Definition A **STATEMENT** is a group of words and symbols that can be classified collectively as true or false.

Exercise #1 (1.1 - #1) Which sentences are statements? If a sentence is a statement, classify it as true or false.

a) Where do you live? not a statement

b) $4+7\neq 5$ statement ; true

c) Washington was the first U.S president. statement ; true

d) $x+3=7$ when $x=5$. statement ; false

Note: We represent statements by letters such as P, Q, and R.

Definition The **NEGATION** of a given statement P makes a claim opposite that of the original statement.

If P is a statement, $\sim P$ (read "not P) indicates its negation.

Exercise #2 (1.1 - #3) Give the negation of each statement.

a) Christopher Columbus crossed the Atlantic Ocean.
C. Columbus did not cross the Atlantic Ocean.

b) All jokes are funny. Some jokes are not funny

c) $2+5=7$ $2+5\neq 7$

d) Some dogs can fly. All dogs cannot fly
(Dogs cannot fly).

Definition A **TRUTH TABLE** is a table that provides the truth values of a statement by considering all possible true/false combinations of the statement's components.

P	$\sim P$
T	F
F	T

→ If P is true, then $\sim P$ is false.
→ When P is false, $\sim P$ is true.

COMPOUND STATEMENTS

Statements can be combined to form compound statements:

Definition A **CONJUNCTION** is a statement of the form ***P* and *Q*.** $\boxed{P \wedge Q}$

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

For the conjunction to be true, it is necessary for P to be true *and* Q to be true.

Exercise #3
(B1,Example 2)

Let P="Babe Ruth played baseball" and Q="4 + 3 <5." Classify as true or false:

a) $P \wedge Q$ $T \wedge F$ False

b) $P \wedge \sim Q$ $T \wedge T$ True

Definition A **DISJUNCTION** is a statement of the form ***P* or *Q*.** $\boxed{P \vee Q}$

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

A disjunction is false only if P and Q are both false.

Example: You can join the Math Club if you have an A average or you are enrolled in a mathematics class.

Exercise #4
(B1,Example 3)

Let P="Babe Ruth played baseball" and Q="4 + 3 <5." Classify as true or false:

a) $P \vee Q$ $T \vee F$ True

b) $P \vee \sim Q$ $T \vee T$ True

Exercise #5
(B1 - #3, 4, 7)

Statement P is true, Q is true, and R is false. Classify each statement as true or false.

a) $P \wedge Q$
$T \wedge T$
T

b) $Q \wedge R$
$T \wedge F$
F

c) $P \wedge (Q \vee R)$
$T \wedge (T \vee F)$
$T \wedge T$
T

Definition An **IMPLICATION or CONDITIONAL** is a statement of the form **"If *P*, then *Q*."**

Note: P is called the *antecedent* (or *hypothesis)*
Q is called the *consequent (*or *conclusion)*

$$\boxed{P \to Q}$$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

The conditional statement makes a promise and fails to satisfy the conditions of this promise only when P is true and Q is false.

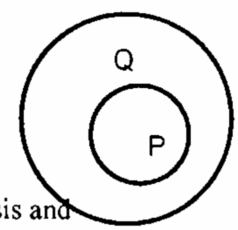
Example Consider the claim, "If you are good, then I'll give you a dollar."
The only way the claim is false is when "you are good, but I don't give you the dollar."

CONVERSE . INVERSE. CONTRAPOSITIVE

Lewis Carroll, the author of *Alice's Adventures in Wonderland* and *Through the looking Glass*, was a mathematician teacher who wrote stories as a hobby. His books contain many amusing examples of both good and deliberately poor logic. Consider the following conversation held at the Mad Hatter's tea Party.

" Then you should say what you mean,", the March Hare went on.
"I do,", Alice hastily replied; "at least – at least I mean what I say – that's the same thing, you know."
" Not the same thing a bit!" said the Hatter. "Why, you might just as well say that 'I see what I eat' is the same thing as 'I eat what I see'!"
"You might just as well say, "added the March Hare, "that 'I like what I get' is the same as 'I get what I like'!"
"You might just as well say," added the Dormouse, who seemed to be talking in his sleep, "that 'I breathe when I sleep' is the same thing as 'I sleep when I breathe'!"
"It *is* the same thing with you," said the Hatter, and here the conversation dropped, and the party went silent for a minute.

Carroll is playing here with pairs of related statements and the Hatter, the Hare, and the Dormouse are right: the sentences in each pair do not say the same thing at all.



Conditional statement: $\boxed{P \to Q}$ If P, then Q. (P implies Q)

Its **CONVERSE** $\boxed{Q \to P}$ If Q, then P.

- The converse of a conditional statement is formed by interchanging its hypothesis and conclusion.
- The converse of a true statement may be false. It is also possible that it may be true, but in either case a statement and its converse do not have the same meaning.

Its **INVERSE**: $\boxed{\sim P \to \sim Q}$ If *not P*, then *not Q*.

- The inverse of a conditional statement is formed by denying both its hypothesis and conclusion.

Its **CONTRAPOSITIVE**: $\boxed{\sim Q \to \sim P}$ If *not Q*, then *not P*.

- The contrapositive of a conditional statement is formed by interchanging its hypothesis and conclusion and denying both.

$P \to Q$ $\quad P \wedge Q$ $\quad P \vee Q$

Exercise #6 (1.1 - #5, 6, 9)

Classify each statement as simple, conditional, a conjunction, or a disjunction.

a) If Alice plays, the volleyball team will win. CONDITIONAL

b) Alice played and the team won. CONJUNCTION

c) Matthew is playing shortstop. SIMPLE

Exercise #7 (1.1 - #11, 12, 16)

State the hypothesis and the conclusion of each statement.

a) If you go to the game, then you will have a great time.

Hypothesis: if you go to the game

Conclusion: you will have a great time.

b) If two cords of a circle have equal lengths, then the arcs of the chords are congruent.

Hypothesis: if two cords of a circle have equal lengths

Conclusion: the arcs of the chords are congruent

c) Vertical angles are congruent when two lines intersect.

Hypothesis: if two lines intersect

Conclusion: Vertical angles are congruent

Exercise #8

1 $P \to Q$

CONVERSE: $Q \to P$

INVERSE: $\sim P \to \sim Q$

CONTRAPOSITIVE $\sim Q \to \sim P$

Identify the relationship of each of the lettered statements to the numbered statement if possible. Write "converse," "inverse," "contrapositive," "original statement," or "none,", as appropriate. P = if a kangaroo is a lady $\quad P \to Q$
Q = it doesn't need a handbag

1. Lady kangaroos do not need handbags.
   a) If a kangaroo is not a lady, it needs a handbag. INVERSE
   b) If it needs a handbag, then it is not a lady kangaroo. CONTRAPOSITIVE
   c) A kangaroo does not need a handbag if it is a lady. ORIGINAL STATEMENT

Exercise #9

Write the inverse, converse, and contrapositive of the following statement:

P → Q

"If you live in Atlantis, then you need a snorkel."

$\sim P \to \sim Q$ a) Inverse: If you don't live in Atlantis, you don't need a snorkel

$Q \to P$ b) Converse: if you need a snorkel, then you live in Atlantis

$\sim Q \to \sim P$ c) Contrapositive: If you don't need a snorkel, then you don't live in Atlantis

Definition Two statements are **logically equivalent** if their truth values are the same for all possible true/false combinations of their components.

Exercise #10 (B1,Example 5) Write the contrapositive of the given statement and then show, using a truth table, that the conditional statement is logically equivalent to its contrapositive.

$P \rightarrow Q$

" If I am sleeping, I am breathing."

Contrapositive: $\sim Q \rightarrow \sim P$: If I am not breathing, then I am not sleeping.

$\boxed{(P \rightarrow Q) \leftrightarrow (\sim Q \rightarrow \sim P)}$

P	Q	$P \rightarrow Q$	$\sim Q$	$\sim P$	$\sim Q \rightarrow \sim P$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Definition A TAUTOLOGIE is a statement that is true for all possible truth value of its components.

Exercise #11 (B1 - #15, 18) Form a truth table and determine all possible truth values for the given statement. Is the given statement a tautology?

a) $(P \vee Q) \rightarrow P$

P	Q	$P \vee Q$	$(P \vee Q) \rightarrow P$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	T

Not a tautologie

b) $[(P \rightarrow Q) \wedge P] \rightarrow Q$

P	Q	$P \rightarrow Q$	$(P \rightarrow Q) \wedge P$	$[(P \rightarrow Q) \wedge P] \rightarrow Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Tautologie.

DEMORGAN'S LAWS

In the study of logic, DeMorgan's Laws (19th century) are used to describe the negation of the conjunction ($\wedge$) and disjunction ($\vee$).

1. $[\sim(P \wedge Q)] \leftrightarrow [\sim P \vee \sim Q]$	The negation of a conjunction is the disjunction of negations.
2. $[\sim(P \vee Q)] \leftrightarrow [\sim P \wedge \sim Q]$	The negation of a disjunction is the conjunction of negations.

Proof of DeMorgan's first law

We need to show that $[\sim(P \wedge Q)]$ and $[\sim P \vee \sim Q]$ have identical truth values.

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Proof of DeMorgan's second law (B1 - #25)

We need to show that $[\sim(P \vee Q)]$ and $[\sim P \wedge \sim Q]$ have identical truth values.

P	Q	$P \vee Q$	$\sim(P \vee Q)$	$\sim P$	$\sim Q$	$\sim P \wedge \sim Q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Exercise #12 Use DeMorgan's Laws to write the negation of the given statement.
(B1, #19, 21, 23)

a) $P \wedge Q$

$[\sim(P \wedge Q)] \leftrightarrow \sim P \vee \sim Q$

b) Mary is an accountant or hamburgers are health food.

Mary is not an accountand and hamburgers are not health food.

c) It is cold and snowing.

It is not cold or it is not snowing.

Exercise #13 Use a truth table to show that $[P \wedge \sim Q]$ is the negation of $P \rightarrow Q$.
(B1 - #29)

$$\boxed{[\sim(P \rightarrow Q)] \leftrightarrow [P \wedge \sim Q]}$$

P	Q	$P \rightarrow Q$	$\sim(P \rightarrow Q)$	$\sim Q$	$P \wedge \sim Q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

identical truth values

Exercise #14 Write the negation of the given statement.
(B1 - #30, 31)

a) If it is medicine, then it tastes bad.

$P \rightarrow Q$

The negation of $(P \rightarrow Q)$ is $(P \wedge \sim Q)$
it is medicine and it doesn't taste bad.

b) If I am good, then I can go to the movie.

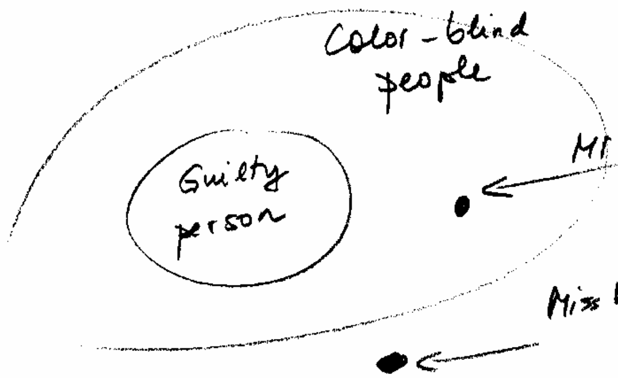
$P \rightarrow Q$

The negation of $(P \rightarrow Q)$ is $(P \wedge \sim Q)$.
I am good and I cannot go to the movie.

VALID ARGUMENTS

Suppose that during a trial a lawyer claims that, from the evidence presented, the guilty person is obviously color-blind and that everyone on the jury accepts this as true. Then he produces proof that Mr. Black is color-blind. Must the jury conclude that Mr. Black is guilty? Suppose also that it is established that Miss White is not color-blind. Must Miss White be innocent?

if a person is guilty, then he/she is color blind.



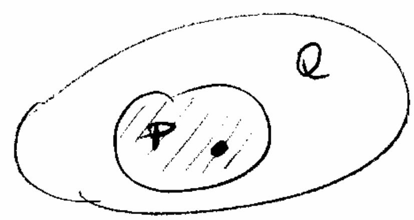
Mr. Black - we can't tell where Mr. Black belongs, so no conclusion is justified.

Miss White - Miss White is outside the larger circle, she cannot be inside the smaller one, and so she is not the guilty person.

Definition An **ARGUMENT** is a set of statements called **premises**, followed by a statement called the **conclusion.** In a **VALID ARGUMENT**, the truth of the premises forces a conclusion that must also be true.

LAW OF DETACHMENT

1. $P \to Q$	Premise 1
2. P	Premise 2
C. Q	Conclusion



Give the symbolic form and prove the Law of Detachment

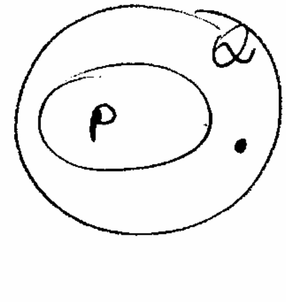
$[(P \to Q) \wedge P] \to Q$ We need to show that the statement is a tautologie.

P	Q	$P \to Q$	$(P \to Q) \wedge P$	$[(P \to Q) \wedge P] \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

tautologie

ATTENTION!!! INVALID ARGUMENT

1. $P \to Q$	Premise 1
2. Q	Premise 2
C. P	Conclusion



Exercise #15 (B2 - #1,2)

Use the Law of Detachment to draw a conclusion.

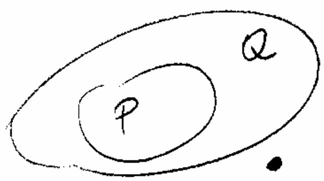
a) If two angles are complementary, the sum of their measures is 90^o. $\angle 1$ and $\angle 2$ are complementary.

CONCLUSION: their sum is $90°$

b) If it gets hot this morning, we will have to turn on the air conditioner. It is hot this morning.

CONCLUSION: We'll have to turn on the air conditioner

LAW OF NEGATIVE INFERENCE



1. $P \to Q$	Premise 1	
2. $\sim Q$	Premise 2	
C. $\sim P$	Conclusion	

Give the symbolic form and prove the Law of Negative Inference

Show that $[(P \to Q) \wedge \sim Q] \to \sim P$ is a tautologic

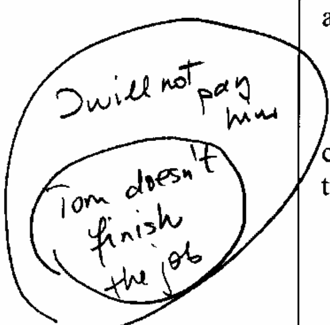
P	Q	$P \to Q$	$\sim Q$	$(P \to Q) \wedge \sim Q$	$\sim P$	$[(P \to Q) \wedge \sim Q] \to \sim P$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Exercise #16 (B2 - #7,8)

Use the Law of Negative Inference to draw a conclusion.

a) If Tom doesn't finish the jobm then I will not pay him. I did pay Tom for the job.

CONCLUSION: Tom finished the job.



c) If the traffic light changes, then you can travel through the intersection. You cannot travel through the intersection.

CONCLUSION: The light did not change.

LAW OF SYLLOGISM

1. $P \to Q$	Premise 1
2. $Q \to R$	Premise 2
C. $P \to R$	Conclusion

Give the symbolic form and prove the Law of Syllogism $[(P \to Q) \wedge (Q \to R)] \to (P \to R)$

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \wedge (Q \to R)$	$P \to R$	$[(P \to Q) \wedge (Q \to R)] \to (P \to R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	F	F	T	T
F	F	F	T	F	F	T	T

tautologie

Exercise #17 (B2 - #9) Use the Law of Syllogism to draw a conclusion.

a) If Izzi lives in Chicago, then she lives in Illionois. If a person lives in Illinois, then she lives in the Midwest.

CONCLUSION: if izzi lives in Chicago, then she lives in the midwest.

Exercise #18 (B2 - #15, 16) Determine which arguments are valid.

a) 1. If Bill and Mary stop to visit, I'll prepare a meal.
   2. Bill stopped to visit at 5 p.m.

C. I prepared a meal.

VALID: YES NO

b) 1. If it turns cold and snows, I'll build a fire in the fireplace.
   2. The temperature began to fall around 3 p.m.
   3. It began snowing before 5 p.m.

C. I built a fire in the fireplace.

VALID: YES NO

LAW OF DENIAL (B2 - #21)

$1. P \vee Q$ Premise 1
$2. \sim Q$ Premise 2

C. P Conclusion

Proof of the Law of Denial

Show that $[(P \vee Q) \wedge \sim Q] \rightarrow P$ is a tautologic

P	Q	$P \vee Q$	$\sim Q$	$(P \vee Q) \wedge \sim Q$	$[(P \vee Q) \wedge \sim Q] \rightarrow P$
T	T	T	F	F	T
T	F	T	T	T	T
F	T	T	F	F	T
F	F	F	T	F	T

Exercise #19 (B2 - # 22)

Use the Law of Denial to draw a conclusion.

Terry is sick or hurt.
Terry is not hurt.

CONCLUSION: Terry is sick.

REASONING

1. INTUITION – With intuition, a sudden insight allows one to make a statement without applying any formal reasoning.

2. INDUCTION – Using specific observations and experiments to draw a general conclusion.
3. DEDUCTION – The type of reasoning in which the knowledge and acceptance of selected assumptions guarantees the truth of a particular conclusion.

Exercise #20 (1.1 - #25, 26)

Name the type of reasoning (if any) used.

a) While participating in an Easter egg hunt, Sarah notices that each of the seven eggs she has found is numbered. Sarah concludes that all eggs used for the hunt are numbered.

Intuition ___ Induction ✓ Deduction ___

b) You walk into your geometry class, look at the teacher, and conclude that you will have a quiz today.

Intuition ✓ Induction ___ Deduction ___

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