## **DEFINITION ON SYMMETRY**

Type of symmetry	How to test for symmetry	What the graph looks like	Geometric meaning
Symmetry with respect to the <i>x</i> -axis	The equation is unchanged when $y$ is replaced by $-y$ .	y (x,y) 0 (x,-y) x	Graph is unchanged when reflected in the <i>x</i> -axis.
Symmetry with respect to the y-axis	The equation is unchanged when $x$ is replaced by $-x$ .	(-x,y) 0 x	Graph is unchanged when reflected in the y-axis.
Symmetry with respect to the origin	The equation is unchanged when $x$ is replaced by $-x$ and $y$ by $-y$ .	y (-x,-y) y (x,y) x	Graph is unchanged when rotated $180^{\circ}$ about the origin. This is the same as a reflection in the <i>x</i> -axis followed by a reflection in the <i>y</i> -axis.

TRANSFORMATION	EQUATION	HOW TO OBTAIN THE GRAPH	WHAT THE GRAPH LOOKS LIKE
Vertical shifts of graphs	y = f(x) + c, $(c > 0)$	Shift graph of $y = f(x)$ <b>upward</b> <i>c</i> units.	y y=f(x)+c y=f(x)
	y = f(x) - c, $(c > 0)$	Shift graph of $y = f(x)$ <b>downward</b> c units.	y=f(x)-c
Horizontal shifts of graphs	y = f(x-c),  (c > 0)	Shift graph of $y = f(x)$ to the <b>right</b> <i>c</i> units.	y = f(x+c) $y = f(x)$ $y = f(x-c)$
	y = f(x+c),  (c > 0)	Shift graph of $y = f(x)$ to the <b>left</b> <i>c</i> units	
Reflecting graphs	y = -f(x)	Reflect the graph of $y = f(x)$ in the <i>x</i> -axis.	y=f(-x) $y$ $y=f(x)$ $y=f(x)$
	y = f(-x)	Reflect the graph of $y = f(x)$ in the <b>y-axis</b> .	
Vertical stretching and shrinking of graphs	$y = af(x), \qquad (a > 1)$	Stretch the graph of $y = f(x)$ vertically by a factor of <i>a</i> .	y y=af(x) $a>1$ $y=f(x)$
	$y = af(x), \qquad (0 < a < 1)$	Shrink the graph of $y = f(x)$ vertically by a factor of <i>a</i> .	0 y=af(x) (0 <a<1 td="" x<=""></a<1>
Horizontal shrinking and stretching of graphs	$y = f(ax), \qquad (a > 1)$	Shrink the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{x}$ .	y y=f(ax) (a>1) y=f(x)
	$y = f(ax), \qquad (0 < a < 1)$	Stretch the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{a}$ .	0 y=f(ax) (1>a>0)

## TRANSFORMATIONS OF FUNCTIONS