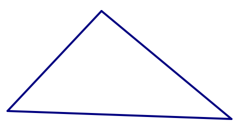
4.1 & 4.2 Properties of a Parallelogram

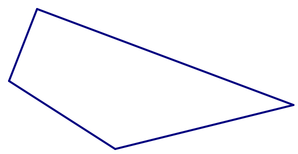
Definition (2.5) A **polygon** is a closed figure whose sides are line segments that intersect only at endpoints. (*Polygon* is a word of Greek origin that means *many angles*; hence, it implies *many sides*).

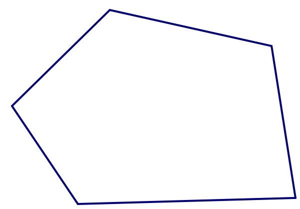
Note: 1. We will be working only with **convex polygons**, polygons in which a line segment joining two points in the interior of the polygon has all its points in the interior of the polygon.

2. The angle measures of convex polygons are between 0° and 180°.

Examples of convex polygons







Definition (2.5) A **diagonal of a polygon** is a line segment that joins two nonconsecutive vertices.

Exercise #1 How many diagonals are in a

a) triangle

b) polygon with 4 sides (quadrilateral)

c) polygon with 5 sides (pentagon)

d) polygon with 6 sides (hexagon)

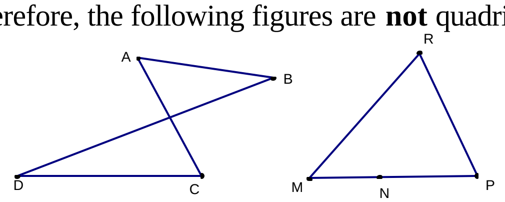
Definition (2.5) A **regular polygon** is a polygon that has its sides and angles congruent.

Definition (4.1) A **quadrilateral** is a polygon that has four sides.

Note: - We will work only with quadrilaterals whose sides are coplanar.
- Special quadrilaterals (squares, rectangles, rhombuses, parallelograms, and trapezoids) occur in various practical circumstances, such as architectural design, construction materials, fabric design, and urban planning.

Important! ABCD is a quadrilateral iff all points are coplanar, no three of which are collinear, and each segment intersects exactly two others, one at each endpoint.

Therefore, the following figures are **not** quadrilaterals:

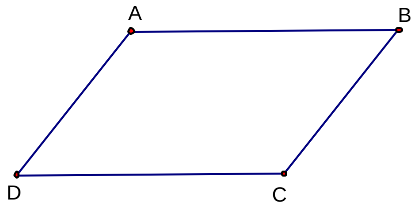


Property (2.5 - C 2.5.4)

The sum of the interior angles of a quadrilateral is 360°.

Definition (4.1)

A **parallelogram** is a quadrilateral whose opposite sides are parallel.



The defining property for a parallelogram is that it si a quadrilateral whose opposite sides are parallel. Many other properties follow from this. The most significant feature of the figure is that for either pair of opposite sides, the other two sides and the diagonals are transversals. Thus, the theory of parallel lines and transversals may be used to prove properties of parallelograms. This theory and that for congruent triangles provide the needed tools for study of parallelograms.

Theorem 1 (4.1 – T 4.1.1)

A diagonal of a parallelogram separates it into two congruent triangles.

Properties of Parallelograms

Corollaries
(4.1 – C 4.1.2,3,4,5)

1. The opposite sides of a parallelogram are congruent (opp sides ▱ ≅).

2. The opposite angles of a parallelogram are congruent (opp $\angle's$ ▱ ≅).

3. Any two consecutive angles of a parallelogram are supplementary (consec $\angle's$ ▱ supp).

4. The diagonals of a parallelogram bisect each other (diags ▱ bisect each other).

Definition The **distance between two parallel lines** is the length of any perpendicular line segment joining the lines.

Definition (4.1) An **altitude of a parallelogram** is a line segment from one vertex that is perpendicular to the opposite side (or to an extension of that side).

Lemma
(4.1 – L 4.1.6)

If two sides of one triangle are congruent to two sides of a second triangle and the included angle of the first triangle is greater than the included angle of the second, then the length of the side opposite the included angle of the first triangle is greater than the length of the side opposite the included angle of the second.

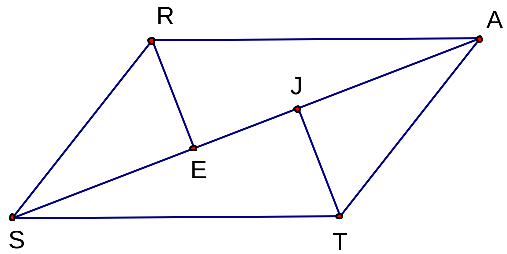
Theorem 2
(4.1 – T 4.1.7)

In a parallelogram with unequal pairs of consecutive angles, the longer diagonal lies opposite the obtuse angle.

Problem #1
(4.1 - #3)

MNPQ is a parallelogram. Suppose that MQ =5, MN = 8, and $m\angle M = 110^\circ$. Find: a) QP; b) NP; c) $m\angle Q$; d) $m\angle P$.

Problem #2



Given: $\square\ STAR$

$\overline{ER} \parallel \overline{JT}$

Prove: $\overline{ER} \cong \overline{JT}$

When is a quadrilateral a parallelogram?

Theorem3
(4.2 – T 4.2.1)

If two sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

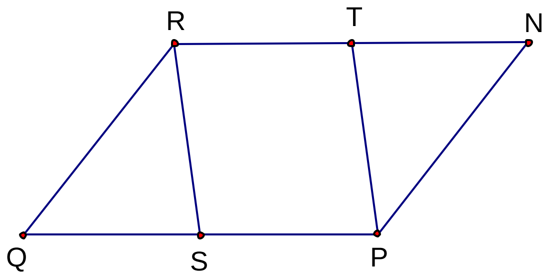
<u>Theorem 4</u>
(4.2 – T 4.2.2)

If both pairs of opposite sides of a quadrilateral are congruent, then it is a parallelogram.

<u>Theorem 5</u>
(4.2 – T 4.2.3)

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Problem #3



Given: ▱$MNPQ$

T midpoint of $\overline{MN}$

S midpoint of $\overline{QP}$

Prove: $\triangle QMS \cong \triangle NPT$

$MSPT$ is a parallelogram

Theorem 6
(4.2 – T 4.2.5)

The segment that joins the midpoints of two sides of a triangle is parallel to the third side and has a length equal to one-half the length of the third side.

Problem #4
(4.2 - #10)

In a triangle ABC, M and N are midpoints of side AC and side BC, respectively. If MN = 7.65, how long is the side AB?