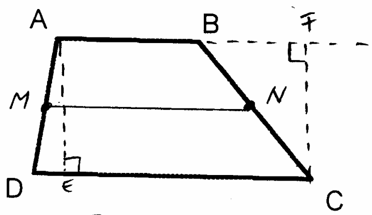
4.4 The Trapezoid

Definition A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. $\overline{AB} \parallel \overline{DC}$



$\overline{AB} \parallel \overline{DC}$

Bases: $\overline{AB}$, $\overline{DC}$

Legs: $\overline{AD}$, $\overline{BC}$

Base angles: $\angle D \,\&\, \angle C$ and $\angle A \,\&\, \angle B$ (base angles exist in pairs)

Median: $\overline{MN}$ (when the midpoints of the legs are joined)

Altitude: $\overline{AE}$, $\overline{CF}$

altitude = line segment from one vertex of one base to the opposite base (or an extension of that base)

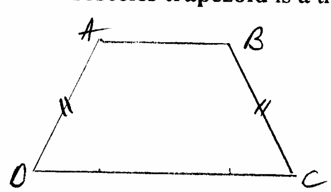
Questions: 1. Can you find any relationships between the angles of the trapezoid?

$m\angle A + m\angle D = 180^\circ$, $m\angle B + m\angle C = 180^\circ$, $m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$

2. Can a trapezoid have all of its angles acute angles? Why or why not?

No. Then the sum of the angles would be less than 360°. (not possible) ✓

Definition An **isosceles trapezoid** is a trapezoid with the nonparallel sides (legs) congruent.



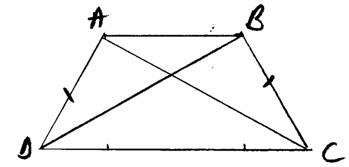
$\overline{AD} \cong \overline{BC}$

$\overline{AB} \parallel \overline{DC}$

Properties of isosceles trapezoids

Theorem 1 (4.4 – T 4.4.1) $\boxed{\text{The base angles of an isosceles trapezoid are congruent.}}$ (base ∡'s isos. trap ≅)

see textbook page 186



Corollary 1 (4.4 – C 4.4.2)

The diagonals of an isosceles trapezoid are congruent.

(diag isos. trap. $\cong$)

Given $ABCD$
$\overline{AB} \parallel \overline{DC}$
$\overline{AD} \cong \overline{BC}$

Prove $\overline{AC} \cong \overline{BD}$

Proof

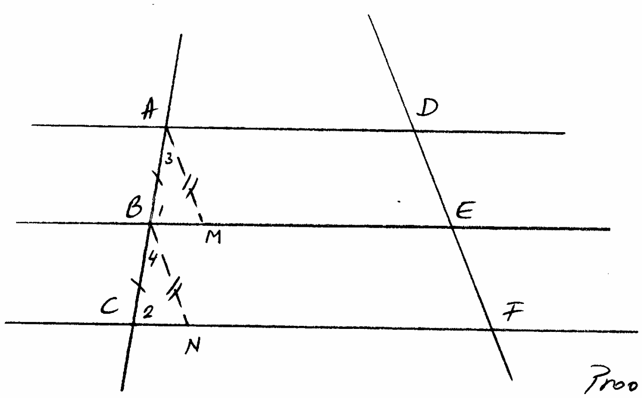
1. $ABCD$ isos trapezoid	1. Given
2. $\angle D \cong \angle C$	2. Base $\angle$'s isos trap $\cong$
3. $\triangle ADC$, $\triangle BCD$: $\overline{DC} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$, $\angle D \cong \angle C$	3. reflexive prop. $\cong$; given; (2)
4. $\triangle ADC \cong \triangle BCD$	4. SAS
5. $\overline{AC} \cong \overline{BD}$	5. CPCTC

Theorem (4.4 – T 4.4.8)

If three (or more) parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

(if 3 || lines cut $\cong$ segm 1 trans, then $\cong$ segm every trans)

Write a formal proof .



Given $\overleftrightarrow{AD} \parallel \overleftrightarrow{BE} \parallel \overleftrightarrow{CF}$
$\overline{AB} \cong \overline{BC}$

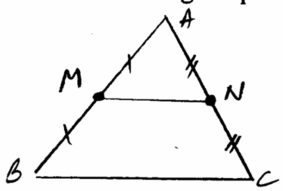
Prove $\overline{DE} \cong \overline{EF}$

Proof

1. $\overleftrightarrow{AD} \parallel \overleftrightarrow{BE} \parallel \overleftrightarrow{CF}$	1. given
2. $\angle 1 \cong \angle 2$	2. corresponding $\angle$'s ($\overleftrightarrow{BE} \parallel \overleftrightarrow{CF}$ with transv. $\overleftrightarrow{AC}$)
3. thru A draw $\overline{AM} \parallel \overline{DE}$; thru B draw $\overline{BN} \parallel \overline{DE}$	3. Parallel Postulate
4. $\overline{AM} \parallel \overline{BN}$	4. 2 lines // 3rd line //
5. $\angle 3 \cong \angle 4$	5. corresponding $\angle$'s ($\overline{AM} \parallel \overline{BN}$ with transv. $\overline{AC}$)
6. $\triangle ABM$, $\triangle BCN$: $\overline{AB} \cong \overline{BC}$, $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	6. given; (2) above; (5) above
7. $\triangle ABM \cong \triangle BCN$	7. ASA
8. $\overline{AM} \cong \overline{BN}$	8. CPCTC
9. $ADEM$ - parallelogram	9. $\square$ iff opp. sides //
10. $\overline{AM} \cong \overline{DE}$	10. opp sides $\square$ $\cong$
11. $BEFN$ - parallelogram	11. $\square$ iff. opp. sides //
12. $\overline{BN} \cong \overline{EF}$	12. opp. sides $\square$ $\cong$
13. $\overline{DE} \cong \overline{EF}$	13. transitivity

(8, 10, 12)

<u>Recall</u> : The segment that joins the midpoints of two sides of a triangle is <u>parallel</u> to the third side and has a length equal to <u>half of the length of the 3rd side</u>.



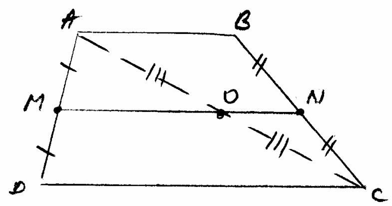
$\triangle ABC$
If M, N = midpoints, then $\overline{MN} \parallel \overline{BC}$ and $MN = \frac{1}{2} BC$

Theorem 2 (4.4 – T 4.4.3)

The length of the median of a trapezoid equals one-half the sum of the lengths of the two bases.

Write a formal proof.

(informal) Proof



Given: $ABCD$ trap; M - midpoint $\overline{AD}$; N - midpoint of $\overline{BC}$

Prove: $MN = \frac{1}{2}(AB + DC)$

M - midpoint of $\overline{AD} \Rightarrow \overline{AM} \cong \overline{MD}$

N - midpoint of $\overline{BC} \Rightarrow \overline{BN} \cong \overline{NC}$

Draw diagonal $\overline{AC}$; let $\overline{AC} \cap \overline{MN} = O$

$\overline{MN}$ - median $\Rightarrow \overline{MN} \parallel \overline{AB} \parallel \overline{DC}$ (theorem 3 - below)

Then, $\overline{AB} \parallel \overline{MN} \parallel \overline{DC}$; $\overline{AD}$ - transv.; $\overline{AM} \cong \overline{MD}$; $\overline{AC}$ - transv $\Rightarrow \overline{AO} \cong \overline{CO}$

(if 3 $\parallel$ lines cut $\cong$ segm / transv, then $\cong$ segm. every transv)

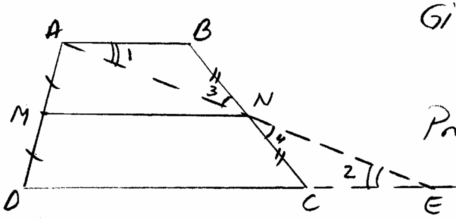
$\triangle ADC$: $MO = \frac{1}{2} DC$

$\triangle ACB$: $NO = \frac{1}{2} AB$

$\Rightarrow MO + NO = MN = \frac{1}{2}(DC + AB)$

Theorem 3 (4.4 – T 4.4.4)

The median of a trapezoid is parallel to each base.



Given: $ABCD$ trap; M - midpoint of $\overline{AD}$; N - midpoint of $\overline{BC}$

Prove: $\overline{MN} \parallel \overline{AB}$; $\overline{MN} \parallel \overline{DC}$

Proof

Draw line $\overleftrightarrow{AN}$ and extend $\overline{DC}$

$\overleftrightarrow{AN} \cap \overleftrightarrow{DC} = E$

$\triangle ANB \cong \triangle ENC$ (AAS) $\left\{ \begin{array}{l} \overline{BN} \cong \overline{CN} \\ \angle 1 \cong \angle 2 \text{ (alt-int } \angle\text{'s formed by } \overline{AB} \parallel \overline{DE} \text{ and transv. } \overline{AE}) \\ \angle 3 \cong \angle 4 \text{ (vertical } \angle\text{'s)} \end{array} \right.$

$\Rightarrow \overline{AN} \cong \overline{EN}$

$\Rightarrow N$ = midpoint of $\overline{AE}$

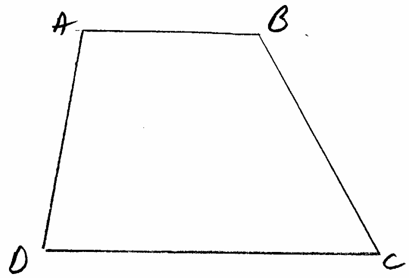
in $\triangle ADE$, $\overline{MN} \parallel \overline{DE}$ ($\overline{MN}$ = joins the midpoints of 2 sides)

Therefore, $\overline{MN} \parallel \overline{DC} \parallel \overline{AB}$

When is a quadrilateral a trapezoid?

Theorem 1 (4.4 – T 4.4.5)

If two of three consecutive angles of a quadrilateral are supplementary, the quadrilateral is a trapezoid



Given: ABCD - quadrilateral; <A & <D - supplementary; <D & <C - not supplementary

Prove: ABCD - trapezoid

Condition: Need to show $\overline{AB} \parallel \overline{DC}$

Proof

1. ABCD - quadrilateral, <A and <D = supplementary	1. given
2. $\overline{AB} \parallel \overline{DC}$	2. ∥ lines iff interior ∠'s same side of transv. supplementary ($\overline{AB}$ and $\overline{DC}$ with transv. $\overline{AD}$)
3. ABCD - trapezoid	3. definition of trapezoid

When is a trapezoid isosceles?

Theorem 1 (4.4 – T 4.4.6)

If two base angles of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.

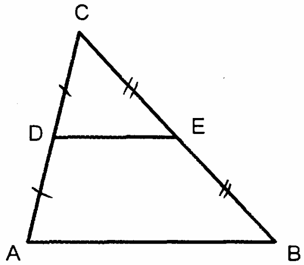
see textbook page 188

Theorem 2 (4.4 – T 4.4.7)

If the diagonals of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.

see textbook page 188.

Problem #1 Use the figure to answer the questions.



Given: D, E midpoints $\Rightarrow$ $\triangle CAB$, $\boxed{\overline{DE} \parallel \overline{AB} \text{ and } DE = \frac{1}{2} AB}$

a) What is DEBA?

DEBA - trapezoid b/c $\overline{DE} \parallel \overline{AB}$

b) If DE = 7 in, find AB.

$DE = \frac{1}{2} AB \Rightarrow AB = 2DE$

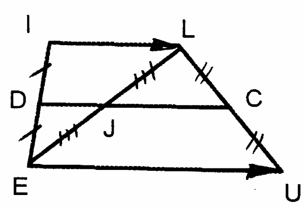
$AB = 2(7) = 14$ in

c) If AB is 23 cm, find DE.

$DE = \frac{1}{2} AB$

$DE = \frac{1}{2} 23 = 11.5$ cm

Problem #2 Use the figure to answer the questions.



Given: trap EULI ($\overline{EU}$, $\overline{IL}$ bases)
D, C midpoints, J midpoint $\overline{EL}$
$\overline{DC} \parallel \overline{EU}$

a) If IL = 43 cm, find DJ.

$\triangle EIL$: $DJ = \frac{1}{2} IL$

$DJ = \frac{1}{2} 43 = 21.5$ cm

b) If EU = 17 in, find JC.

$\triangle LEU$: $JC = \frac{1}{2} EU$

$JC = \frac{1}{2} 17 = 8.5$ in

c) If JC = 12.5 cm, find EU.

$\triangle LEU$: $JC = \frac{1}{2} EU \Rightarrow$

$EU = 2JC = 2(12.5) = 25$ cm

e) If DJ = 6.3 cm, find IL.

$\triangle EIL$: $DJ = \frac{1}{2} IL \Rightarrow$

$IL = 2DJ =$

$= 2(6.3) = 13$ cm

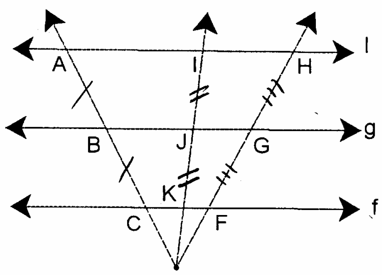
f) If EU = 21 in and IL = 16 in, find DC.

ILUE - trapezoid with DC - median

$DC = \frac{1}{2}(IL + EU)$

$DC = \frac{1}{2}(16 + 21) = \frac{1}{2}(37) = 18.5$ in

Problem #3 Use the figure to answer the questions.



Given: $l \parallel g \parallel f$

$\overline{IJ} \cong \overline{JK}$

a) If AB = 14 cm, find AC.

$AB = BC = 14$ cm (if 3 ∥ lines cut ≅ seg m one transv, then ≅ seg m any transv)

$AC = AB + BC = 28$ cm

b) If FG = 3 in, find FH.

$FG = GH = 3$ in

$FH = 2\,GH = 6$ in

c) If AC = 36 cm, find BC.

$AC = AB + BC$

$AC = 2\,BC$ (b/c $BC = AB$)

$36 = 2\,BC \Rightarrow BC = 18$ cm

d) If GH = 22 in, find HF.

$GH = GF = 22$ in

$HF = 2\,GH$

$= 44$ in

e) If BC = 4 in and GF = 6 in, find AC + HF.

$AC + HF = 2\,BC + 2\,GF$

$= 2(4) + 2(6)$

$= 8 + 12 = 20$ in

Problem #4 Given: RSTV trapezoid

(4.4 - #18)

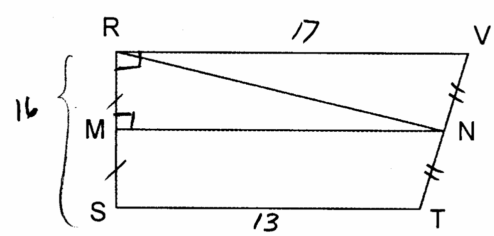
$\overline{RV} \parallel \overline{ST}$

$m\angle SRV = 90°$

M, N midpoints

ST = 13 in, RV = 17 in, RS = 16 in

Find: RN.



Proof

M, N midpoints $\Rightarrow MN \parallel RV$

$\Rightarrow m\angle RMN = 90°$

$\triangle RMN$ - right $\triangle$ $\quad RN^2 = RM^2 + MN^2$

$RM = \frac{1}{2} RS = 8$ in

$MN = \frac{1}{2}(RV + ST) = 15$ in

$\Rightarrow RN^2 = 8^2 + 15^2$

$RN^2 = 289$

$RN = \sqrt{289}$

$RN = 17$