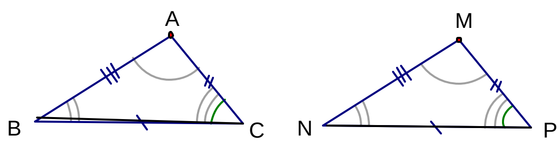
02/21/06

3.1 & 3.2 Congruent Triangles
Applications

Definition Two triangles are congruent if and only if there is a one-to-one correspondence between their vertices such that three pairs of corresponding sides are congruent and the three pairs of corresponding angles are congruent.



$\triangle ABC \cong \triangle MNP$ iff $\overline{AB} \cong \overline{MN}$ $\quad \angle A \cong \angle M$

$\overline{BC} \cong \overline{NP}$ $\quad \angle B \cong \angle N$

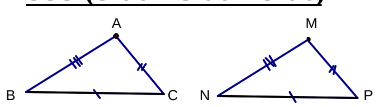
$\overline{AC} \cong \overline{MP}$ $\quad \angle C \cong \angle P$

Properties of congruent triangles

1. Reflexive Property $\triangle ABC \cong \triangle ABC$
2. Symmetric Property If $\triangle ABC \cong \triangle MNP$, then $\triangle MNP \cong \triangle ABC$.
3. Transitive Property If $\triangle ABC \cong \triangle MNP$ and $\triangle MNP \cong \triangle QWS$, then $\triangle ABC \cong \triangle QWS$.

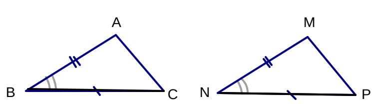
Postulates

<u>SSS (Side – Side – Side)</u>



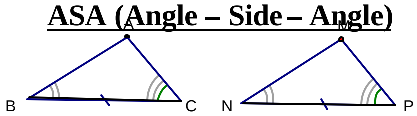
If the three sides of one triangle are congruent with the three sides of a second triangle, then the triangles are congruent.

<u>SAS (Side – Angle – Side)</u>



If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.

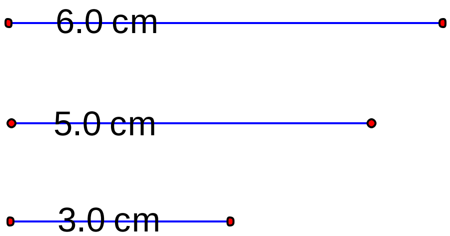
<u>ASA (Angle – Side – Angle)</u>



If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the triangles are congruent.

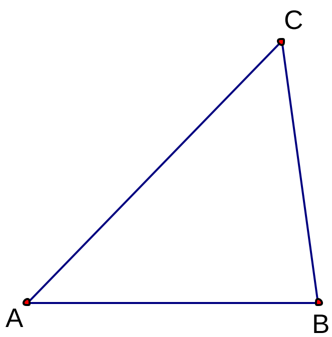
Construction 1
(3.1 - Example 1)

Construct a triangle whose sides have the given lengths.



Construction 2

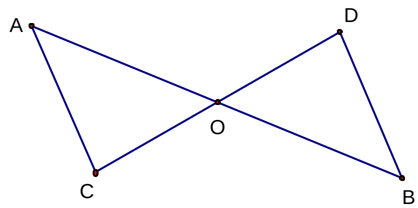
Construct a triangle having its sides congruent to the corresponding parts of a given triangle.



Problem #1

Given an isosceles triangle ABC with base $\overline{BC}$ and M the midpoint of the base, show that $\triangle ABM \cong \triangle ACM$.

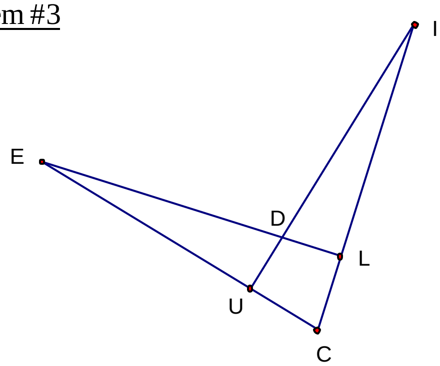
Problem #2



Given $\overline{AB}$ bisects $\overline{CD}$
$\overline{CD}$ bisects $\overline{AB}$

Prove $\triangle AOC \cong \triangle BOD$

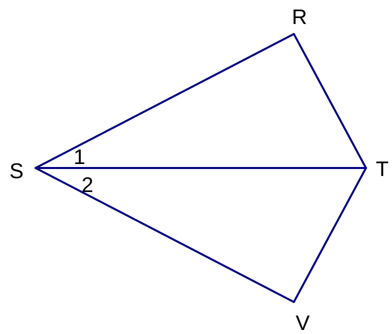
Problem #3



Given $\overline{IU} \perp \overline{EC}$
$\overline{EL} \perp \overline{IC}$
$\overline{CL} \cong \overline{CU}$

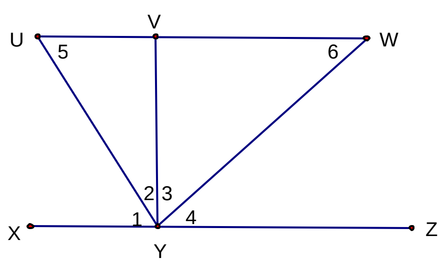
Prove $\triangle ECL \cong \triangle ICU$

Problem #4
(3.2 - #5)



If $\angle R$ and $\angle V$ are right angles and $\angle 1 \cong \angle 2$, prove that $\triangle RST \cong \triangle VST$.

Problem #5
(3.2 - #10)



Given $\overline{UW} \parallel \overline{XZ}$ $\quad m\angle 1 = m\angle 4 = 4x + 3$

$\overline{VY} \perp \overline{UW}$ $\quad m\angle 2 = 6x - 3$

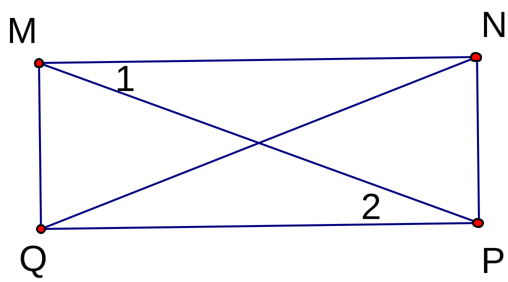
$\overline{VY} \perp \overline{XZ}$

Find The measures of angles 1 through 6.

Problem #6
(3.2 - #24)

In a right triangle *FDG* with right angle *D*, the bisector of angle *D* intersects the hypotenuse at *E*. The acute angles of the triangle are congruent. Prove that *E* is the midpoint of the hypotenuse.

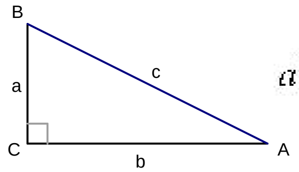
Problem #7
(3.2 - #27)



Given $\angle 1 \cong \angle 2$
$\overline{MN} \cong \overline{QP}$

Prove $\overline{MQ} \parallel \overline{NP}$

The Pythagorean Theorem In any right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.



$$a^2 + b^2 = c^2$$

Note that the <u>converse of the Pythagorean theorem is also true</u>; that is, if the lengths *a, b,* and *c* of the three sides of a triangle are such that $a^2 + b^2 = c^2$, then the triangle is a right triangle with its right angle opposite side *c*.

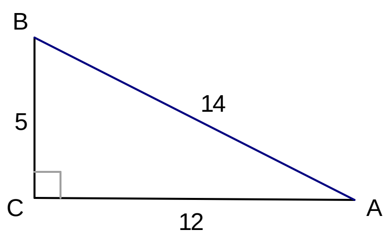
<u>Problem #8</u>

a) In a right triangle the hypotenuse is 13 in and one leg is 12 in. Find the other leg.
b) In a right triangle, one leg is 8cm and the other one is 15 cm. Find the hypotenuse.

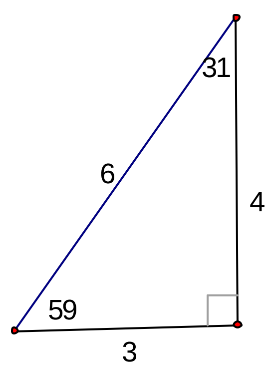
<u>Problem #9</u>

Explain what is wrong in each figure.

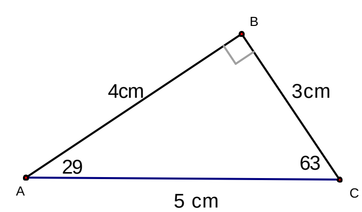
a)



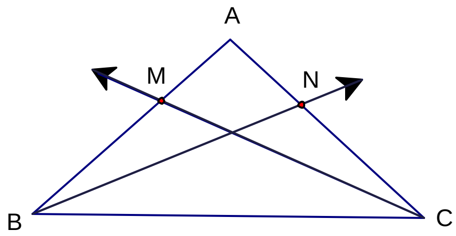
b)



c)



Problem #10



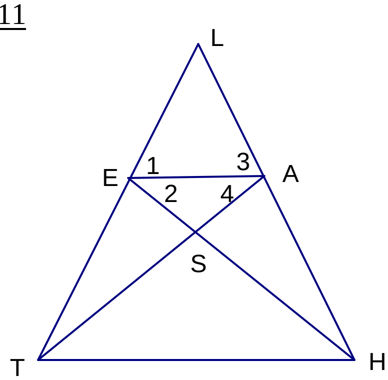
Given $\angle ABC \cong \angle ACB$

$\overrightarrow{BN}$ bis $\angle ABC$

$\overrightarrow{CM}$ bis $\angle ACB$

Prove $\triangle BMC \cong \triangle CNB$

Problem #11



Given $\angle 3 \cong \angle 1$

$\angle 4 \cong \angle 2$

$\triangle EAL$ isosceles ($\overline{EA}$ base)

Prove $\overline{TA} \cong \overline{HE}$