1.3 Early Definitions and Postulates
1.4 Angles and Their Relationships

A **Mathematical System** consists of

1. *Undefined Terms*
   - -point
   - -line
   - -plane
2. *Defined Terms*
3. *Axioms or Postulates*
4. *Theorems*

After some simple terms such as "point", "line", and "plane" have been accepted as undefined, we can begin to define other terms by using them.

When is a statement a definition?

A **good definition** will possess these qualities:

1. It names the term being defined.
2. It places the term into a set or category.
3. It distinguishes the defined term from other terms without providing unnecessary facts.
4. It is reversible.

Definition A **line segment** is the part of a line that consists of two points (endpoints) and all points between them.

$\overline{AB}$

Question Is the above definition a good definition?

4. i) A line segment is the part of a line between and including two points.
ii) The part of a line between and including two points is a line segment.

Exercise #1

a) You have learned that the following statement is true:
If a statement is a definition, then its converse is true.

Does it necessarily follow that if its converse is not true, a statement cannot be a definition? Explain. Yes. This is the contrapositive of the previous statement, so it must also be true.

Decide which of the following true statements are good definitions of the italicized words by determining whether their converses are true.

b) If something is *cold*, then it has a low temperature.
A good definition, because if something has a low temperature it is cold.

c) A *mandolin* is a stringed musical instrument.
A bad definition, because a stringed musical instrument is not necessarily a mandolin.

d) A *kitten* is a young cat.
A good definition, because a young cat is a kitten.

e) An *isosceles triangle* is a triangle that has two congruent sides.
A good definition, because a triangle with two congruent sides is an isosceles triangle.

Note: When both a statement and its converse are true, there is a convenient way to combine the two into one. It is by means of the phrase ***"if and only if".***

When we say
"P if and only if Q",
we mean both
"if P, then Q" and "if Q, then P".

We can represent the phrase "if and only if" by the symbol $\leftrightarrow$.

To write $P \leftrightarrow Q$ means that both $P \rightarrow Q$ and $Q \rightarrow P$ are true.

Postulates

Geometry, or any deductive system, is very much like a game. Before playing the game, it is necessary to accept some basic rules, which we will call *postulates*. The postulates in geometry are man-made, just as the rules of football are, and what the subject will be like depends upon the nature of the postulates used. We will study the geometry called Euclidean, named after Euclid. For many centuries, it was the only geometry known, because it took man a long time to realize that more than one set of rules were possible.

Geometry has very few rules. We will need to supplement them with some of the rules of algebra with which you are already familiar. The rules, or postulates, of algebra concern numbers and operations performed on them.

Properties of Equality (1.5: tables 1,3 & 1.4)

Reflexive Property	Any real number is equal to itself. $a = a$
Symmetric Property	If $a = b$, then $b = a$
Transitive Property	If $a = b$ and $b = c$, then $a = c$
Addition Property	If $a = b$, then $a + c = b + c$ $a - c = b - c$.
Multiplication Property	If $a = b$, then $a \cdot c = b \cdot c$ $\frac{a}{c} = \frac{b}{c}, \forall c \neq 0$.
Distributive Property	$a(b + c) = ab + ac$

The postulates of geometry deal with sets of points and their relationships.

Question Consider a single point. How many lines can pass through, or contain, it?

An unlimited number.

Question Now consider two points. How many lines can contain them?

Only one line.

<u>Postulate 1:</u>

> Through two distinct points, there is exactly one line.
> (Two points determine a line.)

$\overleftrightarrow{AB}$

<u>Definition</u> Points that lie on the same line are called **collinear points.**

<u>Exercise #2</u>
(1.3 - # 7,8)

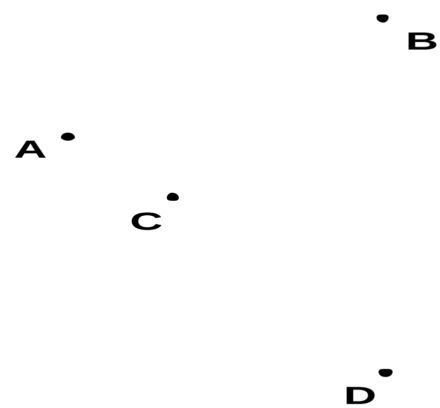
a) Name three points that appear to be collinear.
A, C, D

b) Name three points that appear to be noncollinear.
A, C, B

c) How many lines can be drawn through point *A*?
unlimited number

d) How many lines can be drawn through points A and *B*?
one line

e) How many lines can be drawn through points *A*, *B*, and *C*?
none



<u>Postulate 2:</u>

> **<u>Ruler Postulate</u>**
> The measure of any line segment is a unique positive number.

Note: The term ***unique*** may be replaced by
- one and only one
- exactly one
- one and no more than one

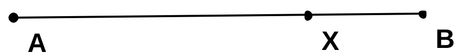
<u>Definition</u> The **distance** between two points is the length of the line segment $\overline{AB}$ that joins the two points.

AB is the length of the segment $\overline{AB}$

Example: Draw two points and find the distance between them.

Postulate 3:

Segment – Addition Postulate
If X is a point of $\overline{AB}$ and $A - X - B$, then $AX + XB = AB$



Definition Two **segments are congruent** if they have the same length.

$\overline{AB} \cong \overline{CD}$ if and only if $AB = CD$

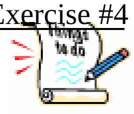
Exercise #3

Given a segment $\overline{AB}$, construct using only a compass and a straightedge, a segment $\overline{CD}$ congruent with $\overline{AB}$.

See textbook page 14, Construction 1.

Definition M is the **midpoint** of a segment $\overline{AB}$ if
A, M, and B are collinear and
$\overline{AM} \cong \overline{MB}$

Exercise #4



Given a segment $\overline{AB}$, construct using only a compass and a straightedge, the midpoint M of the given segment.

See textbook page 15, Construction 2.

Exercise #5
(1.3 - #13)

Given: M is the midpoint of $\overline{AB}$
$AM = 2x + 1$ and $MB = 3x - 2$
Find: x and AM.

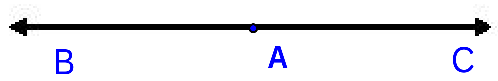
Proof:

STATEMENTS	**REASONS**
1. M = midpoint of segment AB	1. Given
2. $\overline{AM} \cong \overline{MB}$	2. Definition of the midpoint of a segment.
3. $AM = MB$	3. Definition of congruent segments.
4. $2x + 1 = 3x - 2$	4. Substituting the given info. about the lengths AM and MB.
5. $x = 3$	5. The addition property of equality.
6. AM=7	6. Substitution.

Definition **Ray** AB, denoted by $\overrightarrow{AB}$, is the union of $\overline{AB}$ (the segment *AB*) and all the points *X* on $\overleftrightarrow{AB}$ (the line *AB*) such that *B* is between *A* and *X*.

Definition Two rays are **opposite rays** if they have a common endpoint and if their union is a straight line.

$\overrightarrow{AB}$ and $\overrightarrow{AC}$



Exercise #6
(1.3 - #17)

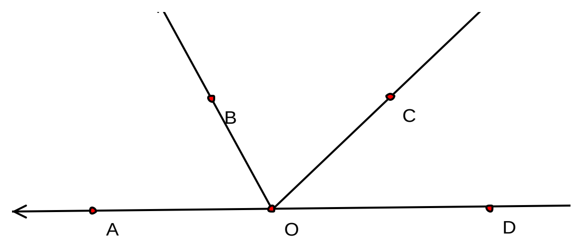
In the figure, name:

a) two opposite rays.

$\overrightarrow{OA}$ and $\overrightarrow{OD}$

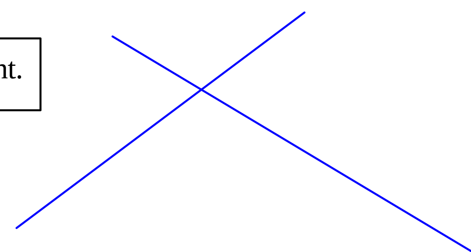
b) two rays that are not opposite.

$\overrightarrow{OB}$ and $\overrightarrow{OD}$

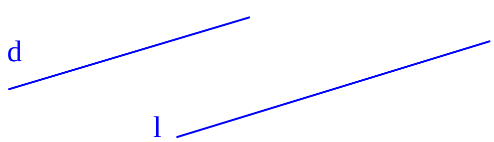


Postulate 4

If two lines intersect, they intersect at a point.



Definition **Parallel lines** are lines that lie in the same plane but do not intersect.



$d \parallel l$

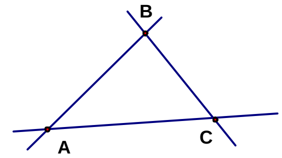
Exercise #7

Draw two lines in a plane. How many common points can they have?

One point,
Or
no point (parallel lines)
or
infinite number of points (lines that coincide or overlap).

Exercise #8
(1.3 - #33)

a) Make a drawing to illustrate **three** noncollinear points A, B, and C, and all of the lines they determine. How many lines are there in all?

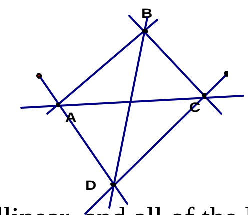


$\overleftrightarrow{AB}$ $\overleftrightarrow{BC}$

$\overleftrightarrow{AC}$

2+1 =3 lines

b) Make a drawing to illustrate **four** points, no three of which are collinear, and all of the lines they determine. How many lines are there in all?



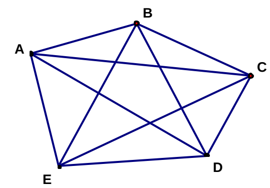
$\overleftrightarrow{AB}$ $\overleftrightarrow{BC}$ $\overleftrightarrow{CD}$

$\overleftrightarrow{AC}$ $\overleftrightarrow{BD}$

$\overleftrightarrow{AD}$

3+2+1=6 lines

c) Make a drawing to illustrate **five points**, no three of which are collinear, and all of the lines they determine. How many lines are there in all?



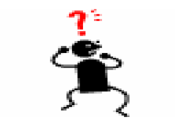
$\overleftrightarrow{AB}$ $\overleftrightarrow{BC}$ $\overleftrightarrow{CD}$ $\overleftrightarrow{DE}$

$\overleftrightarrow{AC}$ $\overleftrightarrow{BD}$ $\overleftrightarrow{CE}$

$\overleftrightarrow{AD}$ $\overleftrightarrow{BE}$

$\overleftrightarrow{AE}$

4+3+2+1 = 10 lines



d) Without making a drawing, can you figure out how many lines are determined by **ten** points, no three of which are collinear?

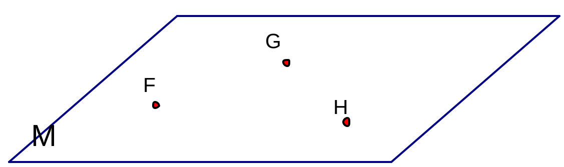
9+8+7+6+5+4+3+2+1= 45 lines

Postulate 5

Through three noncollinear points, there is exactly one plane. (Three noncollinear points determine a plane).

Definition Points that lie in the same plane are called **coplanar points.**

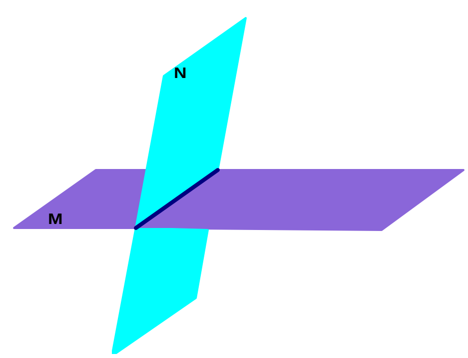
$F, G, H \in M$



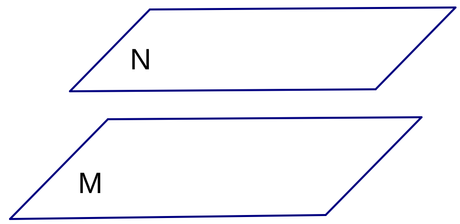
Postulate 6

If two planes intersect, then their intersection is a line.

$M \cap N = l$



Definition Two planes are **parallel** if they do not intersect.

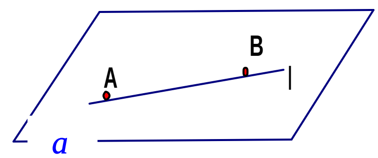


$M \parallel N$ iff $M \cap N = \varnothing$

Postulate 7

> Given two distinct points in a plane, the line containing these points also lies in the plane.

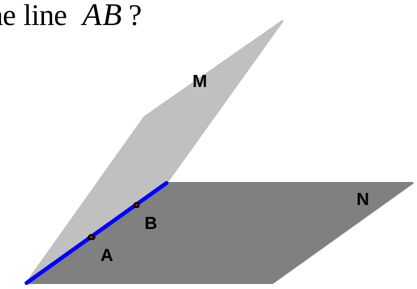
If $A, B \in \alpha$, then $\overleftrightarrow{AB} \subset \alpha$.



Exercise #9 (1.3 - #21)

Suppose that planes *M* and *N* intersect, point *A* lies in both planes *M* and *N*, and point *B* lies in both planes *M* and *N*. What can you conclude regarding the line $\overleftrightarrow{AB}$?

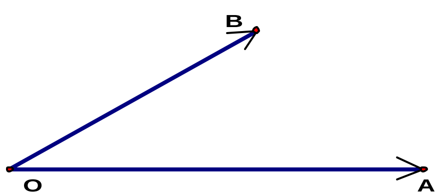
$\overleftrightarrow{AB}$ is the intersecting line



Theorem

> The midpoint of a line segment is unique.

Definition An **angle** is the union of two rays that share a common endpoint.



$\angle AOB$

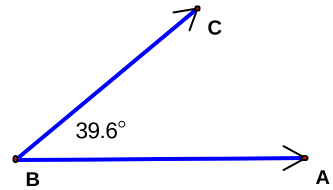
Example Draw an angle, name it, and measure it.

Postulate 8

> **Protractor Postulate**
> The measure of an angle is a unique positive number.

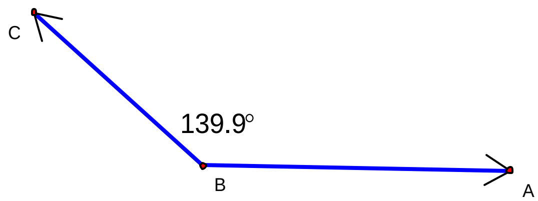
Types of Angles (1.4 – Table 1.2)

ACUTE ANGLE – an angle whose measure is less than 90°.

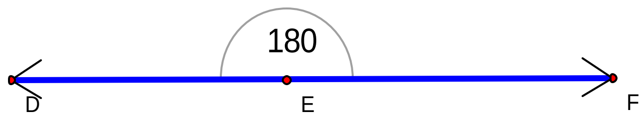


RIGHT ANGLE – an angle whose measure is exactly 90°.

OBTUSE ANGLE – an angle whose measure is between 90° and 180°.



STRAIGHT ANGLE – an angle whose measure is exactly 180°.

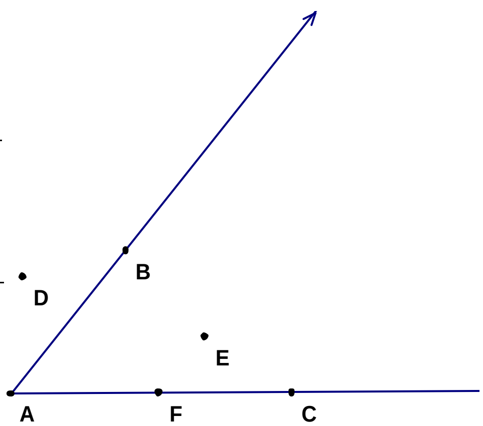


Exercise #10 | Given the figure, which points lie

in the **interior** of $\angle BAC$? ________E________

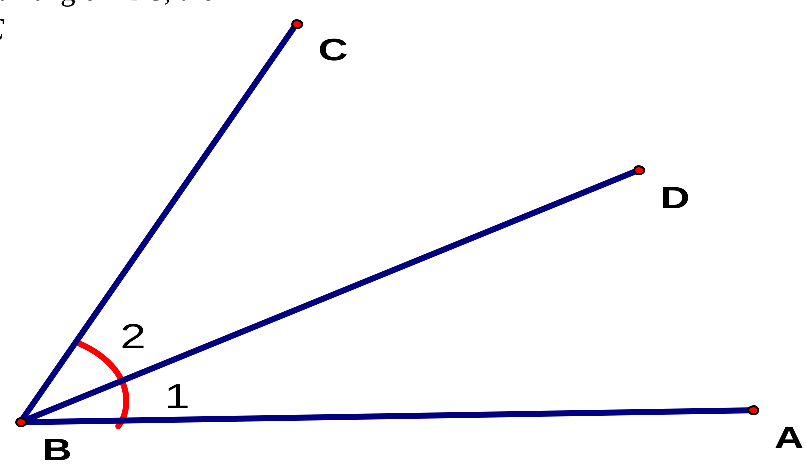
on $\angle BAC$? ________B, F, C________

in the exterior of $\angle BAC$? ________D________



Postulate 9

Angle – Addition Postulate
If a point D lies in the interior of an angle ABC, then
$m\angle ABD + m\angle DBC = m\angle ABC$

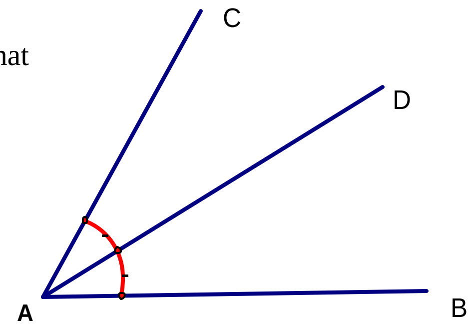


Classifying Pairs of Angles

Definition Two angles are **congruent** if they have the same measure.

$\angle A \cong \angle B$ **iff** $m\angle A = m\angle B$

Definition The **bisector of an angle** $\angle BAC$ is the ray $\overrightarrow{AD}$ such that
D is in the interior of $\angle BAC$ and
$\angle BAD \cong \angle DAC$

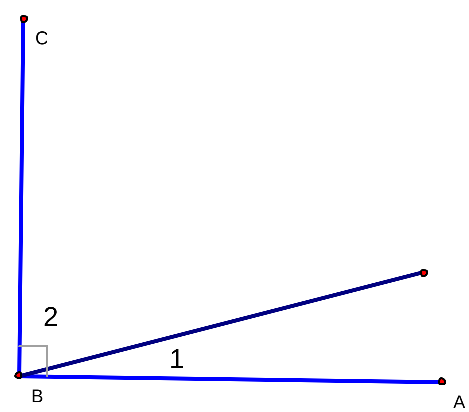


Theorem There is one and only one angle bisector for a given angle.

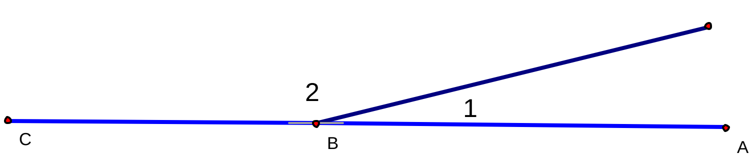
Exercise #11 Given an angle $\angle BAC$, construct using only a compass and a straightedge, the bisector $\overrightarrow{AD}$ of the given angle.

See textbook section 1.4, page 35.

Definition Two angles are **complementary** if their sum is 90°.

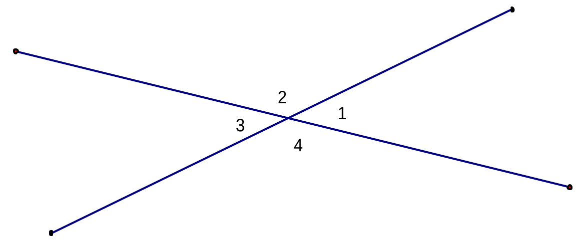


Two angle are **supplementary** if their sum is 180°.



Definition When two lines intersect, the pairs of nonadjacent angles formed are known as **vertical angles.**

Example Draw two intersecting lines.



a) Which angles are vertical angles?

$\angle 1$ and $\angle 3$

$\angle 2$ and $\angle 4$

b) Which angles are supplementary?

$\angle 1$ and $\angle 2$

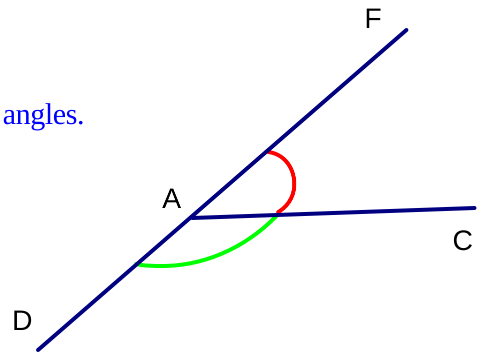
$\angle 2$ and $\angle 3$

$\angle 3$ and $\angle 4$

$\angle 4$ and $\angle 1$

Exercise #11 (1.4 - # 13) $\angle FAC$ and $\angle CAD$ are adjacent and $\overrightarrow{AF}$ and $\overrightarrow{AD}$ are opposite rays. What can you conclude about $\angle FAC$ and $\angle CAD$?

They are supplementary angles.

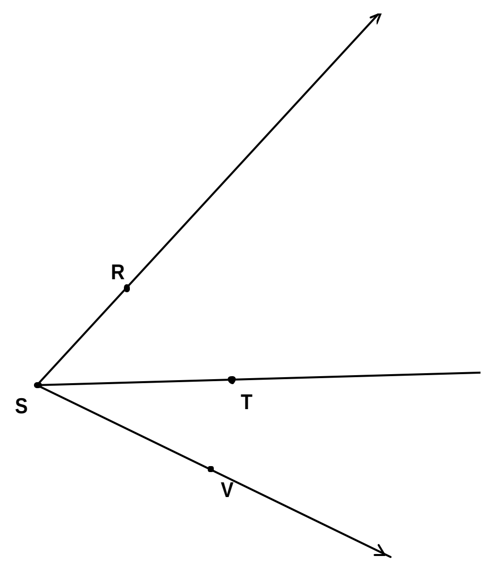


Exercise #12 (1.4 - #16) Given: $m\angle RST = 2x + 9$

$m\angle TSV = 3x - 2$

$m\angle RSV = 67^{\circ}$

Find: x.



STATEMENTS	REASONS
1. T is in the interior of $\angle RSV$.	1. Given.
2. $m\angle VST + m\angle TSR = m\angle RSV$	2. Angle – addition postulate
3. $2x + a + 3x - 2 = 67$	3. Substitution
4. $5x + 7 = 67$	4. Substitution (combining like terms)
5. $5x = 60$	5. Addition property of equality
6. $x = \frac{60}{5}$ =12	6. Multiplication property of equality