Sections 4.5 & 4.6
Exponential and Logarithmic Equations and Their Applications

In class work: Solve each problem.

Solve for x in Problems 1 – 9.

1) $10^{x+3} = 5e^{7-x}$ 2) $2e^{3x} = 4e^{5x}$ 3) $2x - 1 = e^{\ln x^2}$ 4) $9^x = 2e^{x^2}$

5) $5^x = 3^{2x-1}$ 6) $3^{x^2-4} = 27$ 7) $\log_8(x+5) - \log_8 2 = 1$

8) $10^{2x} + 3(10^x) - 10 = 0$ 9) $\log_2(\log_3 x) = -1$ 10) $e^x - e^{-x} = 1$

For problems 11 – 12, solve for t. Assume a and b are positive constants and k is nonzero.

11) $P = P_0 e^{kt}$ 12) $ae^{kt} = e^{bt}$, where $k \neq b$.

Simplify the expressions in 13 – 16 completely.

13) $5e^{\ln(A^2)}$ 14) $\ln(e^{2ab})$ 15) $\ln\left(\frac{1}{e}\right) + \ln AB$ 16) $2\ln(e^A) + 3\ln B^e$

Convert the functions in 17 – 18 into the form $P = P_0 a^t$. Which represent exponential growth and which represent exponential decay?

17) $P = 2e^{-0.5t}$ 18) $P = 15e^{0.25t}$

Convert the functions in 19 – 20 into the form $P = P_0 e^{kt}$.

19) $P = 15(1.5)^t$ 20) $P = 4(0.55)^t$

21) Find the inverse of $f(t) = 50e^{0.1t}$.

22) Find the inverse of $f(t) = 1 + \ln t$.

24) The air in a factory is being filtered so that the quantity of pollutant, P (measured in mg/liter) is decreasing according to the equation $P = P_0 e^{-kt}$, where t represents time in hours. If 10% of the pollution is removed in the first five hours:

a) What percentage of the pollution is left after 10 hours?
b) How long will it take before the pollution is reduced by 50%?

25) If the size of a bacteria colony doubles in 5 hours, how long will it take for the number of bacteria to triple?

26) Suppose a certain radioactive substance has a half-life of 5 years. An object starts with 20 kg of the radioactive material.

a) How much of the radioactive material is left after 10 years?
b) The object can be moved safely when the quantity of the radioactive material is 0.1 kg or less. How much time must pass before the object can be moved?

27) The number of bacteria present in a culture after t hours is given by the formula $N = 1000e^{0.69t}$.

a) How many bacteria will be there after ½ hour?
b) How long will it be before there are 1,000,000 bacteria?
c) What is the doubling time?

28) You place $800 in an account that earns 4% annual interest, compounded annually. How long will it be until you have $2000?

29) (Textbook 4.5 # 66) At the World Championship races held at Rome's Olympic Stadium in 1987, American sprinter Carl Lewis ran the 100-m race in 9.86 sec. His speed in meters per seoncd after t seconds is closely modeled by the function defined by $f(t) = 11.65\left(1 - e^{-\frac{t}{1.27}}\right)$.

a) How fast was he running as he crossed the finish line?
b) After how many seconds was he running at the rate of 10 m per sec?

30) Textbook 4.5 # 68

Answers:

1) 0.515
2) -0.347
3) x=1
4) 1.81, 0.38
5) $\dfrac{\log_5 3}{2\log_5 3 - 1}$
6) $\pm\sqrt{7}$
7) 11
8) $\log 2$
9) $\sqrt{3}$
10) $\ln\dfrac{1+\sqrt{5}}{2}$
11) $\ln\left(P/P_0\right)/k$
12) $\dfrac{\ln a}{b-k}$
13) $5A^2$
14) 2ab
15) -1+lnA+lnB sec.
16) $2A + 3e\ln B$
17) $P = 2\left(0.61\right)^t$; decay;
18) $15(1.284)^t$
19) $15e^{0.41t}$
20) $P = 4e^{-0.6t}$
21) $f^{-1}(t) = 10\ln\left(\dfrac{t}{50}\right)$
22) $y = e^{t-1}$
24) a) 81%; b) 33 hours
25) 7.925 hours;
26) a) 5 kg; b) 38.2 years;
27) a) 1412 bacteria; b) 10 hours; c) 1 hour;
28) 23.4 years
29) a) 11.6451 m per sec; b) 2.4823 sec
30) a) b) 984 ft; c) 39 ft