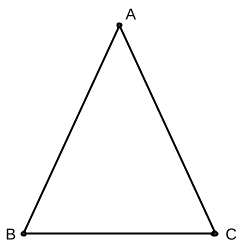
# 3.3 Isosceles Triangles
# Special Line Segments and Triangles

**Definition** A <u>triangle is isosceles</u> if and only if at least two of its sides are congruent.

**Isosceles Triangle**




$\triangle ABC$ - isosceles

$\overline{BC}$ = base

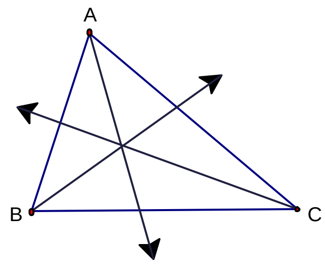
$\overline{AB} \cong \overline{AC}$ - $\overline{AB}, \overline{AC}$ = legs

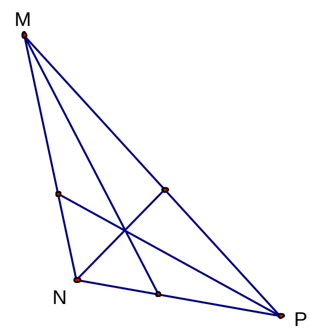
$\angle A$ = vertex

$\angle B, \angle C$ = base angles

**Special Line Segments**

**Definition** The bisectors of the angles of a triangle are called the **<u>angle bisectors of a triangle</u>**.

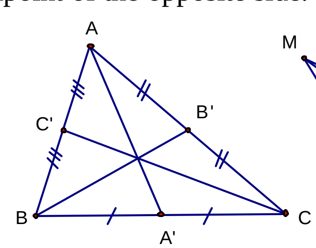


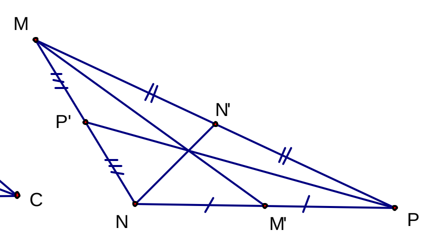




Note that the bisectors are shown as rays in the first figure and as line segments with endpoints on the triangle's sides in the second figure.

Note that the bisectors always intersect at one point in the interior of the triangle.

**Definition** A line segment is **<u>a median of a triangle</u>** if and only if its endpoints are a vertex and the midpoint of the opposite side.

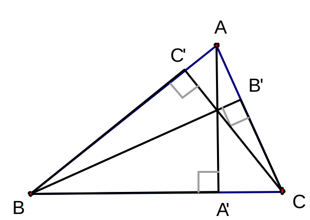


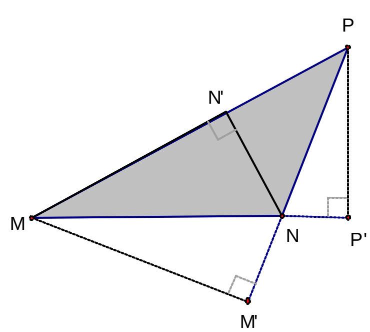




Note that the medians always intersect at one point in the interior of the triangle.

Note that a median is not, in general, the angle bisector. Only in special cases do they coincide.

**Definition** An **<u>altitude of a triangle</u>** is a line segment from one vertex perpendicular to the line containing the opposite side.

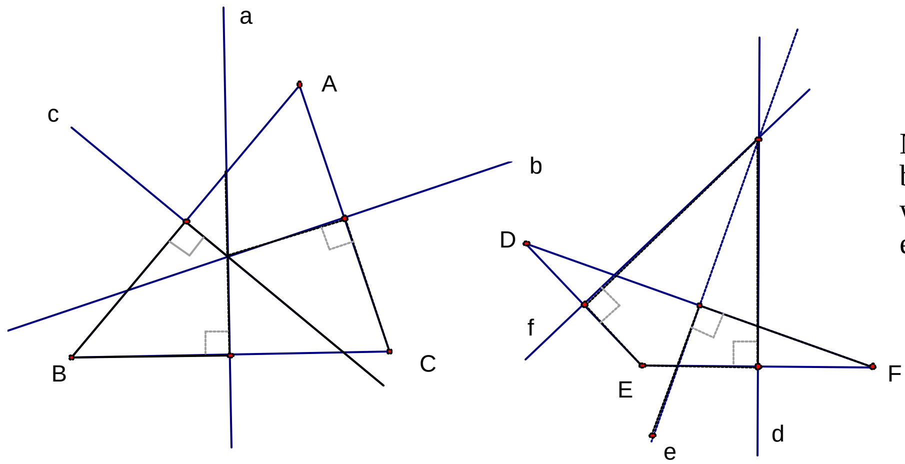



Note that an altitude does not always lie in the interior of a triangle.

Note that the altitudes of the first triangle intersect at one point. The altitudes of the second triangle would also intersect at one point if they were extended.

**Definition** **<u>A perpendicular bisector of a side of a triangle</u>** is the line that perpendicularly bisects the side of the triangle.




Note that the perpendicular bisectors always meet at a point which can be in the interior or exterior of the triangle.

<u>Auxiliary Lines</u> Some proofs in geometry require the addition of lines, line segments, or rays to the given figure. These are called auxiliary lines (helping lines). Their relation to the given figure must be clearly stated and justified in the proof. You must account for the uniqueness of the line, segment or ray as it is introduced into the existing drawing.

**<u>Theorem</u>**
(3.3– T. 3.3.3) If two sides of a triangle are congruent, then the angles opposite the congruent sides are congruent.

**<u>Theorem</u>** (Converse of Theorem 3.3.3)
(3.3 – T. 3.3.4) If two angles of a triangle are congruent, then the sides opposite the congruent angles are congruent.

In conclusion, a triangle is isosceles if and only if

________________________________________________

**Definition** A triangle is equilateral if and only if all three of its sides are congruent.

**Theorem** An equilateral triangle is also equiangular.
(3.3 – C. 3.3.5)

**Theorem** (Converse of Theorem 3.3.5)
(3.3 – C. 3.3.6) An equiangular triangle is also equilateral.

In conclusion, a triangle is equilateral if and only if ________________________________________________

<u>Problem #1</u>
(3.3 - #17)

In an isosceles triangle, one of the base angles is $68^\circ$. Find the other two angles of the triangle.

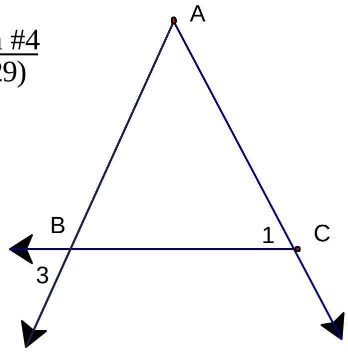
<u>Problem #2</u>
(3.3 - #18)

In an isosceles triangle ABC (base $\overline{BC}$), $m\angle B = 68^\circ$. Find the measure of the angle formed by the angle bisectors of $\angle B$ and $\angle C$.

<u>Problem #3</u>
(3.3 - #21)

In an isosceles triangle ABC with vertex A, each base angle is 12 degrees larger than the vertex angle. Find the measure of each angle.

Problem #4
(3.3 - #29)




Given $\angle 3 \cong \angle 1$

Prove $\overline{AB} \cong \overline{AC}$

Problem #5
(3.4 – Example 3)

Construct the bisector of an angle. Justify your construction.

Problem #6
(3.4 – Example 5)

Construct the perpendicular bisector of a given segment. Justify your construction.