segment represents your ladder. Its length can't change because the radius of the circle is fixed at 2 inches.
8. Hide the circle and point D'.
9. Drag point D back and forth. You should see the top of the ladder move up and down the wall.

Select, in order, points E, D, and B. Then, in the Measure menu, choose **Angle**. Select points E and A; then, in the Measure menu, choose **Distance**. Repeat for AD.

10. Measure $\angle EDA$, EA, and AD. (EA represents the height on the wall that your ladder is reaching.) Calculate EA/AD.

Q1 Drag point D. Given the constraints in the problem, how high can the ladder reach? What angle does it make with the floor?

Q2 Confirm your answers using trigonometry. Show your work.

Explore More

1. Suppose a ladder is propped against one wall in the corner of a room. To one side of the ladder is another wall. A wet paintbrush rests on the center rung of the ladder, just touching the side wall. Suddenly, the foot of the ladder slips and the paintbrush falls with it, painting a streak on the side wall as it falls! What does the streak look like? To model this in your sketch, construct the midpoint of your ladder. While it's selected, choose **Trace Point** in the Display menu. Animate point D along $\overline{BC}$.
2. Select the measurements for EA and AD and choose **Plot As (x, y)** in the Graph menu. Drag the foot of the ladder. What kind of graph do you get? If you were to drag the foot of a ladder away from a wall at a constant rate, would the top of the ladder fall at a constant rate? Why or why not?
3. Write one or more other problems that could be modeled with this sketch.