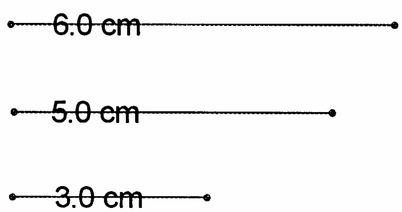
Construction 1
(3.1 - Example 1)

Construct a triangle whose sides have the given lengths.

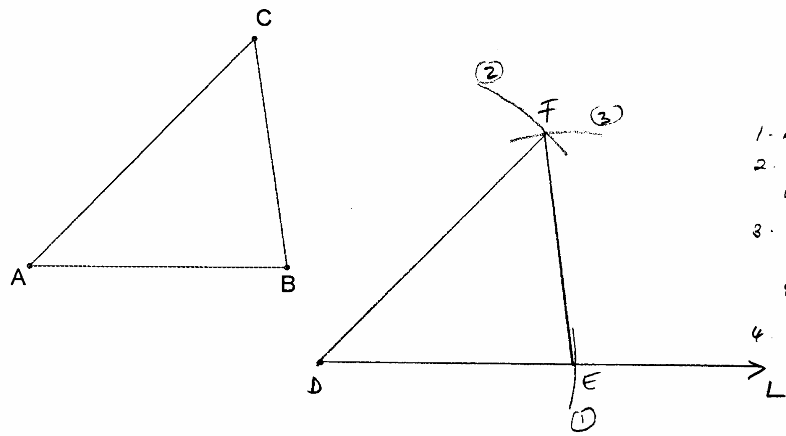


see textbook
section 3.1, page 111
example 1

Construction 2

Construct a triangle having its sides congruent to the corresponding parts of a given triangle.

Given: $\triangle ABC$
construct: $\triangle DEF$ with $\overline{DE} \cong \overline{AB}$, $\overline{EF} \cong \overline{BC}$, $\overline{FD} \cong \overline{CA}$

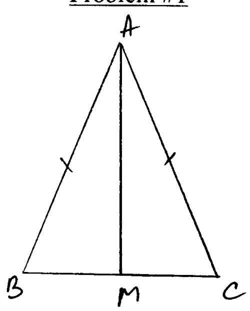


steps:

1. draw $\overrightarrow{DL}$ and construct $\overline{DE} \cong \overline{AB}$
2. with D as center and $\overline{AC}$ as radius, draw an arc on one side of $\overrightarrow{DL}$
3. with E as center and $\overline{BC}$ as radius, draw an arc intersecting the arc of step 2, this determines vertex F.
4. draw $\overline{EF}$ and $\overline{DF}$

Problem #1

Given an isosceles triangle ABC with base $\overline{BC}$ and M the midpoint of the base, show that $\triangle ABM \cong \triangle ACM$.



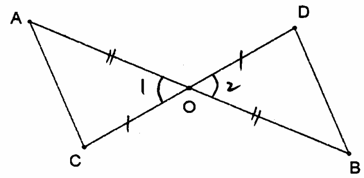
Given: $\triangle ABC$ isosceles ($\overline{BC}$ base)
M - midpoint of $\overline{BC}$

Prove: $\triangle ABM \cong \triangle ACM$

Proof

Statements	Reasons
1. $\triangle ABC$ - isosceles	1. given
2. $\overline{AB} \cong \overline{AC}$	2. definition of isosceles triangle
3. M = midpoint of $\overline{BC}$	3. given
4. $\overline{BM} \cong \overline{MC}$	4. definition of midpoint
5. $\triangle ABM$, $\triangle ACM$: $\overline{AB} \cong \overline{AC}$; $\overline{BM} \cong \overline{MC}$; $\overline{AM} \cong \overline{AM}$	5. (2) above; (4) above; reflexive property of $\cong$
6. $\triangle ABM \cong \triangle ACM$	6. SSS

Problem #2



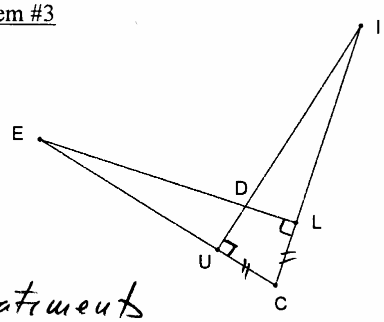
Given $\overline{AB}$ bisects $\overline{CD}$
$\overline{CD}$ bisects $\overline{AB}$

Prove $\triangle AOC \cong \triangle BOD$

Proof

Statements	Reasons
1. $\overline{AB}$ bisects $\overline{CD}$	1. given
2. $\overline{CO} \cong \overline{DO}$	2. definition of segment bisector
3. $\overline{CD}$ bisects $\overline{AB}$	3. given
4. $\overline{AO} \cong \overline{BO}$	4. definition of segment bisector
5. $\overline{AB} \cap \overline{CD} = \{O\}$	5. given
6. $\angle 1 \cong \angle 2$	6. vertical angles
7. $\triangle AOC$, $\triangle BOD$ $\begin{cases} \overline{CO} \cong \overline{DO} \\ \overline{AO} \cong \overline{BO} \\ \angle 1 \cong \angle 2 \end{cases}$	7. $\begin{cases} (2) \\ (4) \\ (6) \end{cases}$
8. $\triangle AOC \cong \triangle BOD$	8. SAS

Problem #3



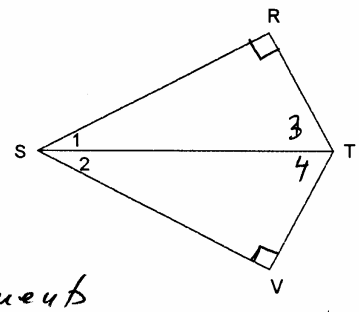
Given $\overline{IU} \perp \overline{EC}$
$\overline{EL} \perp \overline{IC}$
$\overline{CL} \cong \overline{CU}$

Prove $\triangle ECL \cong \triangle ICU$

Proof

Statements	Reasons
1. $\overline{IU} \perp \overline{EC}$	1. given
2. $m\angle U = 90^\circ$	2. $\perp$ lines form right $\angle$'s ($\perp$ iff right $\angle$)
3. $\overline{EL} \perp \overline{IC}$	3. given
4. $m\angle L = 90^\circ$	4. $\perp$ lines form right $\angle$'s ($\perp$ iff right $\angle$)
5. $m\angle U = m\angle L$ (2,4)	5. transitivity
6. $\angle U \cong \angle L$	6. definition of congruent angles
7. $\triangle ICU$, $\triangle ECL$ $\begin{cases} \angle U \cong \angle L \\ \overline{CU} \cong \overline{CL} \\ \angle C \cong \angle C \end{cases}$	7. $\begin{cases} (6) \\ \text{given} \\ \text{reflexive property of} \cong \end{cases}$
8. $\triangle ICU \cong \triangle ECL$	8. ASA

Problem #4
(3.2 - #5)



If $\angle R$ and $\angle V$ are right angles and $\angle 1 \cong \angle 2$, prove that $\triangle RST \cong \triangle VST$.

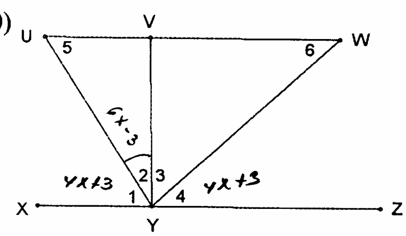
Given: $\angle R$ and $\angle V$ = right $\angle$'s
$\angle 1 \cong \angle 2$

Prove: $\triangle RST \cong \triangle VST$

Proof

Statements	Reasons
1. $\angle R$ and $\angle V$ = right angles	1. given
2. $\angle R \cong \angle V$	2. All right angles are congruent
3. $\triangle RST$, $\triangle VST$: $\overline{ST} \cong \overline{ST}$; $\angle R \cong \angle V$; $\angle 1 \cong \angle 2$	3. reflexive property of $\cong$; (2); given
4. $\triangle RST \cong \triangle VST$	4. AAS

Problem #5
(3.2 - #10)



Given $\overline{UW} \parallel \overline{XZ}$ $\quad m\angle 1 = m\angle 4 = 4x+3$
$\overline{VY} \perp \overline{UW}$ $\quad m\angle 2 = 6x-3$
$\overline{VY} \perp \overline{XZ}$

Find The measures of angles 1 through 6.

Proof

$\overline{VY} \perp \overline{XZ} \Rightarrow m\angle VYX = 90^\circ$

$m\angle 1 + m\angle 2 = 90^\circ$

$4x+3+6x-3 = 90$

$10x = 90$

$x = 9$

$m\angle 1 = 4x+3 = 39^\circ$

$m\angle 4 = 39^\circ$ ($m\angle 1 = m\angle 4$ given)

$m\angle 2 = 6x-3 = 51^\circ$

$m\angle 3 + m\angle 4 = 90^\circ$

$m\angle 3 = 90-39$

$m\angle 3 = 51^\circ$

$m\angle 5 = m\angle 1$ (alternate interior)

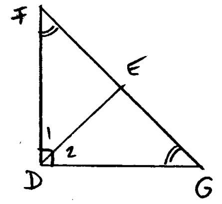
$m\angle 5 = 39^\circ$

$m\angle 6 = m\angle 4$ (alt int)

$m\angle 6 = 39^\circ$

Problem #6
(3.2 - #24)

In a right triangle *FDG* with right angle *D*, the bisector of angle *D* intersects the hypotenuse at *E*. The acute angles of the triangle are congruent. Prove that *E* is the midpoint of the hypotenuse.



Given: $\triangle FDG$
$\angle D$ = right
$\overrightarrow{DE}$ bisects $\angle FDG$
$E \in \overline{FG}$
$\angle F \cong \angle G$

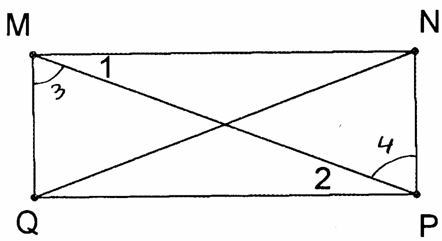
Prove: E = midpoint of $\overline{FG}$

(condition: $\overline{FE} \cong \overline{EG}$)

Proof

Statements	Reasons
1. $\overrightarrow{DE}$ bisects $\angle FDG$	1. given
2. $\angle 1 \cong \angle 2$	2. definition of angle bisector
3. $\triangle FDE$, $\triangle GDE$: $\angle 1 \cong \angle 2$; $\angle F \cong \angle G$; $\overline{DE} \cong \overline{DE}$	3. (2); given; reflexive prop
4. $\triangle FDE \cong \triangle GDE$	4. AAS
5. $\overline{FE} \cong \overline{GE}$	5. CPCTC
6. E = midpoint of $\overline{FG}$	6. definition of midpoint.

Problem #7
(3.2 - #27)



Given $\angle 1 \cong \angle 2$
$\overline{MN} \cong \overline{QP}$

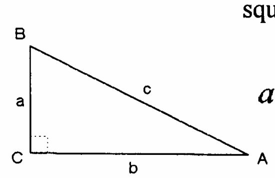
Prove $\overline{MQ} \parallel \overline{NP}$

(condition: with MP-transversal, show $\angle 3 \cong \angle 4$)

Proof

Statements	Reasons
1. $\triangle MNP$, $\triangle PQM$: $\angle 1 \cong \angle 2$; $\overline{MN} \cong \overline{PQ}$; $\overline{MP} \cong \overline{MP}$	1. given; given; reflexive prop of $\cong$
2. $\triangle MNP \cong \triangle PQM$	2. SAS
3. $\angle 4 \cong \angle 3$	3. CPCTC
4. $\overline{MQ} \parallel \overline{NP}$	4. Alt. int. $\angle$'s congruent iff parallel lines cut by transversal

The Pythagorean Theorem In any right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.



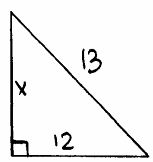
$$a^2 + b^2 = c^2$$

Note that the converse of the Pythagorean theorem is also true; that is, if the lengths *a, b*, and *c* of the three sides of a triangle are such that $a^2 + b^2 = c^2$, then the triangle is a right triangle with its right angle opposite side *c*.

Problem #8

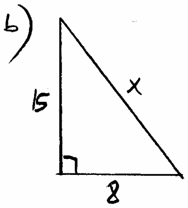
a) In a right triangle the hypotenuse is 13 in and one leg is 12 in. Find the other leg.
b) In a right triangle, one leg is 8cm and the other one is 15 cm. Find the hypotenuse.

a)



let x = the other leg
Pythagorean th:
$x^2 + 12^2 = 13^2$
$x^2 + 144 = 169$
$x^2 = 25$
$\boxed{x = 5}$

b)

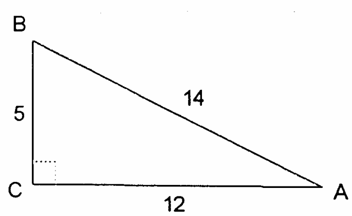


let x = the hypotenuse
Pythagorean theorem:
$8^2 + 15^2 = x^2$
$x^2 = 64 + 225$
$x^2 = 289$
$\boxed{x = 17}$

Problem #9

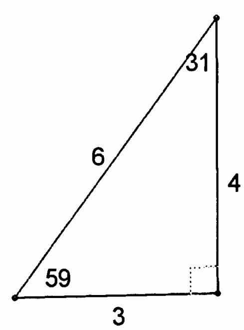
Explain what is wrong in each figure.

a)



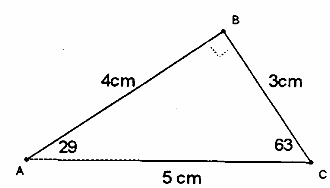
$12^2 + 5^2 = 169 \neq 14^2$

b)



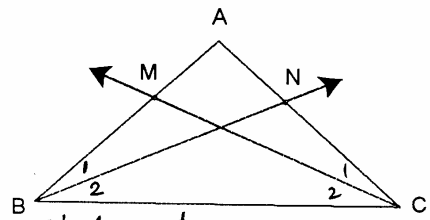
$3^2 + 4^2 = 25 \neq 6^2$

c)



$29^\circ + 63^\circ = 92^\circ \neq 90^\circ$

Problem #10

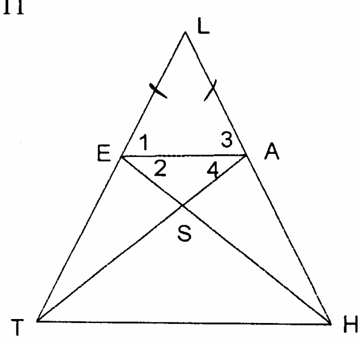


Given $\angle ABC \cong \angle ACB$
$\overrightarrow{BN}$ bis $\angle ABC$
$\overrightarrow{CM}$ bis $\angle ACB$

Prove $\triangle BMC \cong \triangle CNB$

Statements	Reasons
1. $\overrightarrow{BN}$ bisects $\angle ABC$	1. given
2. $m\angle B_2 = \frac{1}{2} m\angle ABC$	2. bisector divides angle into 2 $\cong$ angles.
3. $\overrightarrow{CM}$ bisects $\angle ACB$	3. given
4. $m\angle C_2 = \frac{1}{2} m\angle ACB$	4. bis ÷ $\measuredangle$ into 2 $\cong$ $\angle$'s
5. $\angle ABC \cong \angle ACB$	5. given.
6. $m\angle ABC = m\angle ACB$	6. def. of $\cong$ angles
7. $m\angle B_2 = m\angle C_2$; $\angle B_2 \cong \angle C_2$ (2,4,6)	7. $\frac{1}{2}$s of $\cong$ $\angle$'s are $\cong$.
8. $\triangle BMC$, $\triangle CNB$: $\overline{BC} \cong \overline{BC}$	8. reflexive prop. of $\cong$
$\angle ABC \cong \angle ACB$	given
$\angle C_2 \cong \angle B_2$	(7)
9. $\triangle BMC \cong \triangle CNB$	9. ASA

Problem #11



Given $\angle 3 \cong \angle 1$
$\angle 4 \cong \angle 2$
$\triangle EAL$ isosceles ($\overline{EA}$ base)

Prove $\overline{TA} \cong \overline{HE}$

Statements	Reasons
1. $\angle 3 \cong \angle 1$	1. given
2. $m\angle 3 = m\angle 1$	2. def. of $\cong$ $\angle$'s
3. $\angle 4 \cong \angle 2$	3. given
4. $m\angle 4 = m\angle 2$	4. def of $\cong$ $\angle$'s
5. $m\angle 3 + m\angle 4 = m\angle 1 + m\angle 2$ (2,3)	5. + prop. of =
6. $m\angle LAT = m\angle 3 + m\angle 4$; $m\angle LEH = m\angle 1 + m\angle 2$	6. Angle-Addition Postulate.
7. $m\angle LAT = m\angle LEH$ (5,6)	7. transitivity / substitution
8. $\angle LAT \cong \angle LEH$	8. def. of $\cong$ $\angle$'s
9. $\triangle EAL$ isosceles ($\overline{EA}$ base)	9. given
10. $\overline{EL} \cong \overline{LA}$	10. def. of isosc. $\triangle$
11. $\triangle LAT$, $\triangle LEH$: $\angle L \cong \angle L$	11. reflexive $\cong$
$\overline{LA} \cong \overline{LE}$	(10)
$\angle LAT \cong \angle LEH$	(8)
12. $\triangle LAT \cong \triangle LEH$	12. ASA
13. $\overline{TA} \cong \overline{HE}$	13. CPCTC