

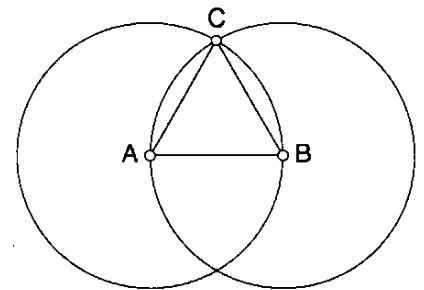
## Euclid's Proposition 1— An Equilateral Triangle

Name(s): \_\_\_\_\_

Euclid, a Greek mathematician born around 300 B.C., wrote a book called the *Elements*, upon which most school geometry books are still based. All of the geometry in the *Elements* is built up sequentially from a few simple constructions and postulates. Each new property that Euclid presents, or new figure that he constructs, is based on properties he has demonstrated previously. The construction that starts it all is the equilateral triangle. Countless other constructions in the *Elements* depend on being able to construct an equilateral triangle with compass and straightedge. In this activity, you'll construct an equilateral triangle using only Sketchpad's freehand tools—the equivalents of Euclid's compass and straightedge.

### Sketch and Investigate

1. Construct  $\overline{AB}$ .
2. Construct circle  $AB$ . (Make sure you use point  $A$  for the center and point  $B$  for the radius-defining point.)
3. Construct circle  $BA$ . (Use point  $B$  for the center and point  $A$  for the radius point this time.)



If you start or finish drawing your segment with the tip of the **Segment** tool directly over the intersection of two objects, both objects will highlight and Sketchpad will construct an endpoint at the intersection.

- 4. Construct  $\overline{AC}$  and  $\overline{CB}$ , where  $C$  is a point of intersection of the two circles.

**Q1** Drag point  $A$  or point  $B$ . What happens to your triangle? Does it appear to stay equilateral?

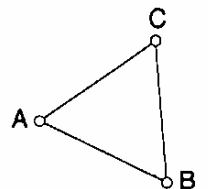
**Q2** Explain why this triangle is always equilateral. (Hints: What roles do the circles play in your construction? How are they related to one another? How are the sides of the triangle related to the circles?)

5. Hide the circles.

To measure an angle, select three points, with the vertex your middle selection. Then, in the Measure menu, choose **Angle**.

- 6. Measure the three angles.

**Q3** Drag point  $A$  or point  $B$ . Make a conjecture about the angles in an equilateral triangle.

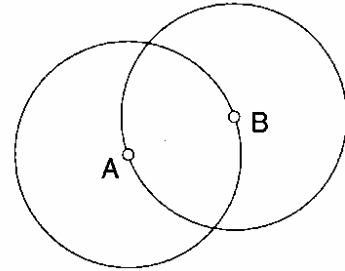


## Daisy Designs

Name(s): \_\_\_\_\_

A daisy design is a simple design that you can create using only a compass. From the basic daisy, you can create more complex designs based on the regular hexagon.

1. Construct circle  $AB$  (a circle with center  $A$  and radius point  $B$ ).
2. Construct circle  $BA$ . Be sure you start your circle with the cursor positioned at point  $B$  and that you finish your circle with the cursor positioned at point  $A$ .

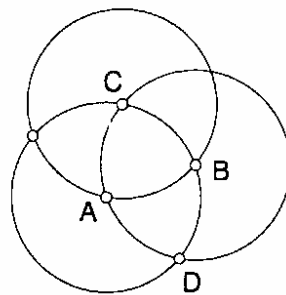


If both circles are not controlled by just points  $A$  and  $B$ , undo and try again.

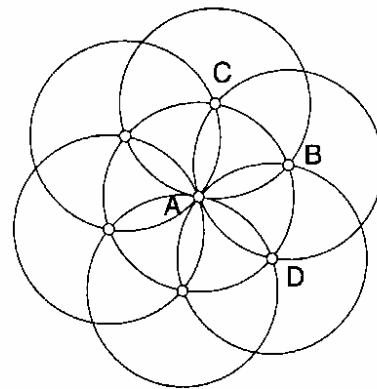
3. Drag point  $A$  and point  $B$  to confirm that both circles are controlled by these two points.
4. Construct point  $C$  and point  $D$ , the two points of intersection of these circles.

Be sure all the circles are connected by constructing them from intersections to existing points in the sketch. Your final daisy should have exactly seven points.

5. Construct a circle from point  $C$  to point  $A$ .
6. Continue constructing circles from new intersection points to point  $A$ . All these circles should have equal radii. The last circle you construct should be centered at point  $D$ . When you're done, your sketch should look like the figure below right. You should be able to drag it without making it fall apart.



Step 5



Step 6

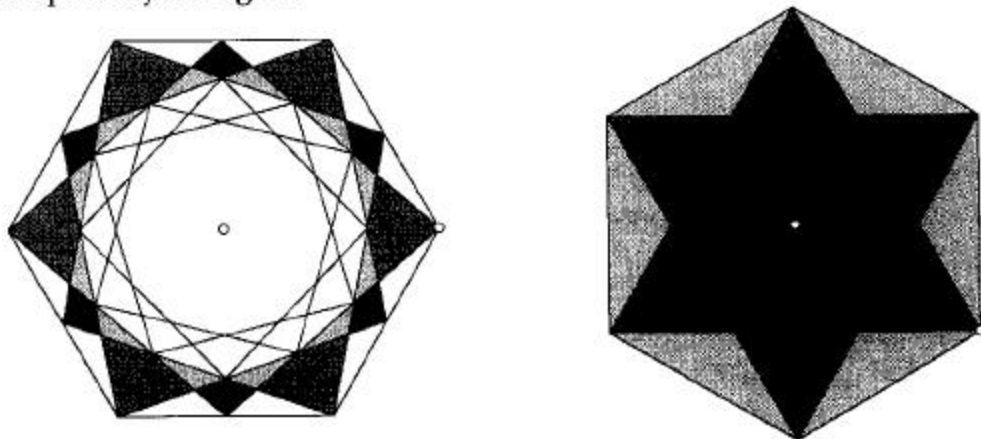
7. Use the **Segment** tool to add some lines to your design; then drag point  $B$  and observe the way your design changes.

The six points of your daisy (besides point  $A$ ) define six vertices of a regular hexagon. You can use these points as the basis for hexagon or star designs like those shown on the next page.

You could construct polygon interiors and experiment with color. You could also construct arcs (select a circle and two points on it) and arc sector and arc segment interiors (select an arc). However, you can

## Daisy Designs (continued)

probably get better results by printing out the basic line design and adding color and shading by hand. Once you have all the lines and polygon interiors you want, you can hide unneeded points. Don't hide your original two points, though, because you can use these points to manipulate your figure.



For tips on making and using custom tools, choose **Toolbox** from the Help menu, then click on the Custom Tools link.

### Explore More

1. Use the daisy construction to create a custom tool for a regular hexagon. Save the new tool in the Tool Folder (next to the application itself on your hard drive) so it can be used in any open sketch.