Circles
Sections 6.1 & 6.2

The many practical uses of the circle range from the wheel to the near-circular orbits of some communication satellites. The mechanical uses of the circle have been known for thousands of years, and the ancient Greeks contributed significantly to our understanding of the circle's mathematical properties. The full moon, ripples in a pond when a stone is dropped in, and the shape of some bird's nests show some of the circles that appear in nature.

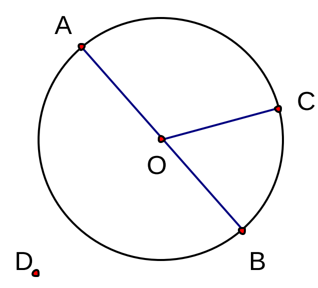
Our study of circles begins with some definitions, an explanation of the standard symbols used, and certain figures related to circles.

Definition A **circle** is the set of all points in a plane that are at a given distance from a given point in the plane.

The given distance = ______________________

The given point = _________________________

Notation: _________________________________



Note: A circle divides the plane into three distinct subsets:

- the interior ______________
- the circle itself ______________
- the exterior ______________

Note: The radius of a circle is defined above as a number. It is standard practice, however, for "radius" to also mean a line segment, as in the following definition. You can usually determine which meaning of the word " radius" is intended by the context in which it is used.

Definition A **radius** of a circle is a segment that joins the center of the circle to a point on the circle. _________

Definition A **diameter** of a circle is a segment whose endpoints are points of the circle and it contains the center of the circle. _________

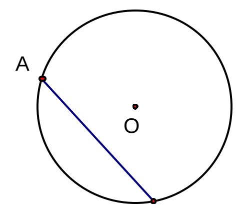
Theorem In any given circle all radii are congruent and all diameters are congruent.
(radii $\odot \cong$ and diams $\odot \cong$)

Definition Two or more **circles are congruent** if ____
($\odot s \cong$ iff radii $\cong$).

Definition Two or more coplanar **circles are concentric** if ______________________________________

Question: How many circles can share the same center? _______________________________________

Definition A line segment is a **chord** of a circle if its endpoints are points of the circle.



Questions: 1) Is a diameter a chord? ____________

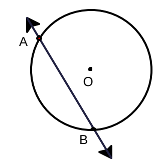
2) Is a radius of chord? ____________

3) What is the longest possible chord? ________________

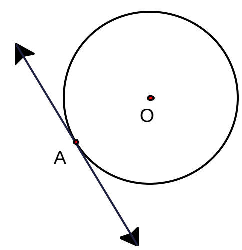
4) How is the length of a chord related to its distance from the center? _____________________

____

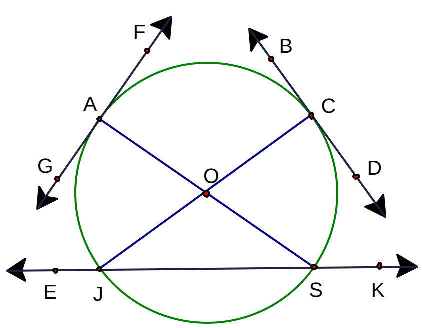
Definition A line (or segment or ray) is a **secant** if it intersects a circle at exactly twp points.



Definition A line is a **tangent** to a circle if it intersects the circle at exactly one point.



Problem #1 In the given figure, name:



a) fours radii

b) two diameters

c) three chords

d) two tangents

e) one secant

Several types of angles associated with circles are seen in the above figure. The next definition describes the most fundamental of these angles.

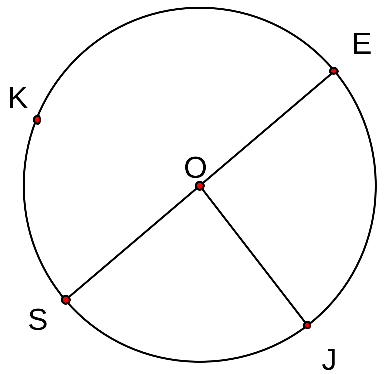
Definition An angle is a **central angle** of a circle if ____

A central angle may be - acute _____________

- right

- obtuse (measure less than 180°) _______________

These angles "cut off" portions of the circle called arcs.



Definition A **minor arc** is the set of points of a circle that are on a central angle or in its _________________.

Example: ___________________________________

Definition The **intercepted arc of an angle** is the minor arc associated with the central angle.

Example: What is the intercepted arc of $\angle SOJ$?

Definition A **major arc** is the set of points of a circle that are on a central angle or in its _________________.

Example: ___________________________________

Definition A **semicircle** is the set of points of a circle that are on, or are on one side of, a line containing a

____

Example: ___________________________________

Definition The degree measure of

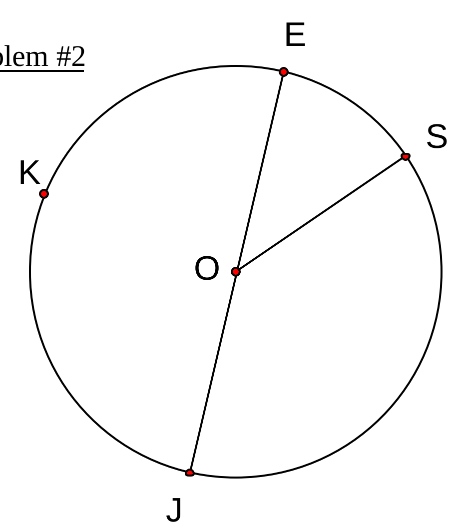
a) a minor arc is the measure of its central angle (also known as **The Central Angle Postulate**),

b) a semicircle is 180°,

c) a circle is 360°,

d) a major arc is 360° minus the measure of its associated minor arc.

Problem #2



Given: $\odot O$

$m\angle EOS = 41^\circ$

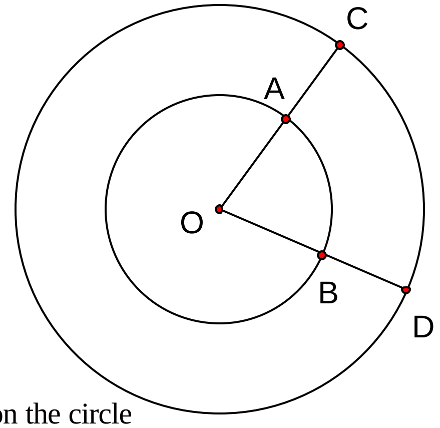
Find: $m\overset{\frown}{ES}$ ______________________________

$m\overset{\frown}{ESJ}$ ______________________________

$m\angle SOJ$ ______________________________

$m\overset{\frown}{SJ}$ ______________________________

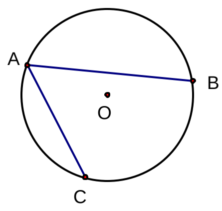
$m\overset{\frown}{EKS}$ ______________________________



Note: The degree measure of an arc is not a measure of the arc's length.

For the concentric circles in the figure,
$m\overset{\frown}{AB} = m\overset{\frown}{CD}$ because the arcs have the same central angle,
but certainly $\overset{\frown}{AB}$ is not as long as $\overset{\frown}{CD}$.

Definition An angle is an **inscribed angle** of a circle if its vertex is a point on the circle and its sides are chords of the circle.



Central Angles, Arcs, and Chords

There are some important properties about central angles, arcs, and chords that are associated with a given circle or with two circles that are the same size. But what is meant by "the same size"? ________________________________

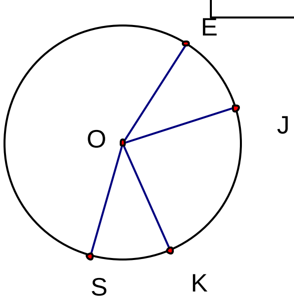
Definition Two **arcs** of a circle or of congruent circles are congruent iff their degree measures are equal.

Note: Since congruent arcs are defined in terms of numbers (degree measures), the addition, subtraction, multiplication, and division properties of congruence may be easily extended to include congruence between arcs.

Theorems relating central angles, arcs, and chords in the same or congruent circles

Theorem 1 (6.1 – T 6.1.3)

If two minor arcs of a circle or of congruent circles are congruent, then their central angles are congruent (if $\overset{\frown}{s} \cong$, central $\angle s \cong$).



Converse 1 (Converse of Theorem 1)
(6.1 – T 6.1.4)

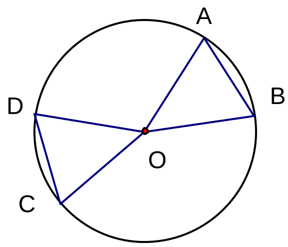
(if central $\angle s \cong$, $\overset{\frown}{s} \cong$)

Theorem 2

If two central angles in a circle or in congruent circles are congruent, then their chords are

_____________________ (if central $\angle s \cong$, chords $\cong$).

Write a formal proof.



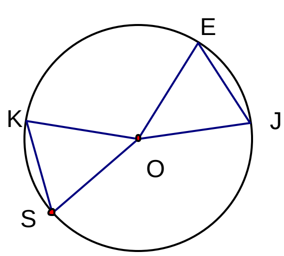
Converse 2 (Converse of Theorem 2)

(if chords $\cong$, central $\angle s \cong$).

Theorem 3
(6.1 – T 6.1.5)

If two chords in a circle or in congruent circles are congruent, then their arcs are _____________ (if chords $\cong$, $\frown s \cong$).

Write a formal proof.



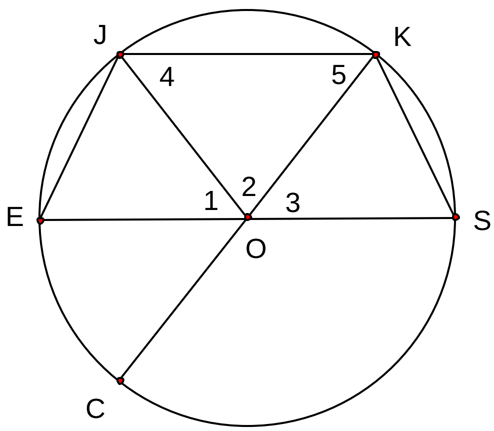
Converse 3 (Converse of Theorem 3)
(6.1 – T. 6.1.6)

(if $\frown s \cong$, chords $\cong$).

The above six theorems are summarized in the following diagram:

$\cong$ central angles $\leftrightarrow$ $\cong$ arcs $\leftrightarrow$ $\cong$ chords.

Problem 4



Given: $\odot O$

$\overline{ES} \parallel \overline{JK}$

$m\overset{\frown}{JK} = 78^\circ$

$JE = 1.5$ cm

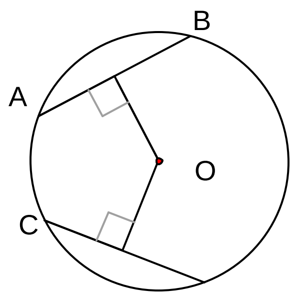
Find:

a) $\angle s1-5$

b) $m\overset{\frown}{JE}$, $m\overset{\frown}{KS}$, KS, $m\overset{\frown}{JC}$

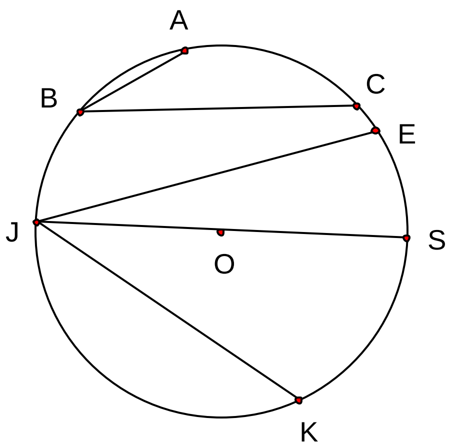
Theorem 4 (6.1 – T. 6.1.7 and T. 6.1.8)

Chords are at the same distance from the center of a circle if and only if they are congruent.



Inscribed Angles

There are three different types of inscribed angles when considered in relation to the center of the circle.



1) One side of the angle may contain a diameter, as do ________________

2) The circle's center may be in the angle's interior as is the case for _____

3) The center may be in the angle's exterior as it is for ________________

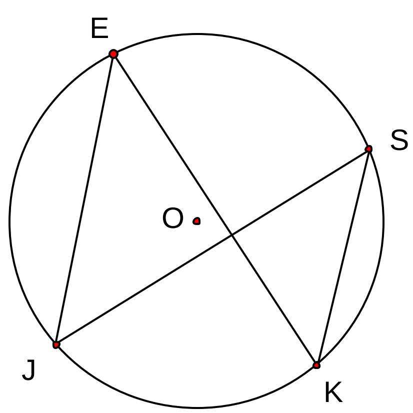
Theorem 1
(6.1 – T 6.1.2)

The measure of an inscribed angle is equal to one-half the degree measure of its intercepted arc. (inscr $\angle = \frac{1}{2}\frown$)

Example:

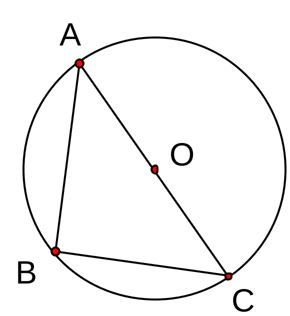
Theorem 2
(6.1 – T 6.1.10)

If two inscribed angles in a circle intercept the same arc or congruent arcs, then the angles are congruent (inscr $\angle s$ intercept same $\frown$ or $\cong$ $\frown s$ are $\cong$).



Theorem 3
(6.1 – T 6.1.9)

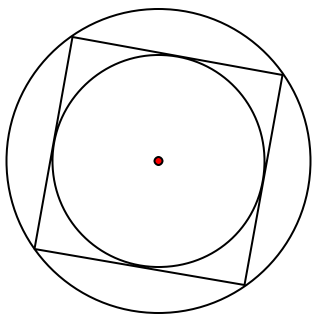
If an inscribed angle intercepts a semicircle, then it is a _______ angle (inscr $\angle$ interc semi$\odot$ is rt$\angle$).



Polygons inscribed in a circle

Definition Any **polygon is inscribed in a circle** if and only if all its vertices are points of the circle; the **circle is** said to be **circumscribed about the polygon**.

Also, a **circle is inscribed in a polygon** if and only if it is tangent to each of the polygon's sides.

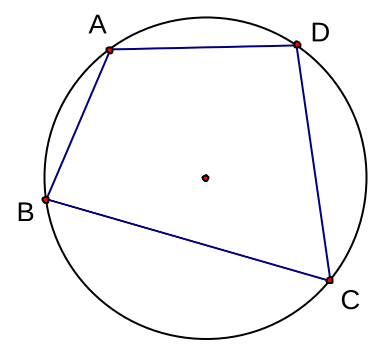


Example:

- the square is inscribed in the ______________________________
- the larger circle is __________________________ about the square.
- the ________________________________ is inscribed in the square.

Theorem (6.2 – T 6.2.1)

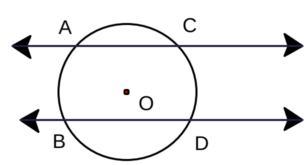
> If a quadrilateral is inscribed in a circle, then its opposite angles are _________________________ (if quad inscr in $\odot$, opp $\angle$s supp).



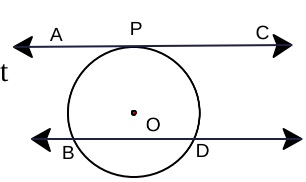
Theorem (6.2 – T 6.2.8)

> If two parallel lines intersect a circle, then the arcs of the circle between the parallel lines are congruent (if || lines intersect $\odot$, $\overset{\frown}{}s$ $\cong$).

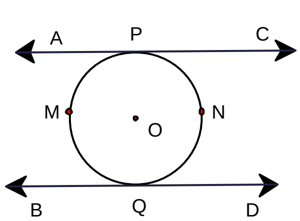
a) two secants



b) secant and tangent



c) two tangents



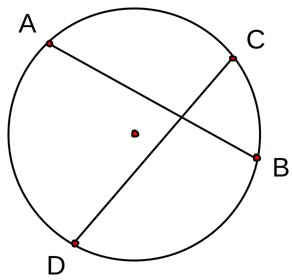
Chords, Tangents, and Secants

Theorem 1
(6.2 – T 6.2.2)

The measure of an angle formed by two chords that intersect within a circle is

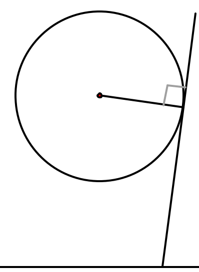
____

(2 chords $\angle = \frac{1}{2}$ sum $\frown$'s).



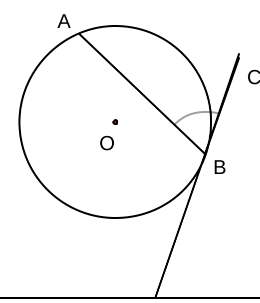
Theorem 2
(6.2 – T 6.2.3)

If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of contact (tan $\perp$ rad to point contact).



Corollary
(6.2 – T 6.2.4)

The measure of an angle formed by a tangent to a circle and a chord drawn to the point of tangency is on-half the measure of its intercepted arc.



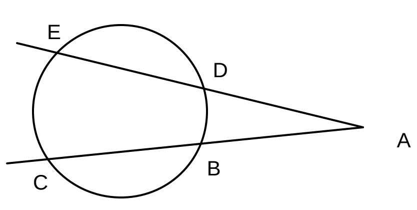
Theorem3
(6.2 – T 6.2.5, T 6.2.6, T 6.2.7)

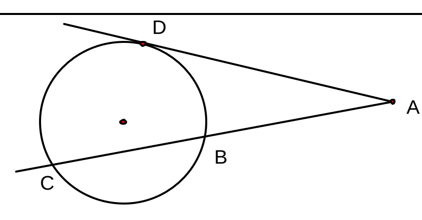
If an angle is formed by

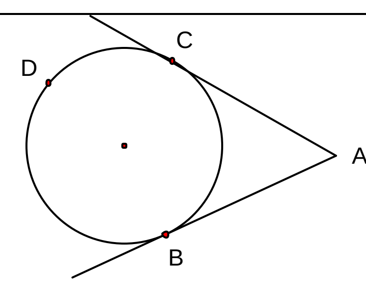
- two secants
  or
- a tangent and a secant
  or
- two tangents

intersecting in the exterior of the circle, then the measure of the angle is ________________________

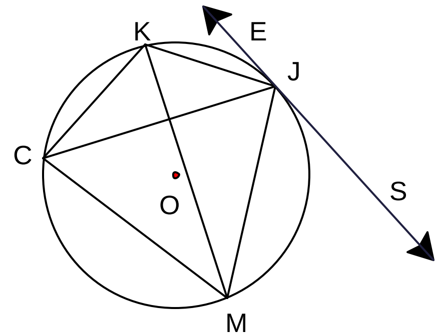
____







Problem 5 Use the figure to answer the questions.



Given $\odot O$

tan $\overleftrightarrow{ES}$

a) Name two angles congruent to $\angle KJE$.

b) Name two angles congruent to $\angle JCM$.

c) Name three angles supplementary to $\angle KJS$.

d) Name one angle supplementary to $\angle KCM$.

Problem 6 Given $\odot O$

$m\widehat{EJ} = 88^\circ$

$m\widehat{KS} = 74^\circ$

$m\angle 8 = \frac{1}{3} m\angle 2$

Find $m\angle 1-8$