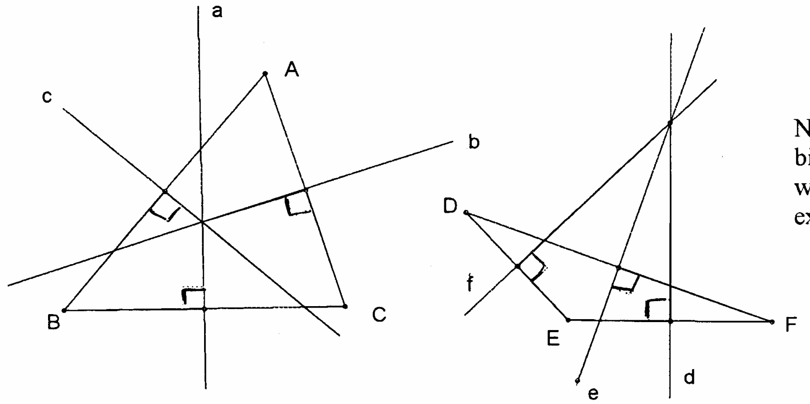
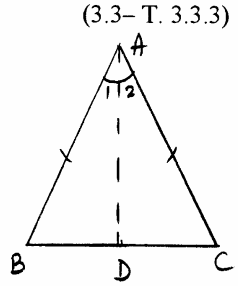
Definition **A perpendicular bisector of a side of a triangle** is the line that perpendicularly bisects the side of the triangle.



Note that the perpendicular bisectors always meet at a point which can be in the interior or exterior of the triangle.

Auxiliary Lines — Some proofs in geometry require the addition of lines, line segments, or rays to the given figure. These are called auxiliary lines (helping lines). Their relation to the given figure must be clearly stated and justified in the proof. You must account for the uniqueness of the line, segment or ray as it is introduced into the existing drawing.

Theorem (3.3– T. 3.3.3) If two sides of a triangle are congruent, then the angles opposite the congruent sides are congruent.



Given: $\triangle ABC$, $\overline{AB} \cong \overline{AC}$

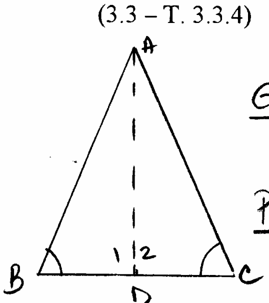
Prove: $\angle B \cong \angle C$

Proof

Statements	Reasons
1. Draw $\overline{AD}$ bisector of $\angle A$, $D \in \overline{BC}$	1. Every angle has one and only one bisector.
2. $\angle 1 \cong \angle 2$	2. definition of bisector
3. $\triangle ABD$, $\triangle ACD$: $\overline{AD} \cong \overline{AD}$; $\overline{AB} \cong \overline{AC}$; $\angle 1 \cong \angle 2$	3. reflexive prop. of $\cong$; given; (2) above
4. $\triangle ABD \cong \triangle ACD$	4. SAS
5. $\angle B \cong \angle C$	5. CPCTC

Note • instead of constructing $\overline{AD}$ - bisector, we could also construct $\overline{AD}$ - median. Then, $\triangle ABD \cong \triangle ACD$ by SSS

Theorem (Converse of Theorem 3.3.3) (3.3 – T. 3.3.4) If two angles of a triangle are congruent, then the sides opposite the congruent angles are congruent.



Given: $\triangle ABC$, $\angle B \cong \angle C$

Prove: $\overline{AB} \cong \overline{AC}$

Proof

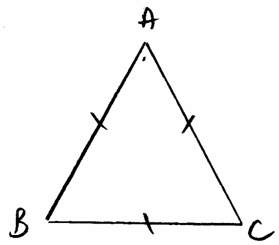
Statements	Reasons
1. Draw $\overline{AD} \perp \overline{BC}$, $D \in \overline{BC}$	1. The $\perp$ from a point to a line is unique.
2. $\angle D_1 \cong \angle D_2$	2. definition of $\perp$ lines. ($\perp$ iff $\cong$ adj. $\angle$'s)
3. $\triangle ABD$, $\triangle ACD$: $\overline{AD} \cong \overline{AD}$; $\angle B \cong \angle C$; $\angle D_1 \cong \angle D_2$	3. reflexive prop. of $\cong$; given; (2) above
4. $\triangle ABD \cong \triangle ACD$	4. AAS
5. $\overline{AB} \cong \overline{AC}$	5. CPCTC

Note • instead of constructing $\overline{AD}$ = altitude, we could also construct $\overline{AD}$ = bisector of $\angle A$ (then AAS)
• Also, we could show $\triangle ABC \cong \triangle ACB$

In conclusion, a triangle is isosceles if and only if it has 2 congruent angles.

Definition A triangle is equilateral if and only if all three of its sides are congruent.

Theorem (3.3 – C. 3.3.5) An equilateral triangle is also equiangular.



Given: $\triangle ABC$ equilateral

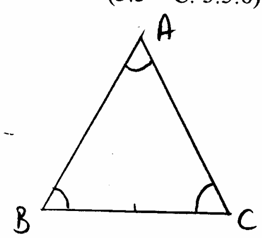
Prove: $\triangle ABC$ - equiangular

(Condition: $\angle A \cong \angle B \cong \angle C$)

Proof

Statements	Reasons
1. $\triangle ABC$ equilateral	1. given
2. $\overline{AB} \cong \overline{AC}$	2. Definition of equil. $\triangle$
3. $\angle C \cong \angle B$	3. $\triangle$, if 2 sides $\cong$, opp. $\angle$'s $\cong$.
4. $\overline{AB} \cong \overline{BC}$	4. Definition of equil. $\triangle$
5. $\angle C \cong \angle A$	5. Same as (3)
6. $\angle B \cong \angle A$ (3,5)	6. Transitivity $\cong$
7. $\triangle ABC$ equiangular	7. Definition of equiangular $\triangle$.

Theorem (Converse of Theorem 3.3.5)
(3.3 – C. 3.3.6) An equiangular triangle is also equilateral.



Given. $\triangle ABC$ equiangular

Prove: $\triangle ABC$ equilateral

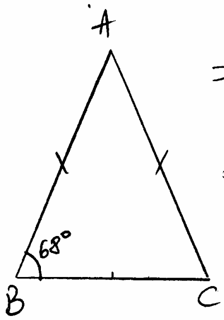
(Condition: $\overline{AB} \cong \overline{BC} \cong \overline{AC}$)

Proof

Statements	Reasons
1. $\triangle ABC$ equiangular	1. given
2 $\angle A \cong \angle B$	2. Definition of equiangular $\triangle$
3 $\overline{BC} \cong \overline{AC}$	3. $\triangle$, if 2 $\angle$'s $\cong$, opp. sides $\cong$.
4. $\angle B \cong \angle C$	4. Same as (2)
5. $\overline{AC} \cong \overline{AB}$	5. Same as (3)
6. $\overline{BC} \cong \overline{AB}$ (3,5)	6. Transitivity $\cong$
7. $\triangle ABC$ equilateral	7. Definition of equilateral $\triangle$.

In conclusion, a triangle is equilateral if and only if it has 3 congruent angles.

Problem #1 (3.3 - #17) In an isosceles triangle, one of the base angles is $68°$. Find the other two angles of the triangle.



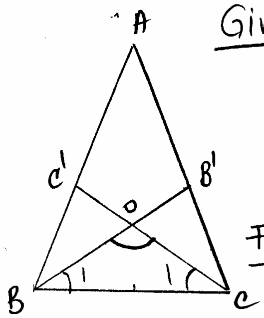
Given: $\triangle ABC$ isosc. $m\angle B = 68°$

Find: $m\angle A = ?$ $m\angle C = ?$

Proof

Statements	Reasons
1. $\triangle ABC$ isosceles	1. given
2. $\overline{AB} \cong \overline{AC}$	2. Definition of isosc. $\triangle$
3. $\angle C \cong \angle B$	3. $\triangle$, if 2 sides $\cong$, opp. $\angle$'s $\cong$
4. $m\angle C = m\angle B$	4. Definition of $\cong$ $\angle$'s.
5. $m\angle B = 68°$	5. given
6. $m\angle C = 68°$ (4,5)	6. transitivity
7. $m\angle A + m\angle B + m\angle C = 180°$	7. $\triangle$, sum $\angle$'s $= 180°$
8. $m\angle A + 68° + 68° = 180°$	8. substitution
9. $m\angle A = 44°$	9. subtraction prop. $=$

Problem #2 (3.3 - #18) In an isosceles triangle ABC (base $\overline{BC}$), $m\angle B = 68°$. Find the measure of the angle formed by the angle bisectors of $\angle B$ and $\angle C$.



Given: $\triangle ABC$ isosc (base $\overline{BC}$), $m\angle B = 68°$, $\overline{BB'}$ bis. $\angle B$, $\overline{CC'}$ bis. $\angle C$

Find: $m\angle BOC = ?$

Proof

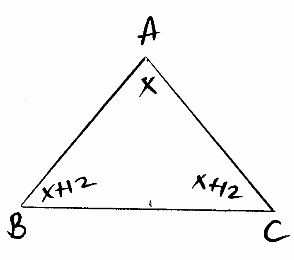
$\triangle ABC$ isosceles $\Rightarrow \overline{AB} \cong \overline{AC}$
$\Rightarrow \angle B \cong \angle C$
$\Rightarrow m\angle C = m\angle B = 68°$

$\overline{BB'}$ bis. $\angle B \Rightarrow m\angle B_1 = \frac{1}{2} m\angle B = 34°$

$\overline{CC'}$ bis. $\angle C \Rightarrow m\angle C_1 = \frac{1}{2} m\angle C = 34°$

In $\triangle BOC$: $m\angle B_1 + m\angle C_1 + m\angle BOC = 180°$
$34° + 34° + m\angle BOC = 180°$
$m\angle BOC = 112°$

Problem #3 (3.3 - #21) In an isosceles triangle ABC with vertex A, each base angle is 12 degrees larger than the vertex angle. Find the measure of each angle.

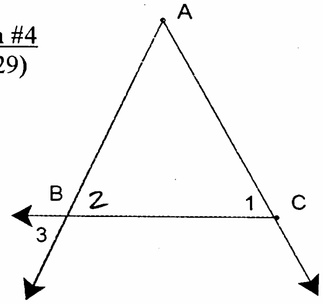


Let $m\angle A = x$
then $m\angle B = x + 12$
$m\angle C = x + 12$

In $\triangle ABC$, $m\angle A + m\angle B + m\angle C = 180°$
$x + x + 12 + x + 12 = 180$
$3x + 24 = 180$
$3x = 156$
$x = 52$

Therefore, $m\angle A = 52°$
$m\angle B = 52° + 12° = 64°$
$m\angle C = 64°$

Problem #4
(3.3 - #29)



Given $\angle 3 \cong \angle 1$

Prove $\overline{AB} \cong \overline{AC}$

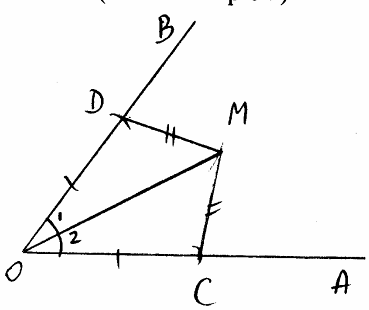
(Condition: $\angle 2 \cong \angle 1$)

Proof

Statements	Reasons
1. $\angle 3 \cong \angle 1$	1. given
2. $\angle 3 \cong \angle 2$	2. vertical angles
3. $\angle 1 \cong \angle 2$ (1,2)	3. transitivity
4. $\overline{AB} \cong \overline{AC}$	4. $\triangle$, 2 $\angle$'s $\cong$, opp sides $\cong$

Problem #5
(3.4 – Example 3)

Construct the bisector of an angle. Justify your construction.



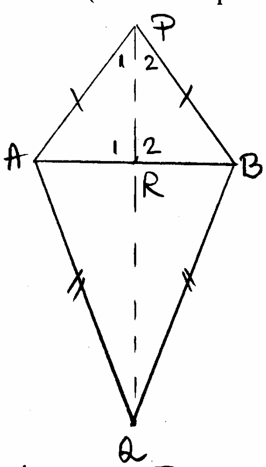
Given: $\angle AOB$
Construct $\overline{OM}$ bis. of $\angle O$
(Condition: $\angle O_1 \cong \angle O_2$)

Proof

Statements	Reasons.
1. $\angle AOB$	1. given
2. Construct $\overline{OC} \cong \overline{OD}$ with $C \in \overrightarrow{OA}$, $D \in \overrightarrow{OB}$	2. radii in circle of center O (by construction)
3. construct $\overline{CM} \cong \overline{DM}$	3. $\overline{CM}$ = radius in circle of center C; $\overline{DM}$ = radius $=\overline{CM}$ in circle of center D (by construction)
4. $\triangle ODM$, $\triangle OCM$: $\overline{OM} \cong \overline{OM}$, $\overline{OD} \cong \overline{OC}$, $\overline{DM} \cong \overline{CM}$	4. reflexive $\cong$; (2) above; (3) above
5. $\triangle ODM \cong \triangle OCM$	5. SSS
6. $\angle O_1 \cong \angle O_2$	6. CPCTC
7. $\overline{OM}$ bisector $\angle O$	7. Definition of bisector.

Problem #6
(3.4 – Example 5)

Construct the perpendicular bisector of a given segment. Justify your construction.



Given: $\overline{AB}$
Construct $\overline{PQ} \perp$ bis
(Condition: $\overline{PQ} \perp \overline{AB}$, $\overline{AR} \cong \overline{RB}$)

Proof

Statements	Reason
1. $\overline{AB}$	1. given
2. Construct $\overline{AP} \cong \overline{BP}$	2. AP – radius in circle of center A; BP – radius in circle of center B (by construction, AP = BP)
3. Construct $\overline{AQ} \cong \overline{BQ}$	3. AQ – radius in circle of center A; BQ – radius in circle of center B (by construction, AQ = BQ)
4. $\triangle PAQ$, $\triangle PBQ$: $\overline{PQ} \cong \overline{PQ}$, $\overline{AP} \cong \overline{BP}$, $\overline{AQ} \cong \overline{BQ}$	4. reflexive $\cong$; (2); (3)
5. $\triangle PAQ \cong \triangle PBQ$	5. SSS
6. $\angle P_1 \cong \angle P_2$	6. CPCTC
7. $\triangle APR$, $\triangle ABR$: $\overline{PR} \cong \overline{PR}$, $\overline{AP} \cong \overline{BP}$, $\angle P_1 \cong \angle P_2$	7. reflexive; (2); (6)
8. $\triangle APR \cong \triangle ABR$	8. SAS

9. $\left\{ \begin{array}{l} \overline{AR} \cong \overline{BR} \\ \angle R_1 \cong \angle R_2 \end{array} \right.$	9. CPCTC
10. $\overline{PR} \perp \overline{AB}$	10. Definition of $\perp$ lines (($\perp$ lines iff $\cong$ adj $\measuredangle$'s))
11. $\overline{PQ}$ = $\perp$ bisector of $\overline{AB}$ (9,10)	11. Definition of $\perp$ bisector