3.5 Inequalities in a Triangle

There are several important order relations involving the measures of the sides and angles of a triangle.

Definition If $a, b \in \mathbb{R}$, then $a > b$ if and only if there is $p > 0$ such that $a = b + p$.

Properties of Inequalities
(A 3)

Addition: If $a < b$, then $a + c < b + c$ for any *c*.

Subtraction: If $a < b$, then $a - c < b - c$ for any *c*.

Multiplication: If $a < b$, then $a \cdot c < b \cdot c$ for any $c > 0$.
$a \cdot c > b \cdot c$ for any $c < 0$.

Division: If $a < b$, then $\frac{a}{c} < \frac{b}{c}$ for any $c > 0$.
$\frac{a}{c} > \frac{b}{c}$ for any $c < 0$.

The Transitive Property of Inequality (Order)
If $a, b, c \in \mathbb{R}$ and if $a < b$ and $b < c$, then $a < c$

Lemma 1
(3.5 – L 3.5.1)

The measure of a line segment is greater than the measure of any of its parts.

Lemma 2
(3.5 – L 3.5.2)

The measure of an angle is greater than the measure of any of its parts.

Lemma 3
(3.5 – L 3.5.3)

The measure of an exterior angle of a triangle is greater than the measure of either nonadjacent interior angle.

Theorem1
(3.5 – T 3.5.6)

If the lengths of two sides of a triangle are unequal, then the measures of the angles opposite them are unequal and the larger angle is opposite the longer side.

(2 sides $\Delta \neq$, opp $\angle 's \neq$ same order)

Note: The relationship described in the above theorem extends to all sides and all angles of a triangle. That is, the largest of the three angles of a triangle is opposite the longest side, and the smallest angle is opposite the shortest side.

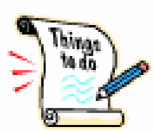
The converse of Theorem 1 is also true.

<u>Theorem 2</u>
(3.5 – T 3.5.7)

If the measures of two angles of a triangle are unequal, then the lengths of the sides opposite them are unequal and the longer side is opposite the larger angle.
($2\ \angle's\ \Delta \neq$, opp sides $\neq$ same order)

<u>Corollary 1</u>
(3.5 – C 3.5.8)

The perpendicular segment from a point to a line is the shortest segment that can be drawn from the point to the line.



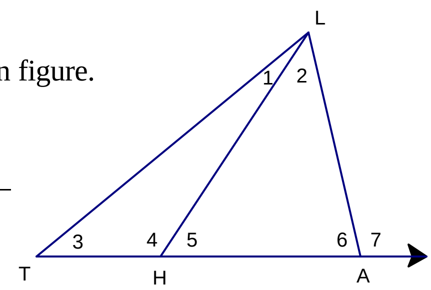
<u>Corollary 2</u>
(3.5 – C 3.5.9)

The perpendicular segment from a point to a plane is the shortest segment that can be drawn from the point to the plane.

Theorem 3 **The Triangle Inequality**

(3.5 – T 3.5.10) The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Problem #1 Write the theorem that justifies each statement. Refer to the given figure.



a) $m\angle 7 > m\angle 5$ ________________________________

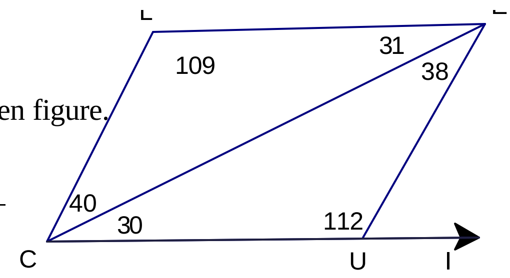
b) $m\angle 4 > m\angle 6$ ________________________________

c) If $LH < TL$, then $m\angle 3 < m\angle 4$ ________________________________

d) If $m\angle 6 > m\angle 3$, then $TL > LA$. ________________________________

e) $LA + TA > TL$ ________________________________

Problem #2 Write the theorem that justifies each statement. Refer to the given figure.



a) $\overline{LE} \nparallel \overline{CU}$ ____________________________________

b) $\overline{LC} \nparallel \overline{EU}$ _____________________________________

c) $LE > LC$ ____

d) $EU + UC > EC$ ____

e) $LE < LC + EC$ ____

f) $EU < UC$ ____

g) $m\angle EUI < m\angle ECU$ ____

Problem #3
(3.5 - #13)

Is it possible to draw a triangle whose sides measure:

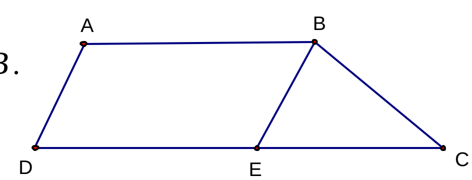
a) 8, 9, and 10 in?

b) 8, 9, and 17 m?

c) 8, 9, and 18 ft?

Problem #4
(3.5 - #24)

Given a quadrilateral ABCD with $\overline{AB} \cong \overline{DE}$, show that $DC > AB$.



Problem #5
(3.5 - #27, #29)

a) The sides of a triangle have lengths of 4, 6, and *x*. Write an inequality that states the possible values of *x*.

b) If the lengths of two sides of a triangle are represented by 2x+5 and 3x+7 (in which x is positive), describe in terms of x the possible lengths of the third side whose length is represented by y.