Sections 2.3 & 2.4
Linear Functions. Equations of Lines. Curve Fitting

In class work: Complete all statements. Solve all exercises.

Linear Equation in Two Variables

Standard form: $ax + by = c$

Slope –Intercept form: $y = mx + b$, where m is the slope of the line, b is the y-intercept

Slope –Point form: $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) a point on the line.

Vertical Line: $x = k$, where k is a constant

Horizontal Line: $y = k$, where k is a constant.

Slope of a Line

$m = \frac{change\ in\ y}{change\ in\ x}$ as we move from one point to another on the line.

$m = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2}$

The slope m is the rate of change of y with respect to x.

Properties of Lines

Two distinct lines are parallel if and only if they have the same slope.

$$l_1 \parallel l_2 \Leftrightarrow m_1 = m_2$$

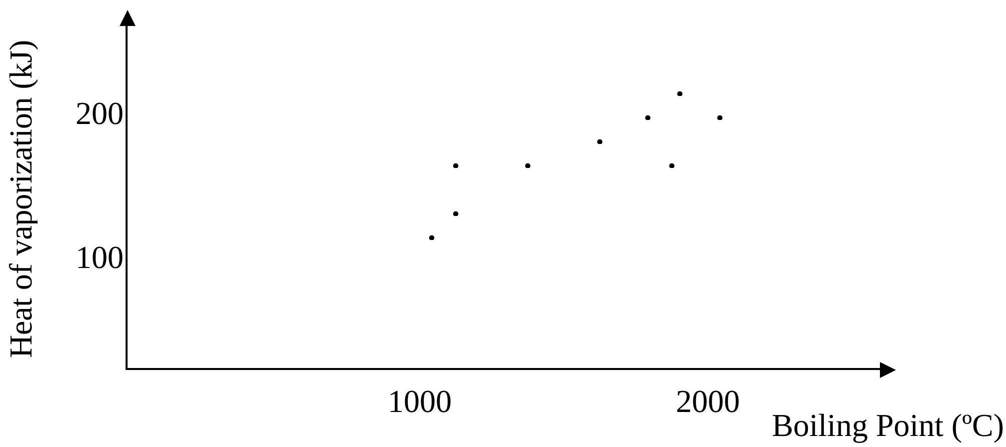
Two lines are perpendicular if and only if the product of their slopes is -1.

$$l_1 \perp l_2 \Leftrightarrow m_1 \cdot m_2 = -1$$

Lines of best fit. Linear regression
Linear Interpolation and Extrapolation

An equation that relates two variables can be used to find values of one variable from the value of the other. We will consider methods for fitting a linear equation to a collection of data points.

For example, the figure below is called a **scatterplot.** Each point on a scatterplot exhibits a pair of measurements about a single event. The points on a scatterplot may or may not show some sort of a pattern. In our example, although the points do not lie on a straight line, they seem to be clustered around some imaginary line.



Linear regression:

If the data in a scatterplot are roughly linear, we can estimate the location of an imaginary "line of best fit" that passes as close as possible to the data points. We can use this line to make predictions about the data (when drawing the line that "fits" the data points as best as we can, we try to end up with roughly equal numbers of data points above and below our line). The process of predicting a value of y based on a straight line that fits the date is called **linear regression**, and the line itself is called **the regression line.** The equation of the regression line is usually used (instead of the graph) to predict values.

Linear Interpolation:

The process of estimating between known data points is called interpolation.

Linear Extrapolation:

The process of making predictions beyond the range of known data is called extrapolation.

1. Write an equation for the line described (2.4: # 7,10, 35, 37):

a) Through $(-5,4)$ and slope $m=-\frac{2}{3}$.

b) Through $(8,-1)$ and $(4,3)$.

c) Through $(-1,4)$, parallel to $x+3y=5$.

d) Through $(1,6)$, perpendicular to $3x+5y=1$.

2. Find an equation for the graph shown and state the significance of the slope in terms of the problem.

a)

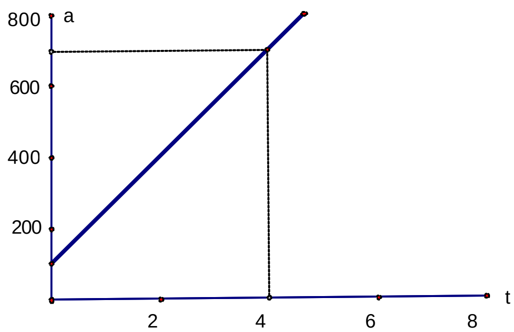
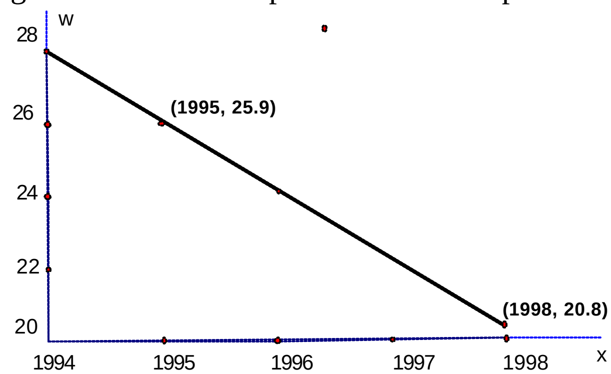


Figure shows the altitude, a, (in feet) of a skier t minutes after getting on a ski lift.

b)



The graph shows the number of food stamp recipients, *y*, (in millions) from 1994 through 1998. (2.3: # 71)

3. (2.4: # 47) The table lists the average annual cost (in dollars) of tuition and fees at private 4-year colleges for selected years.

Year	Tuition and Fees (in $)
1990	9,340
1992	10,448
1994	11,719
1996	12,994
1998	14,709
2000	16,233
2002	18,116

a) Determine a linear function that models the cost in in terms of the number of years since 1990 using the points (0,9340) and (12, 18116).

b) Use your equation to approximate tuition and fees in 1995. Compare it with the actual value.

c) What is the slope of your line and what does it represent?

4. (2.4: #49) When the Celsius temperature is 0°, the corresponding Fahrenheit temperature is 32°. When the Celsius temperature is 100°, the corresponding Fahrenheit temperature is 212°. Let C reperesent the Celsius temperature and F the Fahrenheit temperature.

a) Express F as an exact linear function of C.
b) For what temperature is $F=C$?

5. 1. A spring is suspended from the ceiling. The table shows the length of the spring in centimeters as it is stretched by hanging various weights from it.

Weight, kg	3	4	8	10	12	15	22
Length, cm	25.75	25.88	26.36	26.6	26.84	27.2	28.04

a) Plot the data. Do the points lie on a straight line? How can you know for sure that the relationship between the two variables is linear?
b) If the relationship is linear, find an equation for the line.
c) If the spring is stretched to 27.56 cm, how heavy is the attached weight?

6. (5.1: #80) Find an equation of the parabola that passes through $(-3,4)$, $(-2,1)$, and $(2,9)$.

7. (5.1 #29, 56, 70) Solve the following systems:

a) $\begin{cases} \frac{2x-1}{3}+\frac{y+2}{4}=4 \\ \frac{x+3}{2}-\frac{x-y}{3}=3 \end{cases}$ b) $\begin{cases} 8x-3y+6z=-2 \\ 4x+9y+4z=18 \\ 12x-3y+8z=-2 \end{cases}$ c) $\begin{cases} \frac{1}{x}+\frac{3}{y}=\frac{16}{5} \\ \frac{5}{x}+\frac{4}{y}=5 \end{cases}$

(A: (5,2)) (A: $\left(\frac{-3}{4},\frac{5}{3},\frac{3}{2}\right)$) (A: (5,1)

8. (5.1 # 59, 64) Solve the system in terms of the variable x.

a) $\begin{cases} x-2y+3z=6 \\ 2x-y+2z=5 \end{cases}$ b) $\begin{cases} x-y+z=-6 \\ 4x+y+z=7 \end{cases}$

(A: (x, -4x+3, -3x+4)) (A: $\left(x,\frac{-3x+13}{2},\frac{-5x+1}{2}\right)$)

9. (5.1 # 95) Pat Summers wins $200,000 in the Louisiana lottery. He invests part of the money in real estate with an annual return of 3% and another part in a money market account at 2.5% interest. He invests the rest, which amounts to $80,000 less than the sum of the other two parts, in certificates of deposit that pay 1.5% . If the total annual interest on the money is $4990, how much was invested at each rate? (A: $100,000, 40,000, 60,000)

10. (5.1 # 92) A glue company needs to make some glue that it can sell for $120 per barrel. It wants to use 150 barrels of glue worth $100 per barrel, along with some glue worth $150 per barrel, and some glue worth $190 per barrel. It must use the same number of barrels of $150 and $190 glue. How much of the $150 and $190 glue will be needed? How many barrels of $120 glue will be produced?

(A: 30, 30, 210)