TEST 2 @ 140 points

Show your work for credit. Write all responses on separate paper. Please write only on one side and clearly label the exercises.

1. Let $y = 3x^2 + 4x + 2$ be the equation of a parabola.

   a) What are the coordinates of the vertex?

   b) Write the equation in vertex form.

   c) What is the domain of the function?

   d) What is the range of the function?

   e) Graph the above parabola showing how its graph is obtained from the basic parabola $y = x^2$.

---

2. a) Find a polynomial function of least degree with leading coefficient 1 and having only real coefficients knowing that 2 and 1 + *i* are zeros of multiplicity one.

   b) Write the polynomial function in standard form.

---

3. Use Descartes' rule of signs to determine the possible number of positive real zeros and negative real zeros for $f(x) = 2x^5 + 5x^4 - 8x^3 - 14x^2 + 6x + 9$.

---

4. Consider the following polynomial function $f(x) = 3x^4 - 4x^3 - 22x^2 + 15x + 18$.
Questions a-g below relate to this polynomial function.

   a) Use the leading term to describe the long-term behavior of this function; that is, what happens as $x \to \pm\infty$.

   b) Use synthetic division to divide $f(x)$ by $x - 1$ and relate dividend, divisor, quotient and remainder in an equation.

   c) Compute and compare the values of $f(1)$ and $f(2)$. What can you conclude using the intermediate value theorem?

   d) State why the condition for the theorem on rational zeros is satisfied and use the theorem on rational zeros to list all possible rational zeros for $f(x)$.

   e) Find all the zeros of the polynomial.

   f) What are the x- and y-intercepts of the graph?

   g) Sketch a graph of $f(x)$ showing how it passes through its intercepts.

5. Identify all the asymptotes for the following functions:

a) $f(x)=\dfrac{8x^2+2x-10}{2x^2-3x-5}$

b) $g(x)=\dfrac{2x^2+5}{x-3}$

---

6. Consider $f(x)=\dfrac{3x}{x^2-x-2}$.

a) Factor the denominator.

b) What are the vertical asymptotes?

c) What is the horizontal asymptote?

d) What are the intercepts for this function?

e) Plot additional points, as necessary, to get the shape of this function and sketch a graph.

MATH 130

TEST #2 – SOLUTIONS

(1) $y = 3x^2 + 4x + 2$

(a) $V(x_v, y_v)$ $\quad x_v = \frac{-b}{2a} = \frac{-4}{2(3)} = \frac{-2}{3}$

$$y_v = 3\left(\frac{-2}{3}\right)^2 + 4\left(\frac{-2}{3}\right) + 2$$

$$y_v = 3 \cdot \frac{4}{9} - \frac{8}{3} + 2$$

$$y_v = \frac{4}{3} - \frac{8}{3} + 2 \qquad y_v = \frac{-4}{3} + 2 = \frac{2}{3}$$

$$\boxed{V\left(\frac{-2}{3}, \frac{2}{3}\right)}$$

(b) $y = a(x - x_v)^2 + y_v$

$$\boxed{y = 3\left(x + \frac{2}{3}\right)^2 + \frac{2}{3}}$$

(c) $\boxed{x \in \mathbb{R}}$

(d) $y = 3x^2 + 4x + 2$ parabola opens upward

the Vertex = the minimum point

$y \in [y_v, \infty)$

$$\boxed{y \in \left[\frac{2}{3}, \infty\right)}$$

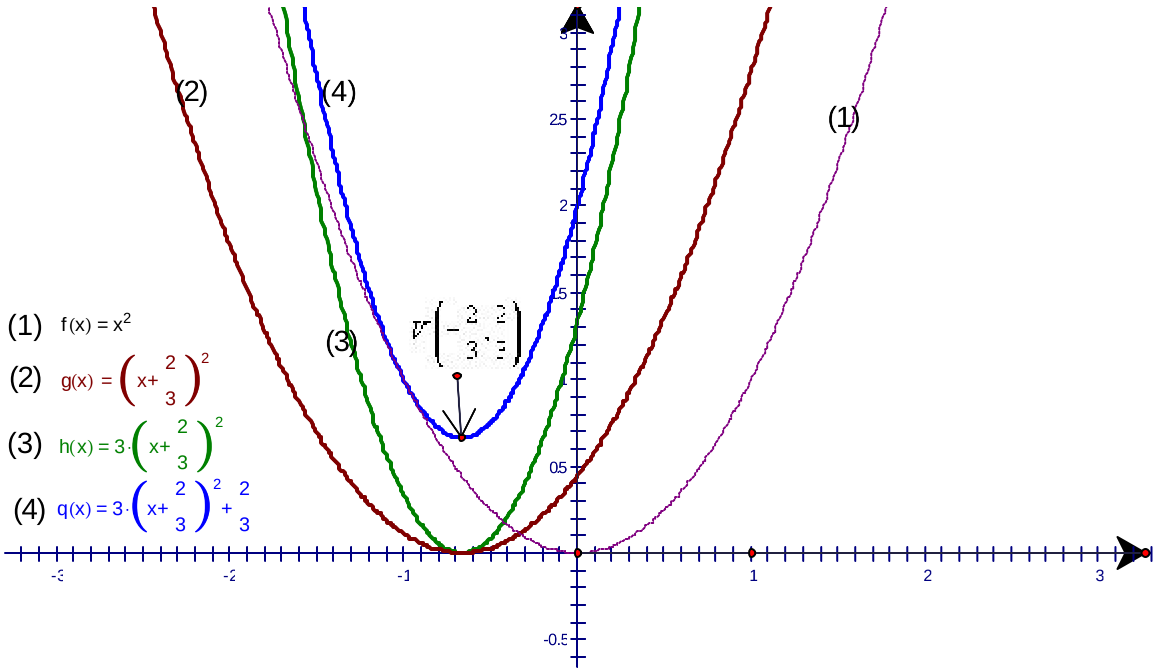
(e) $y = 3\left(x + \frac{2}{3}\right)^2 + \frac{2}{3}$

1st $\quad y = x^2$ basic parabola

2nd $\quad y = \left(x + \frac{2}{3}\right)^2$ shift $y = x^2$ to the left $\frac{2}{3}$

3rd $\quad y = 3\left(x + \frac{2}{3}\right)^2$ stretch $y = \left(x + \frac{2}{3}\right)^2$ vertically by a factor of 3

4th $\quad y = 3\left(x + \frac{2}{3}\right)^2 + \frac{2}{3}$ shift $y = 3\left(x + \frac{2}{3}\right)^2$ up $\frac{2}{3}$



(1) $f(x) = x^2$
(2) $g(x) = \left(x+\frac{2}{3}\right)^2$
(3) $h(x) = 3\cdot\left(x+\frac{2}{3}\right)^2$
(4) $q(x) = 3\cdot\left(x+\frac{2}{3}\right)^2+\frac{2}{3}$
$V\left(-\frac{2}{3};\frac{2}{3}\right)$
(1)
(2)
(3)
(4)

(2) a) $x=2$ zero of multiplicity 1
$x=1+i$ zero of multiplicity 1
$\Rightarrow$ $x=1-i$ is also a zero of multiplicity 1
(Conjugates Theorem)

So, $x-2 \mid f(x)$
$x-(1+i) \mid f(x)$
$x-(1-i) \mid f(x)$
$\Rightarrow$ $\boxed{f(x)=(x-2)(x-(1+i))(x-(1-i))}$

b) $f(x)=(x-2)(x-1-i)(x-1+i)$
$f(x)=(x-2)((x-1)-i)((x-1)+i)$
$f(x)=(x-2)((x-1)^2-i^2)$
$f(x)=(x-2)(x^2-2x+1+1)$
$f(x)=(x-2)(x^2-2x+2)$
$\boxed{f(x)=x^3-4x^2+6x-4}$

(3) $f(x)=2x^5\underbrace{+5x^4-8x^3}_{1}\underbrace{-14x^2+6x}_{2}+9$

2 variations in sign in $f(x)$ $\Rightarrow$ $\boxed{\text{2 or 0 positive roots}}$

$f(-x)=2(-x)^5+5(-x)^4-8(-x)^3-14(-x)^2+6(-x)+9$
$f(-x)=\underbrace{-2x^5+5x^4}_{1}\underbrace{+8x^3-14x^2}_{2}\underbrace{-6x+9}_{3}$

3 variations in sign in $f(-x)$ $\Rightarrow$ $\boxed{\text{3 or 1 negative root}}$

(4) $f(x) = 3x^4 - 4x^3 - 22x^2 + 15x + 18$

(a) Since the degree is even and the leading coefficient is positive, when $\boxed{\begin{array}{ll} x \to -\infty, & f(x) \to \infty \\ x \to \infty, & f(x) \to \infty. \end{array}}$

(b)

x	3	-4	-22	15	18	
1	3	-1	-23	-8	10	R

$\boxed{f(x) = (x-1)(3x^3 - x^2 - 23x - 8) + 10.}$

(c) $f(1) = 10$ ($f(1)$ = remainder when dividing $f(x)$ by $x-1$)

$f(2) = -24$ ($f(2)$ = remainder when dividing $f(x)$ by $x-2$)

	3	-4	-22	15	18
2	3	2	-18	-21	-24

Since $f(x)$ changes sign on the interval $[1,2]$, there is some $r \in (1,2)$ such that $f(r) = 0$

(d) Since the coefficients of $f(x)$ are all integers, we may conclude that the rational zeros of f are in the set $\boxed{\pm\{1, 2, 3, 6, 9, 18, \frac{1}{3}, \frac{2}{3}\}}$

$\frac{p}{q} = \frac{\text{factors of } 18}{\text{factors of } 3} = \pm \frac{1, 2, 3, 6, 9, 18}{1, 3}$

(e) $f(x) = 3x^4 - 4x^3 - 22x^2 + 15x + 18$

	3	-4	-22	15	18
3	3	5	-7	-6	0
$-\frac{2}{3}$	3	3	-9	0	

$f(x) = (x-3)(3x^3+5x^2-7x-6)$

$f(x) = (x-3)(x+\frac{2}{3})(3x^2+3x-9)$

$f(x) = 3(x-3)(x+\frac{2}{3})(x^2+x-3)$

$f(x) = (x-3)(3x+2)(x^2+x-3)$

$x^2+x-3=0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{1+12}}{2} = \frac{-1 \pm \sqrt{13}}{2} \begin{cases} \approx 1.3 \\ \approx -2.3 \end{cases}$$

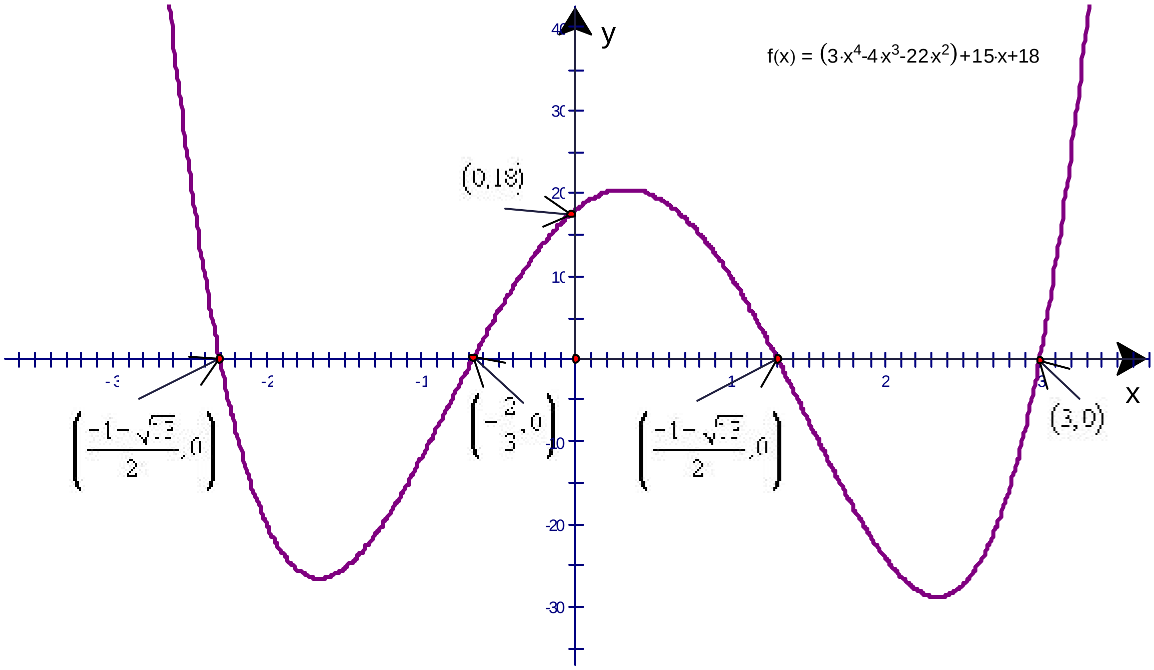
The zeros of f are $\begin{cases} x = 3 \\ x = -\frac{2}{3} \\ x = \frac{-1+\sqrt{13}}{2} \\ x = \frac{-1-\sqrt{13}}{2} \end{cases}$

(f) The x-intercepts are the zeros of f
(the solutions of $f(x)=0$)
Therefore, the x-intercepts are

x-n $\boxed{\begin{array}{c} (3,0) \\ (-\frac{2}{3},0) \\ (\frac{-1+\sqrt{13}}{2},0) \\ (\frac{-1-\sqrt{13}}{2},0) \end{array}}$

The y-intercept: $x=0$, $f(0) = 18$

y-n $\boxed{(0,18)}$



f(x) = (3·x4-4·x3-22·x2)+15·x+18
y
x
(0,18)
(3,0)

(5) $f(x) = \dfrac{8x^2+2x-10}{2x^2-3x-5}$

(a) $f(x) = \dfrac{8x^2+2x-10}{(2x-5)(x+1)}$

Vertical asymptotes: $\boxed{x = \frac{5}{2} \quad x = -1}$

Horizontal asymptote: $\boxed{y = 4}$ (degree numerator = = degree denominator)

(b) $g(x) = \dfrac{2x^2+5}{x-3}$

Vertical asymptote: $\boxed{x=3}$

No horizontal asymptote because degree numerator > > degree denominator

Oblique asymptote

$\boxed{y = 2x+6}$

$$\begin{array}{r|l} & 2x + 6 \\ \hline x-3 & 2x^2 + 5 \\ & -2x^2 + 6x \\ \hline & \quad 6x + 5 \\ & \quad -6x + 18 \\ \hline & \qquad 23 \end{array}$$

(6) $f(x) = \dfrac{3x}{x^2 - x - 2}$

(a) $f(x) = \dfrac{3x}{(x-2)(x+1)}$

(b) The vertical asymptotes are along the lines $\boxed{\begin{matrix} x=2 \\ x=-1 \end{matrix}}$

(c) The horizontal asymptote is along the line $\boxed{y=0}$
(degree numerator < degree denominator)

(d) x-int: $y = 0 \iff 3x = 0 \iff x = 0$

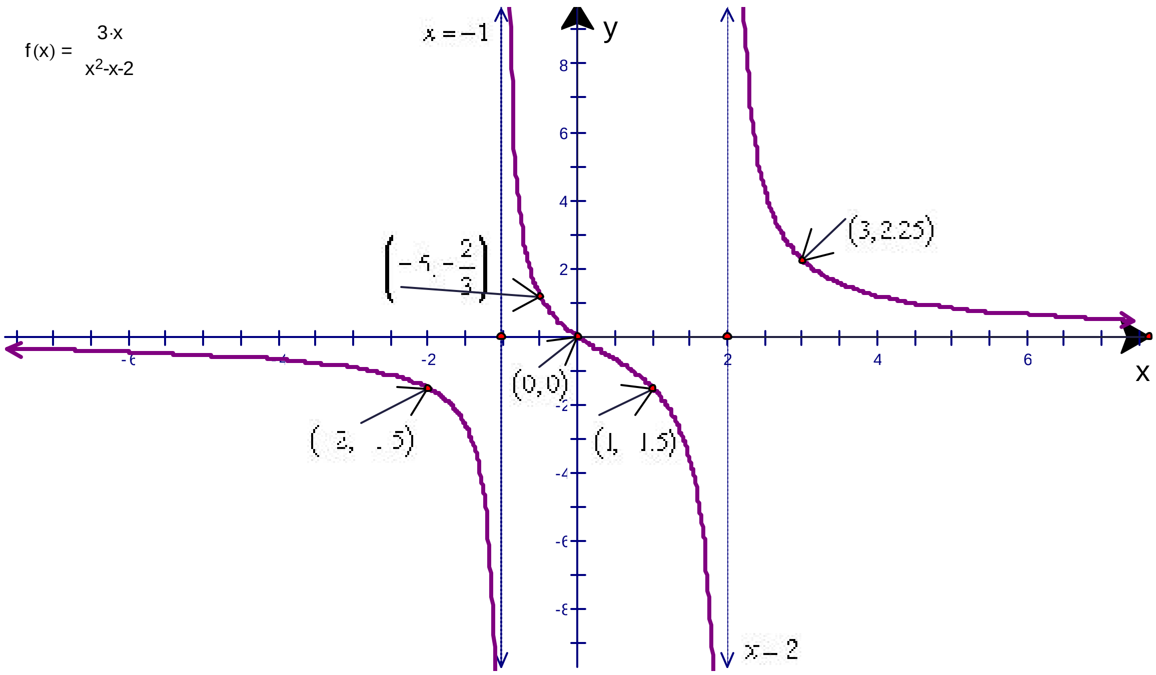
The x-int and y-int is $\boxed{(0,0)}$

(e)

x	$-\infty$	-2	-1	-0.5	0	1	2	3	∞
$f(x)$	0	-1.5	$-\infty \mid \infty$	$\frac{2}{3}$	0	-1.5	$-\infty \mid \infty$	2.25	0
	$\boxed{y=0}$ H.A		$\boxed{x=-1}$ V.A				$\boxed{x=2}$ V.A.		$\boxed{y=0}$ H.A

Behavior near V.A. $x=2$ $\begin{cases} x \to 2^+, \ f(x) \to \infty & \left(\dfrac{6}{(+0)(3)}\right) \\ x \to 2^-, \ f(x) \to -\infty & \left(\dfrac{6}{(-0)3}\right) \end{cases}$

Behavior near V.A. $x=-1$ $\begin{cases} x \to -1^+, \ f(x) \to \infty & \left(\dfrac{-3}{(-3)(+0)}\right) \\ x \to -1^-, \ f(x) \to -\infty & \left(\dfrac{-3}{(-3)(-0)}\right) \end{cases}$


f(x) = 3·x / (x^2-x-2)
x = −1
y
x
(0,0)
(3,2.25)
8
6
4
2
-2
-4
-6
-8
2
4
6