PROJECT # 2 - DUE 03/07 25 POINTS TOTAL

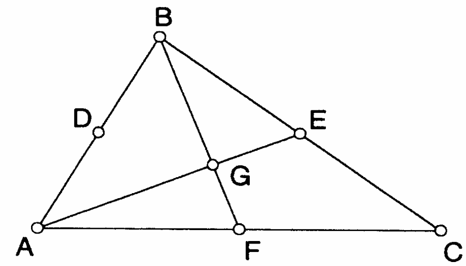
## Medians in a Triangle 5 points

**Name(s):** ____________________

A median in a triangle connects a vertex with the midpoint of the opposite side. In previous investigations, you may have discovered properties of angle bisectors, perpendicular bisectors, and altitudes in a triangle. Would you care to make a guess about medians? You may see what's coming, but there are new things to discover about medians, too.

### Sketch and Investigate

1. Construct triangle *ABC*.
2. Construct the midpoints of the three sides.
3. Construct two of the three medians, each connecting a vertex with the midpoint of its opposite side.



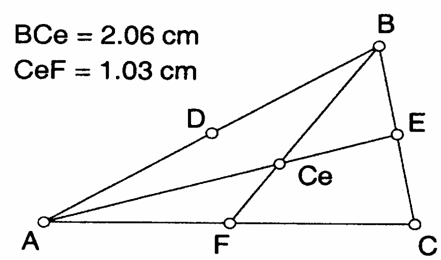

If you've already ›onstructed three medians, select ›o of them. Then, in the Construct menu, choose **Intersection**.

4. Construct the point of intersection of the two medians.
5. Construct the third median.

**Q1** What do you notice about this third median? Drag a vertex of the triangle to confirm that this conjecture holds for any triangle.

Using the **Text** ›ol, click once on ›he point to show its label. Double-click the label to change it.

6. The point where the medians intersect is called the *centroid*. Show its label and change it to *Ce* for centroid.




| BCe | CeF |
|---|---|
| 2.17 cm | 1.08 cm |
| 1.00 cm | 0.50 cm |
| 3.24 cm | 1.62 cm |
| 2.06 cm | 1.03 cm |

To measure the ›istance between ›wo points, select ›hem; then, in the Measure menu, ›ose **Distance**.

7. Measure the distance from *B* to *Ce* and the distance from *Ce* to the midpoint *F*.
8. Drag vertices of $\Delta ABC$ and look for a relationship between *BCe* and *CeF*.

Select the two measurements; then, in the Graph menu, ›ose **Tabulate**.

9. Make a table with these two measures.
10. Change the triangle and double-click the table values to add another entry.
11. Keep changing the triangle and adding entries to your table until you can see a relationship between the distances *BCe* and *CeF*.

Choose **Calculate** from the Measure menu to open the Calculator. Click once on a measurement to enter it into a calculation.

12. Based on what you notice about the table entries, use the Calculator to make an expression with the measures that will remain constant even as the measures change.

**Q2** Write the expression you calculated in step 12.

**Q3** Write a conjecture about the way the centroid divides each median in a triangle.

13. Plot the table data on a piece of graph paper or using the **Plot Points** command in the Graph menu. You should get a graph with several collinear points.

14. Draw or construct a line through any two of the data points and measure its slope.

**Q4** Explain the significance of the slope of the line through the data points.

## Explore More

1. Make a custom tool that constructs the centroid of a triangle. Save your sketch in the **Tool Folder** (next to the Sketchpad application itself on your hard drive) so that the tool will be available for future investigations of triangle centers.

PRINT OUT A COPY OF YOUR SKETCHPAD WORK FOR EACH INVESTIGATION

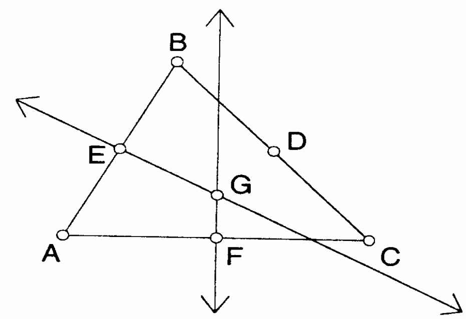
## Perpendicular Bisectors in a Triangle

5 points

**Name(s):** ______________________

In this investigation, you'll discover properties of perpendicular bisectors in a triangle. You'll also learn how to construct a circle that passes through each vertex of a triangle.

### Sketch and Investigate




1. Construct triangle *ABC*.
2. Construct the midpoints of the sides.
3. Construct two of the three perpendicular bisectors in the triangle.

   *Select a side and its midpoint; then, in the Construct menu, choose **Perpendicular Line**.*

4. Construct point *G*, the point of intersection of these lines.

   *Click at the intersection with the **Selection Arrow** tool or with the **Point** tool.*

5. Construct the third perpendicular bisector.

**Q1** What do you notice about this third perpendicular bisector (not shown)? Drag a vertex of the triangle to confirm that this conjecture holds for any triangle.

6. Drag a vertex around so that point *G* moves into and out of the triangle. Observe the angles of the triangle as you do this.

**Q2** In what type of triangle is point *G* outside the triangle? inside the triangle?

**Q3** Drag a vertex until point *G* falls on a side of the triangle. What kind of triangle is this? Where exactly does point *G* lie?

7. Measure the distances from point *G* to each of the three vertices.

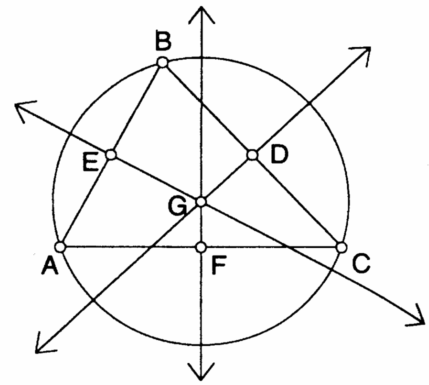
   *elect point G and a vertex; then, in the Measure menu, choose **Distance**. Repeat for the other two vertices.*

8. Drag a vertex of the triangle and observe the distances.

**Q4** The point of intersection of the three perpendicular bisectors is called the *circumcenter* of the triangle. What do you notice about the distances from the circumcenter to the three vertices of the triangle?

Make sure you start your circle at point *G* and finish it with the cursor directly over point *A*. Otherwise, the circle may not stay circumscribed when you drag. (If it doesn't, undo and try again.)

9. Construct a circle with center $G$ and radius endpoint $A$. This is the circumscribed circle of $\Delta ABC$.




## Explore More

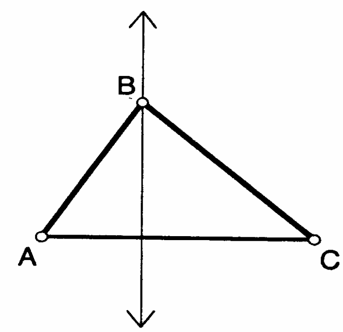
1. Make a custom tool that constructs the circumcenter of a triangle. Save your sketch in the **Tool Folder** (next to the Sketchpad application itself on your hard drive) so that the tool will be available for future investigations of triangle centers.
2. Explain why the circumcenter is the center of the circumscribed circle. *Hint:* Recall that any point on a segment's perpendicular bisector is equidistant from the endpoints of the segment. Why would the circumcenter be equidistant from the three vertices of the triangle?
3. See if you can circumscribe other shapes besides triangles. Describe what you try and include below any additional conjectures you come up with.

## Altitudes in a Triangle 5 points

**Name(s):** ____________________

In this activity, you'll discover some properties of altitudes in a triangle. An *altitude* is a perpendicular segment from a vertex of a triangle to the opposite side (or to a line containing the side). The side where the altitude ends is the *base* for that altitude, and the length of the altitude is the *height* of the triangle from that base. Because a triangle has three sides, it also has three altitudes. You'll construct one altitude and make a custom tool for the construction. Then you'll use your tool to construct the other two.

### Sketch and Investigate




1. Construct triangle $ABC$.

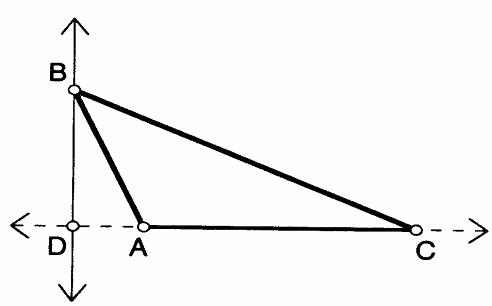
Select point $B$ and $\overline{AC}$; then, in the Construct menu, choose **Perpendicular Line**.

2. Construct a line perpendicular to $\overline{AC}$ through point $B$.

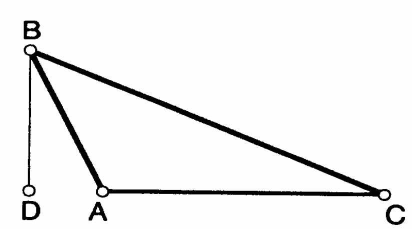
**Q1** As long as your triangle is acute, this perpendicular line should intersect a side of the triangle. Drag point $B$ so that the line falls outside the triangle. Now what kind of triangle is it?

Hold down the mouse button on the **Segment** tool and drag right to choose the **Line** tool. Construct your line through the endpoints of the triangle side.

3. With the perpendicular line outside the triangle, use a line to extend side $AC$ so that it intersects the perpendicular line.
4. Construct point $D$, the point of intersection of the extended side and the perpendicular line.
5. Hide the lines.
6. Construct $\overline{BD}$. Segment $BD$ is an altitude.




Steps 3 and 4




Steps 5 and 6

7. Drag vertices of the triangle and observe how your altitude behaves.

**Q2** Where is your altitude when $\angle A$ is a right angle?

## Altitudes in a Triangle (continued)

8. Drag your triangle so that it is acute again (with the altitude falling inside the triangle).

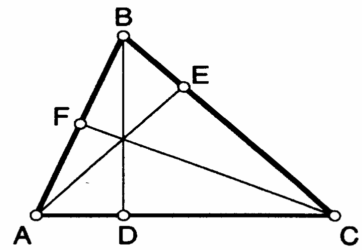
Select everything in your sketch. Then, press the **Custom** tools icon (the bottom tool in the Toolbox) and choose **Create New Tool** from the menu that appears.

9. Make a custom tool for this construction. Name the tool "Altitude."
10. Use your custom tool on the triangle's vertices to construct a second altitude. Don't worry if you accidentally construct the altitude that already exists. Just use the tool on the vertices again in a different order until you get another altitude.

Starting with three points (the tool's "givens"), the Altitude tool will construct a triangle and an altitude from one of the vertices. To use the tool, click on the **Custom** tools icon to select the most recently created tool, then click on the three triangle vertices.

11. Use your Altitude tool to construct the third altitude in the triangle.
12. Drag the triangle and observe how the three altitudes behave.

**Q3** What do you notice about the three altitudes when the triangle is acute?



Step 11

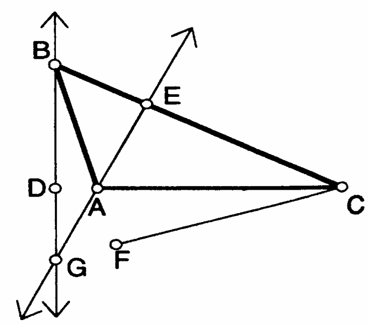
**Q4** What do you notice about the altitudes when the triangle is obtuse?

When the triangle is obtuse, the three altitudes don't intersect. But do you think they would if they were long enough? Follow the steps below to investigate that question.

13. Make sure the triangle is obtuse. Construct two lines that each contain an altitude.

If you've already constructed three lines, select two of them; then, in the Construct menu, choose **Intersection**.

14. Construct their point of intersection. This point is called the *orthocenter* of the triangle.
15. Construct a line containing the third altitude.
16. Drag the triangle and observe the lines.



Steps 13 and 14

**Q5** What do you notice about the lines containing the altitudes?

### Explore More

1. Hide everything in your sketch except the triangle and the orthocenter. Make and save a custom tool called "Orthocenter." You can use this tool in other investigations of triangle centers.

# Angle Bisectors in a Triangle

**Name(s):** ____________________

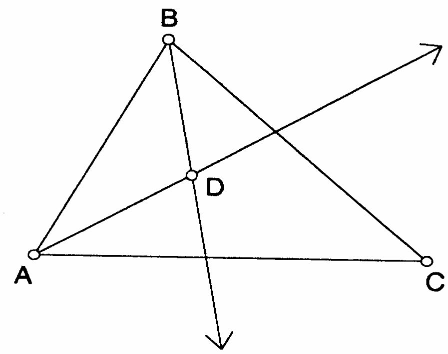
In this investigation, you'll discover some properties of angle bisectors in a triangle.

## Sketch and Investigate

**Select** three points, with the vertex your middle selection. Then, in the Construct menu, choose **Angle Bisector**.

**Click** at the intersection with the **Arrow** or the **Point** tool. Or select the two bisectors, then, in the Construct menu, choose **Intersection**.

1. Construct triangle $ABC$.
2. Construct the bisectors of two of the three angles: $\angle A$ and $\angle B$.
3. Construct point $D$, the point of intersection of the two angle bisectors.
4. Construct the bisector of $\angle C$.




Steps 1–3

**Q1** What do you notice about this third angle bisector (not shown)? Drag each vertex of the triangle to confirm that this observation holds for any triangle.

**Select** point $D$ and one side of the triangle. Then, in the Measure menu, choose **Distance**. **Repeat** for the other two sides.

5. Measure the distances from $D$ to each of the three sides.
6. Drag each vertex of the triangle and observe the distances.

**Q2** The point of intersection of the angle bisectors in a triangle is called the *incenter*. Write a conjecture about the distances from the incenter of a triangle to the three sides.

## Explore More

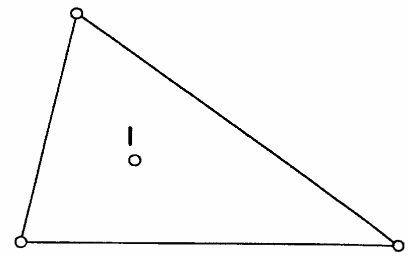
1. An inscribed circle is a circle inside a triangle that touches each of the three sides at one point. Construct an inscribed circle that stays inscribed no matter how you drag the triangle. (*Hint:* You'll need to construct a perpendicular line.)
2. Make and save a custom tool for constructing the incenter of a triangle (with or without the inscribed circle). You can use this tool when you investigate properties of other triangle centers.
3. Explain why the intersection of the angle bisectors would be the center of the inscribed circle. *Hint:* Recall that any point on an angle bisector is equidistant from the two sides of the angle. Why would the incenter be equidistant from the three sides of the triangle?

## The Euler Segment

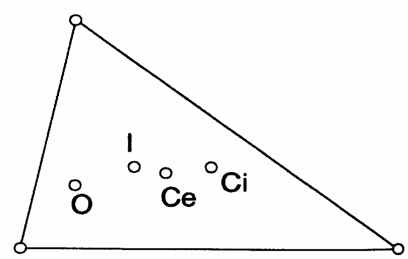
**Name(s):** ______________________

In this investigation, you'll look for a relationship among four points of concurrency: the incenter, the circumcenter, the orthocenter, and the centroid. You'll use custom tools to construct these triangle centers, either those you made in previous investigations or pre-made tools.

### Sketch and Investigate




1. Open a sketch (or sketches) of yours that contains tools for the triangle centers: incenter, circumcenter, orthocenter, and centroid. Or open **Triangle Centers.gsp**.
2. Construct a triangle.
3. Use the **Incenter** tool on the triangle's vertices to construct its incenter.
4. If necessary, give the incenter a label that identifies it, such as *I* for incenter.
5. You need only the triangle and the incenter for now, so hide anything extra that your custom tool may have constructed (such as angle bisectors or the incircle).




6. Use the **Circumcenter** tool on the same triangle. Hide any extras so that you have just the triangle, its incenter, and its circumcenter. If necessary, give the circumcenter a label that identifies it.
7. Use the **Orthocenter** tool on the same triangle, hide any extras, and label the orthocenter.
8. Use the **Centroid** tool on the same triangle, hide extras, and label the centroid. You should now have a triangle and the four triangle centers.

**Q1** Drag your triangle around and observe how the points behave. Three of the four points are always collinear. Which three?

9. Construct a segment that contains the three collinear points. This is called the *Euler segment*.

**Q2** Drag the triangle again and look for interesting relationships on the Euler segment. Be sure to check special triangles, such as isosceles and right triangles. Describe any special triangles in which the triangle centers are related in interesting ways or located in interesting places.

**Q3** Which of the three points are always endpoints of the Euler segment and which point is always between them?

To measure the distance between two points, select the two points. Then, in the Measure menu, choose **Distance**. (Measuring the distance between points is an easy way to measure the length of part of a segment.)

10. Measure the distances along the two parts of the Euler segment.

**Q4** Drag the triangle and look for a relationship between these lengths. How are the lengths of the two parts of the Euler segment related? Test your conjecture using the Calculator.

## Explore More

1. Construct a circle centered at the midpoint of the Euler segment and passing through the midpoint of one of the sides of the triangle. This circle is called the *nine-point circle*. The midpoint it passes through is one of the nine points. What are the other eight? (*Hint:* Six of them have to do with the altitudes and the orthocenter.)
2. Once you've constructed the nine-point circle, drag your triangle around and investigate special triangles. Describe any triangles in which some of the nine points coincide.

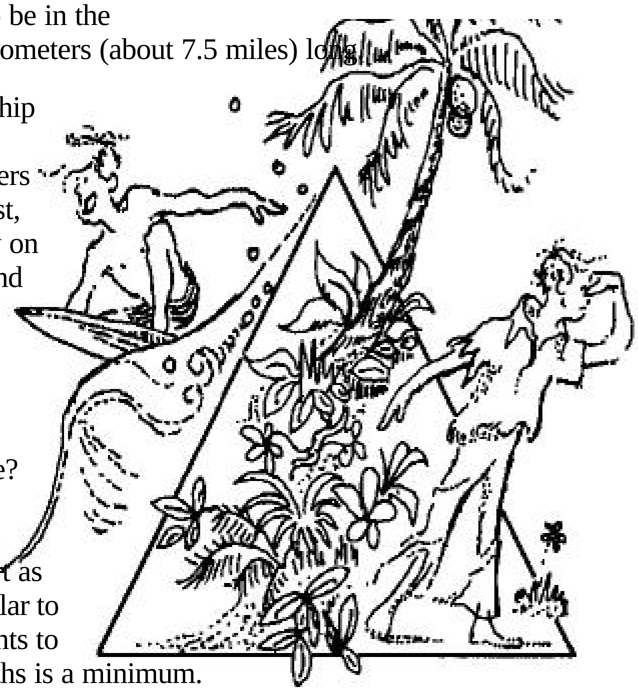
Extra Credit – 5 points

Answer the questions of the puzzle using the Sketchpad investigations. Show clearly which sketchpad investigation and , in particular, which sketchpad conjecture you're using in answering the puzzle questions.

# The Puzzles of the Surfer and the Spotter

One night a ship is wrecked in a storm at sea and only tow members of the crew survive. They manage to swim to a deserted tropical island where they fall asleep exhausted. After exploring the island the next morning, one of the men decides that he would like to stay there and spend the rest of his life surfing on the beaches. The other man, however, wants to escape and decides to use his time looking for a ship that might rescue him.

The island is overgrown with vegetation and happens to be in the shape of an **equilateral triangle,** each side being 12 kilometers (about 7.5 miles) long.



Wanting to be in the best possible position to spot any ship that might sail by, the man who hopes to escape ( we will call him the "spotter") goes to one of the corners of the island. Since he doesn't know which corner is best, he decides to rotate from one to another, spending a day on each. He wants to build a shelter somewhere on the island and a path from it to each corner so that the sum of the lengths of the three paths is a minimum. (Digging up the vegetation to clear the paths is not an easy job).
**Where should the spotter build his house?**

What about the surfer? Where should he build his house? He likes the beaches along all three sides of the island and decides to spend an equal amount of time on each. To make the paths from his house to each beach as short as possible, he constructs them so that they are perpendicular to the lines of the beaches. The surfer, like the spotter, wants to locate his house so that the sum of the lengths of the paths is a minimum.
**Where is the best place on the island for him?**