Math 130 Spring 06

January 9 & 11, 2006

REVIEW

Sections 1.4&1.5 Quadratic Equations

Applications and Modeling with Quadratic Equations

In class work: Complete all statements. Solve all exercises.

Definition The standard form of a quadratic or second degree equation in one variable is $ax^2 + bx + c = 0$ where $a, b, c \in \mathbb{R}; a \neq 0$.

Solving quadratic equations

(1) THE FACTORING METHOD – used to solve equations of the form $ax^2 + bx + c = 0$ that are factorable.

Zero-Factor Property: The product of two factors equals zero if and only if one of the factors (or both) is zero.

$$ab = 0 \Leftrightarrow a = 0 \ \ or \ \ b = 0$$

(2) EXTRACTION OF ROOTS – used to solve equations of the form

$$x^2 = k \text{ or } (x - p)^2 = k.$$

(3) COMPLETING THE SQUARE $ax^2 + bx + c = 0$

Step 1: Coefficient of x^2 equal to 1.

Step 2: Constant isolated.

Step 3: Complete the square by adding $\left(\frac{1}{2} \cdot coeficient\ of\ x\right)^2$ to both sides of the equation.

(4) QUADRATIC FORMULA If $ax^2 + bx + c = 0$, then the solutions are given by:

$$\boxed{x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

<u>Definition</u> The discriminant of a quadratic equation is ______________________________.

<u>Properties</u> **(1)** If $a,b,c \in \mathbb{R}$, then:

If $\Delta > 0$, the equation has two distinct real solutions.

If $\Delta = 0$, the equation has one real (rational) solution.

If $\Delta < 0$, the equation has complex (nonreal) solutions.

(2) If $a,b,c \in \mathbb{Q}$, then:

If Δ is a perfect square greater than 0, the equation has **rational solutions**.

If Δ is not a perfect square, then the equation has **irrational solutions.**

<u>Exercise #1</u> Solve by factoring:

a) $x^2 + 2x - 8 = 0$

c) $(3\mathrm{x}+1)^2 - 9x^2 = 31$

b) $-20\mathrm{y}^2 + 6 = -7y$

d) $(2a+3)(3a-5) = -10$

<u>Exercise #2</u> Solve (in $\mathbb{C}$) by extracting roots:

a) $\dfrac{2x^2}{3} = 4$

b) $\left(t - \dfrac{1}{2}\right)^2 = \dfrac{3}{4}$

c) $1 - 3(x-1)^2 = 10$

<u>Exercise #3</u> Solve (in $\mathbb{C}$) by completing the square. Give exact answers.

a) $x^2 - \dfrac{5}{3}x = 1$

b) $2p^2 - 6p - 5 = 0$

c) $\dfrac{x^2}{3} + \dfrac{25}{3} - \dfrac{5}{3}x = 0$

d) $-4m^2 - 36m - 65 = 0$

e) $x^2 + \sqrt{3}x - \dfrac{1}{4} = 0$

Exercise #4 Solve (in $\mathbb{C}$) by the quadratic formula. Give exact answers.

a) $x^2 - \frac{x}{2} + 1 = 0$

b) $\frac{1}{2}a^2 + 1 = \frac{3}{2}a$

c) $3z^2 = 4.2z + 1.5$

Exercise #5 a) Write (in standard form) a quadratic equation with rational coefficients that has $2+\sqrt{3}$ as a solution.

b)Write (in standard form) a quadratic equation with integer coefficients that has 2 and $-\frac{1}{2}$ as solutions.

c) Write (in standard form) a quadratic equation with real coefficients that has 1+i as a solution.

Exercise #6 a) Determine k such that the solutions of $3x^2 + 4x = k$ are nonreal complex numbers.

b) Find the value(s) of k that will make the solutions of the following equation equal:

$$(k-1)x^2 + (k-1)x + 1 = 0$$

Exercise #7 Solve each equation for the indicated variable:

a) $3x^2 + xy + y^2 = 2$, for y;

b) $A = 2w^2 + 4lw$, for w;

c) $a^2 + b^2 = c^2$, for b.

<u>Definition</u> The graph of a quadratic function $y = ax^2 + bx + c$ is a parabola.

Note: If a>0, the parabola opens upward.

If a<0, the parabola opens downward.

<u>The Main Elements of a Parabola</u>

VERTEX - the lowest point if parabola opens upward.

- the highest point if parabola opens downward

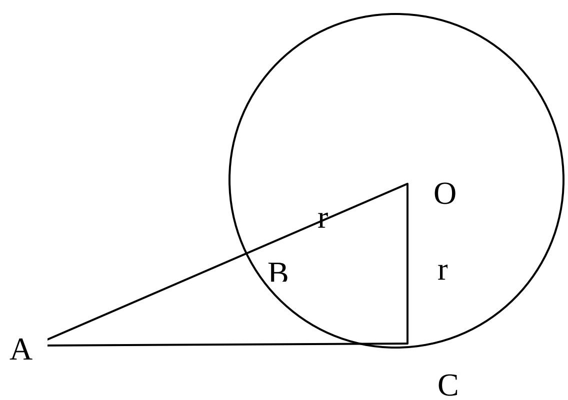
$V\left(x_v, y_v\right)$, where $x_v = -\dfrac{b}{2a}$

y-INTERCEPT ____

x-INTERCEPTS (if any) ____

AXIS OF SYMMETRY ____

Exercise #8 . In the diagram, $\triangle AOC$ is a right triangle with right angle at *C*. Solve for r if $AC = 12$ and $AB = 8$



Exercise #9 The total profit Kiyoshi makes from producing and selling "x" floral arrangements is

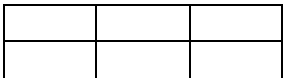
$$P = -0.4x^2 + 36x \text{ .}$$

a) How many floral arrangements should Kiyoshi produce and sell to maximize his profit?
b) What is his maximum profit? Explain how do you know for sure you found the maximum profit.

Exercise #10 A new electronics firm is considering marketing a line of telephones. The cost per phone for producing x telephones is $C = 0.001x^2 - 3x + 2270$. A) How many telephones should they produce in order to minimize the cost per phone? Explain how do you know for sure you found the minimum cost per phone. B) What will their total cost be at that production level?

Exercise #11 A rancher has 1000 feet of fencing to construct 6 rectangular corrals (as shown in the figure below).

a) Write an equation that represents the total area "A" in terms of the width "x" of the whole structure.



b) Find the dimensions of the outer structure that will maximize the enclosed area. Explain how you know for sure these dimensions produce the larger area.

Exercise #12 A stone is thrown upward so that its height h after t seconds is given by $h = -16t^2 + 56t + 6$, where h is measured in feet.

a) When will the stone reach its maximum height?
b) What is the maximum height?
c) When will the stone reach the ground?

Exercise #13 A shopping center has a rectangular area of 40,000 square yards enclosed on three sides for a parking lot. The length is 200 yd more than twice the width. What are the dimensions of the lot?

Exercise #14 A skydiver jumps out of an airplane at 11,000 feet. While she is in free-fall, her altitude in feet t seconds after jumping is given by $h = -16t^2 - 16t + 11{,}000$.

a) If the skydiver must open her parachute at an altitude of 1000 feet, how long can she free-fall? Write and solve an equation to find the answer.
b) If the skydiver drops a marker just before she opens her parachute, how long will it take the marker to hit the ground?

Exercise #15 The base of an isosceles triangle is one inch shorter than the equal sides, and the altitude of the triangle is two inches shorter than the equal sides. What is the length of the equal sides?