PROJECT #3

The Pythagorean Theorem

Q1: This construction uses the following property: A square has right angles and congruent sides.

Q2: This construction uses the following property: A right triangle has one side perpendicular to another side.

Q3: The sum of the areas of the two smaller squares equals the area of the largest square.

Q4: $a^2 + b^2 = c^2$

Explore more:

1. The areas of the regions built on the legs of a right triangle will always sum to the area of the region built on the hypotenuse, as long as all three regions are similar.
2. If the areas of squares on two sides of a triangle sum to the area of the square on the third side, the triangle must be a right triangle.

Visual Demonstration of the Pythagorean Theorem

Q1: In this sketch, the squares on the sides of a right triangle are sheared, without changing their areas, so that a shape on the legs is congruent to a shape on the hypotenuse. This shows that the sum of the areas of the original squares on the legs of a right triangle is equal to the area of the original square on the hypotenuse, thus demonstrating the Pythagorean theorem.

Pythagorean triples

Q1: The eact length of the hypotenuse is $\sqrt{2}$.

Q2: No. If the lengths of the legs are 1cm and 2 cm, the hypotenuse noes not have a whole-number length. Its length is $\sqrt{5}$.

Q3:

a	b	c
3	4	5
4	3	5
5	12	13
6	8	10
8	6	10
9	12	15
12	16	20
15	20	25

Q4: Any reordering of the three numbers of a triple shows a triangle congruent to the original. In the list above, (3,4,5) and (4,3,5) are triples of congruent triangles, as are (6,8,10) and (8,6,10).

Q5 In the list above, (3,4,5), (6,8,10), (9, 12, 15), and (15, 20, 25) represent triples of triangles similar to each other.

Q6: There are an infinite number of Pythagorean triples because you can multiply the numbers in any truple by the same positive integer to create another triple. For example, multiply (3,4,5) by 2 to get (6,8,10). Any triple that is not the integer multiple of another triple is called a primitive Pythagorean triple.

Q7: For example,

$3^2 + 4^2 = 5^2$ because $9+16=25$

$5^2 + 12^2 = 13^2$ because $25+144=169$

$9^2 + 12^2 = 15^2$ because $81+144=225$.

Explore more:

If m=2 and n=1, Euclid's formula for Pythagorean triples gives the triple (3,4,5).
Similarly, m=3 and n=2 generate the triple (5,12,13).
You can use this formula to generate infinitely many Pythagorean triples because there are infinitely many choices for m and n.
Euclid's formula generates every primitive Pythagorean triple, but it does not generate every triple that's a multiple of a primitive. For example, you cannot generate (9, 12, 15) using Euclid's formula because there are no whole-number values of m and n such that $m^2 + n^2 = 15$.