3.5 Graphs of Rational Functions

Example 1 Graph the reciprocal function $f(x)=\frac{1}{x}$ Answer the following questions:

a) What is the domain of the function? _____

b) What is the range of the function? _____

c) What are the x-and y-intercepts? _____

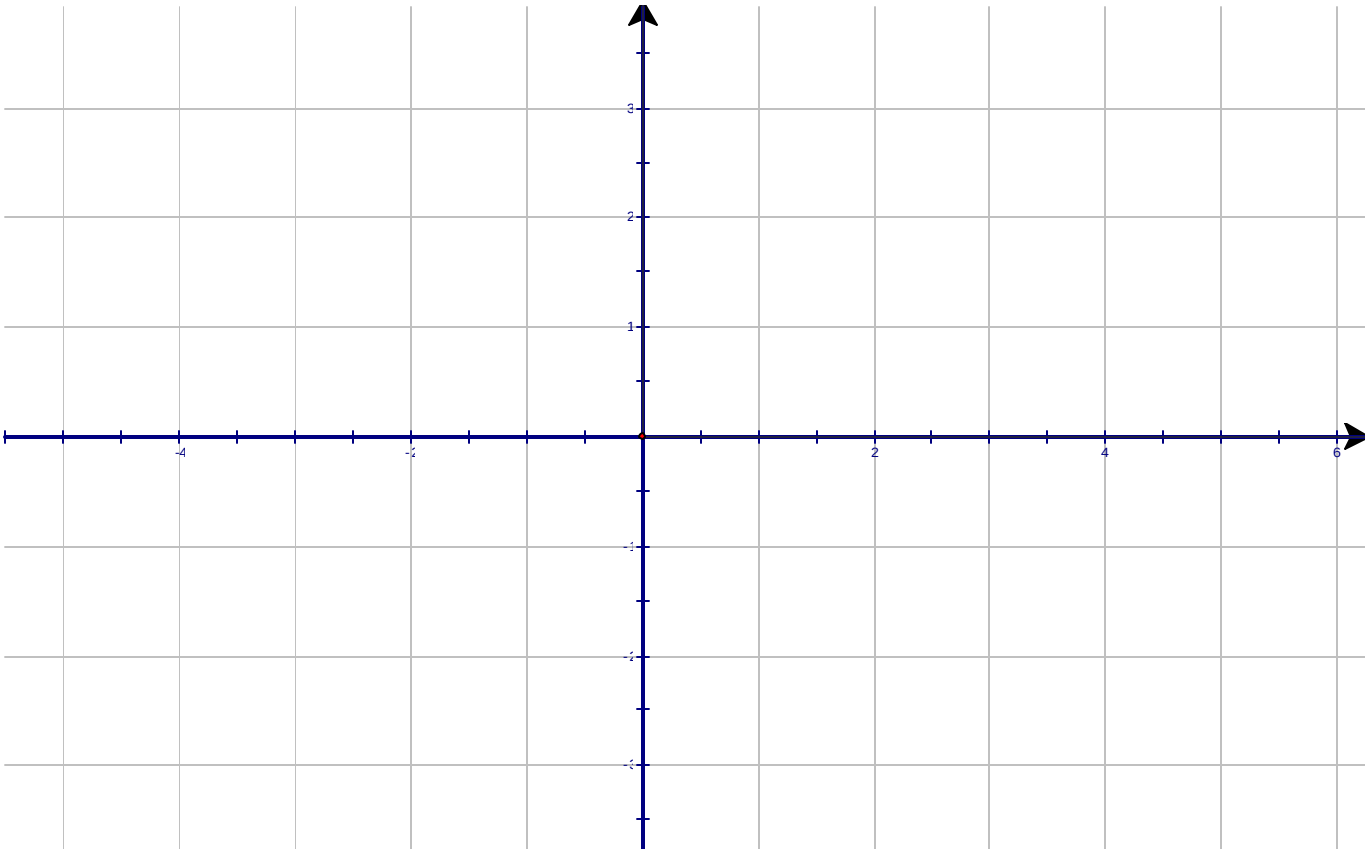
d) What is the end- behavior of the function, that is, what happens with the values of y as x goes to ∞ and $-\infty$?

_____

e) What is the behavior of the function when x approaches 0 ?

_____

x



Definition A **rational function** is a function f of the form $f(x)=\dfrac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials, with $q(x)\neq 0$.

Notations:

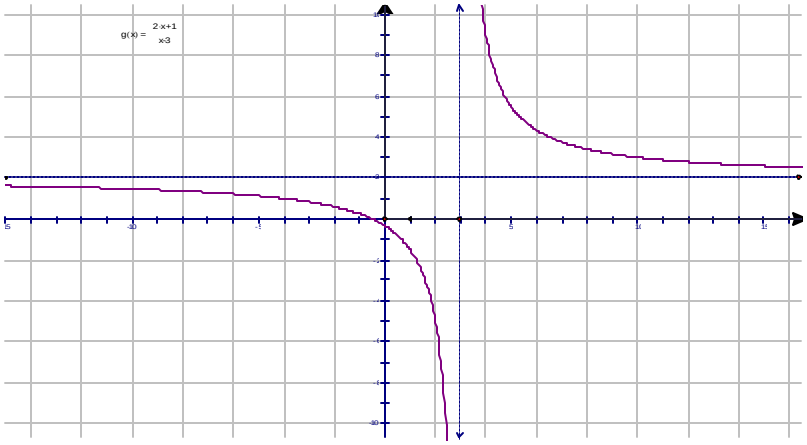
$x\to\infty$	x approaches infinity (x increases without bound)
$x\to-\infty$	x approaches negative infinity (x decreases without bound)
$x\to a^{+}$	x approaches a from the right
$x\to a^{-}$	x approaches a from the left

Definition The line $\boxed{x=a}$ is a **vertical asymptote** for the graph of $f(x)$ if, when $x\to a$, $y\to\pm\infty$.

The line $\boxed{y=b}$ is a **horizontal asymptote** for the graph of $f(x)$ if, when $x\to\pm\infty$, $y\to b$.

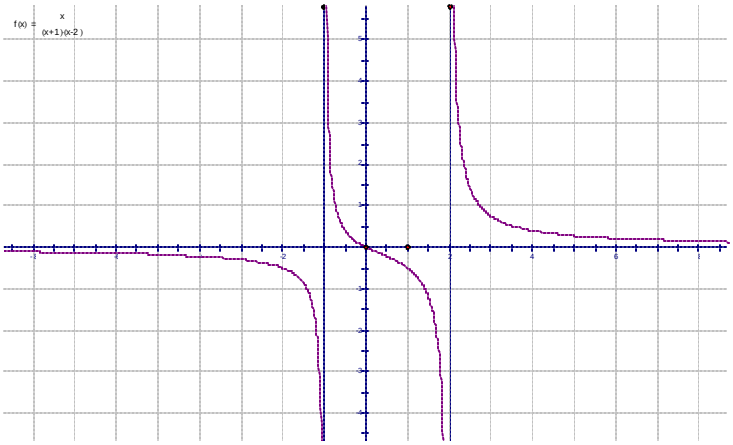
Exercise #1 Identify all the vertical and horizontal asymptotes of the following graphs.
How can the vertical asymptotes be found? What about the horizontal asymptotes?

a)

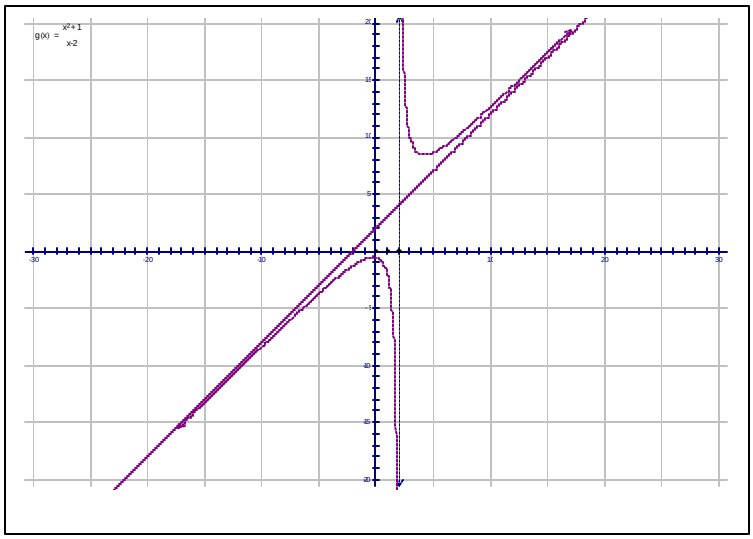


$$f(x)=\frac{2x+1}{x-3}$$

b)



$$f(x)=\frac{x}{(x+1)(x-2)}$$

c) 

$$f(x)=\frac{x^2+1}{x-2}$$

Asymptotes for a rational function $f(x)=\frac{p(x)}{q(x)}=\frac{a_n x^n+...+a_0}{b_m x^m+...b_0}$

1. The vertical asymptotes are the lines $x=c$, where c is a zero of the denominator.

2. If $n<m$, then $y=0$ (the x-axis) is the **horizontal asymptote.**

If $n=m$, then $y=\frac{a_n}{b_n}$ is the **horizontal asymptote.**

If $n>m$, there are **no horizontal asymptotes.**

If, however, $n=m+1$, then there is an oblique asymptote . Divide the numerator by the denominator and disregard the remainder.

y = quotient is the oblique asymptote

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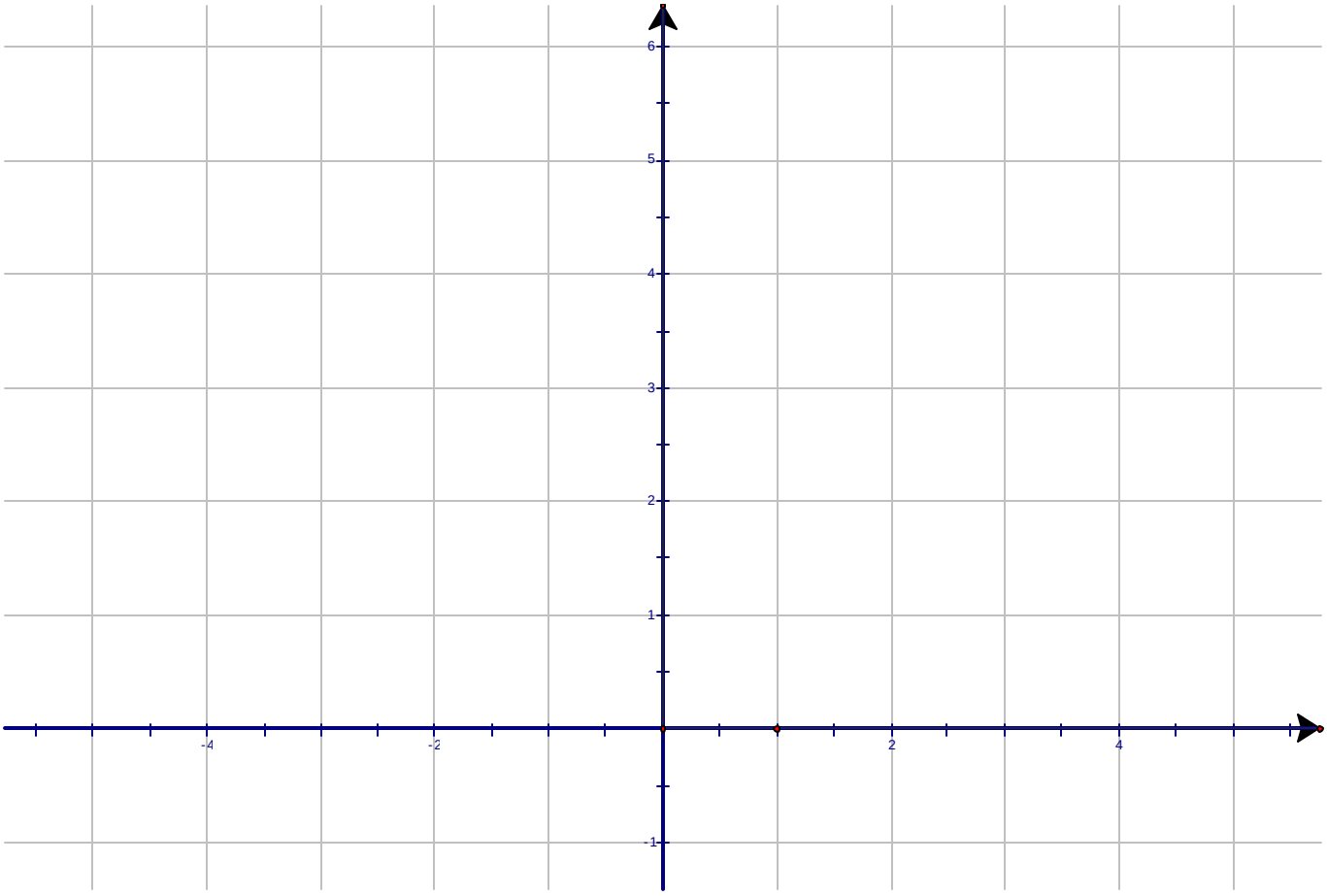
Exercise #2 Identify all the asymptotes for the following functions:

$f(x)=\frac{2x+7}{x-5}$ | $g(x)=\frac{4x^2+x-5}{2x^2-3x-5}$ | $h(x)=\frac{x^2+6}{x-3}$ | $l(x)=\frac{1}{2x^2-2}$

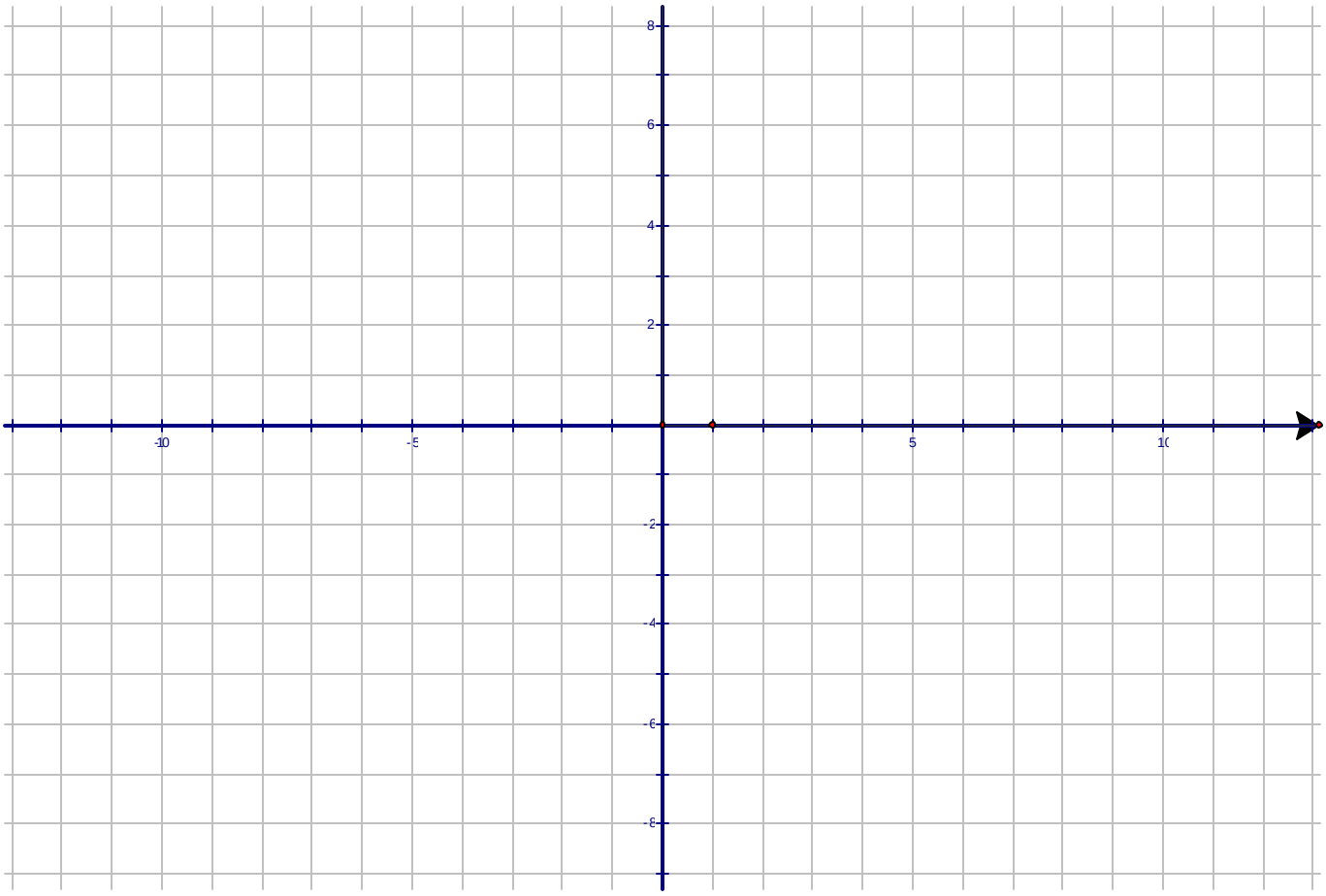
Exercise #3 Graph the function $f(x)=\frac{1}{x^2}$. Find the domain, the asymptotes, and the *x*- and *y*-intercepts.



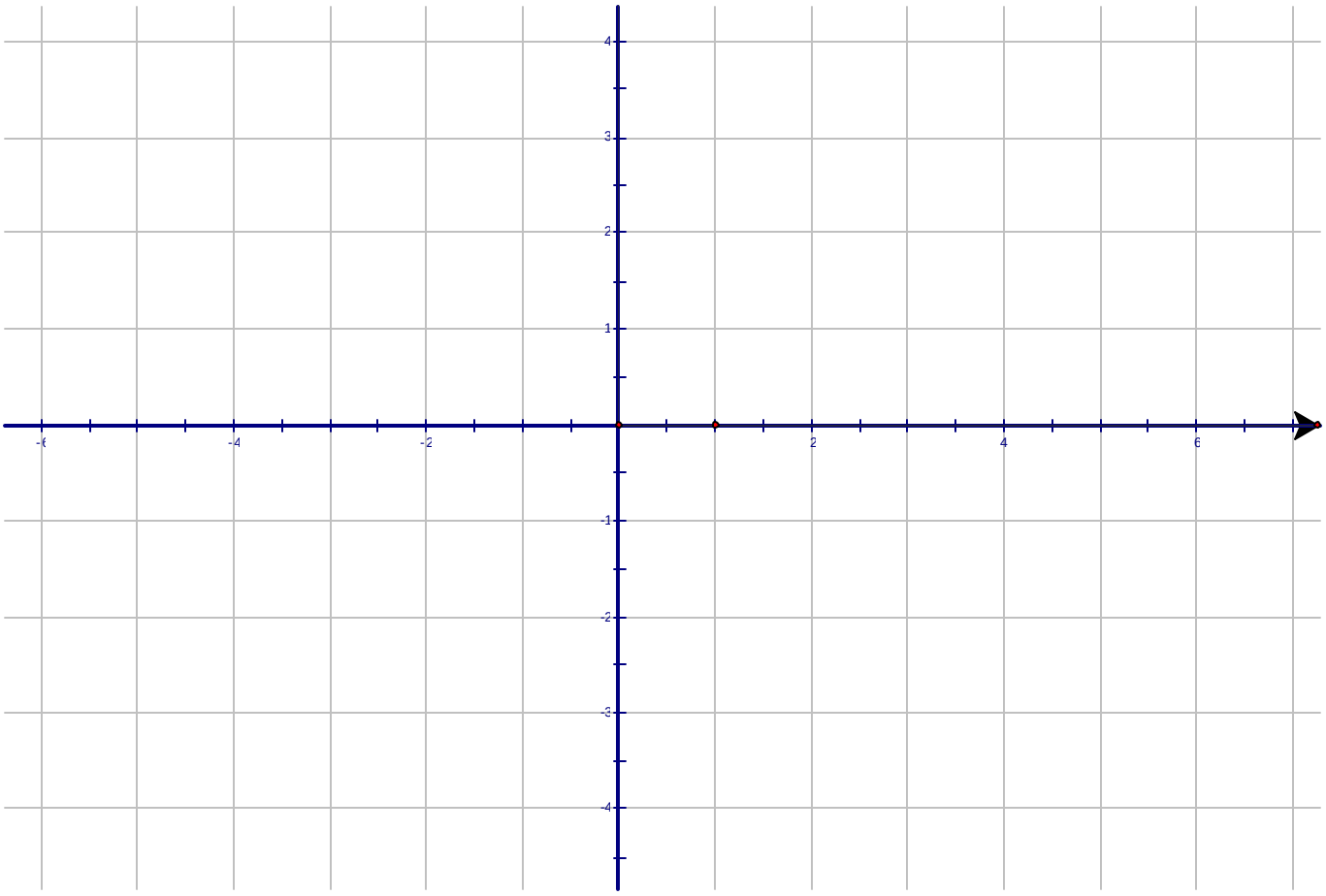
Exercise #4 Show how to obtain the graph of $g(x)=\frac{1}{(x+1)^2}+1$ from the graph of $f(x)=\frac{1}{x^2}$.

What are the asymptotes of $g(x)$?

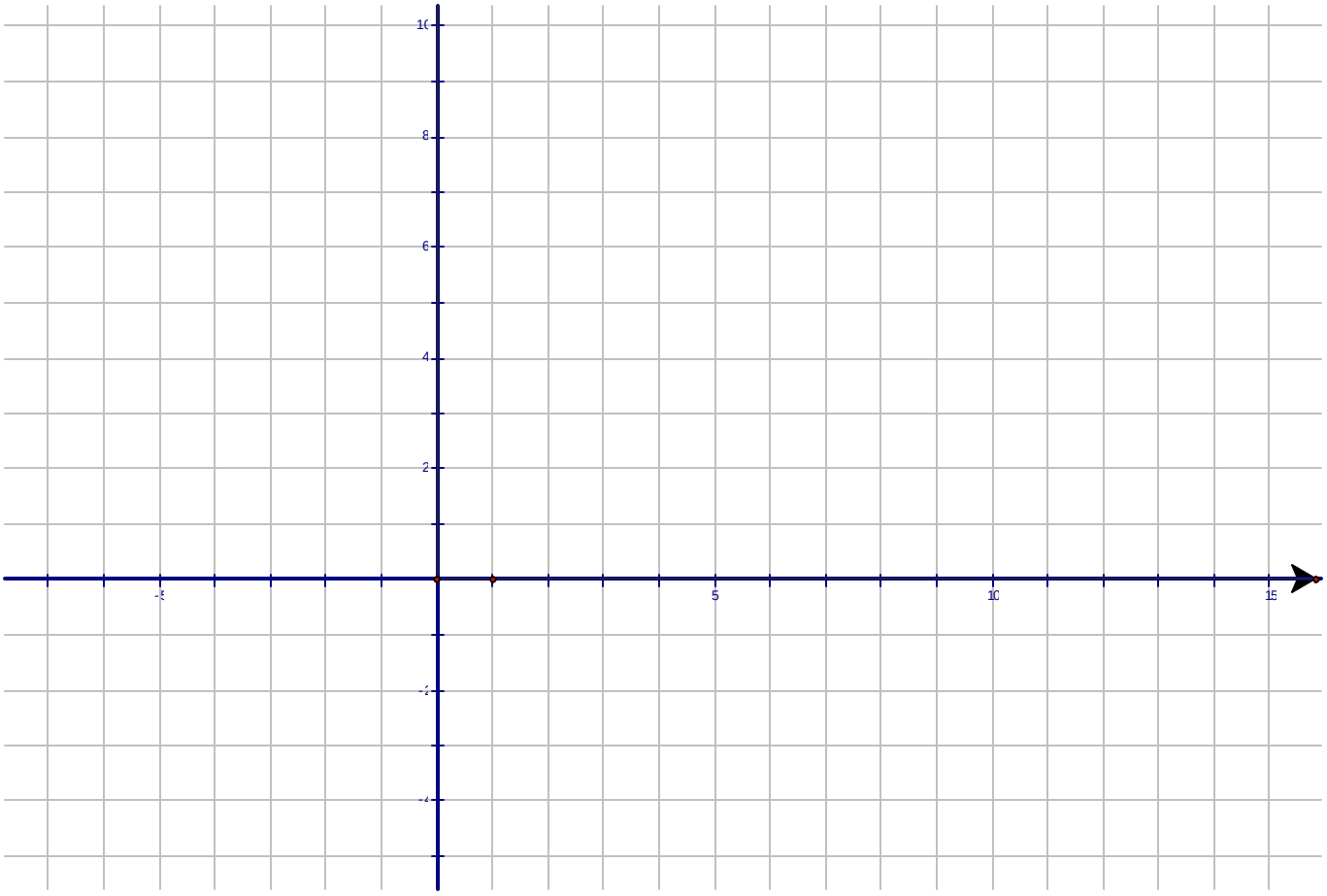
Exercise #5 Sketch the graph of $f(x)=\dfrac{x+1}{x-4}$. Find the domain, all the asymptotes, the *x*- and *y*-intercepts Determine if the graph intersects its nonvertical asymptote. Plot additional test points, as needed.



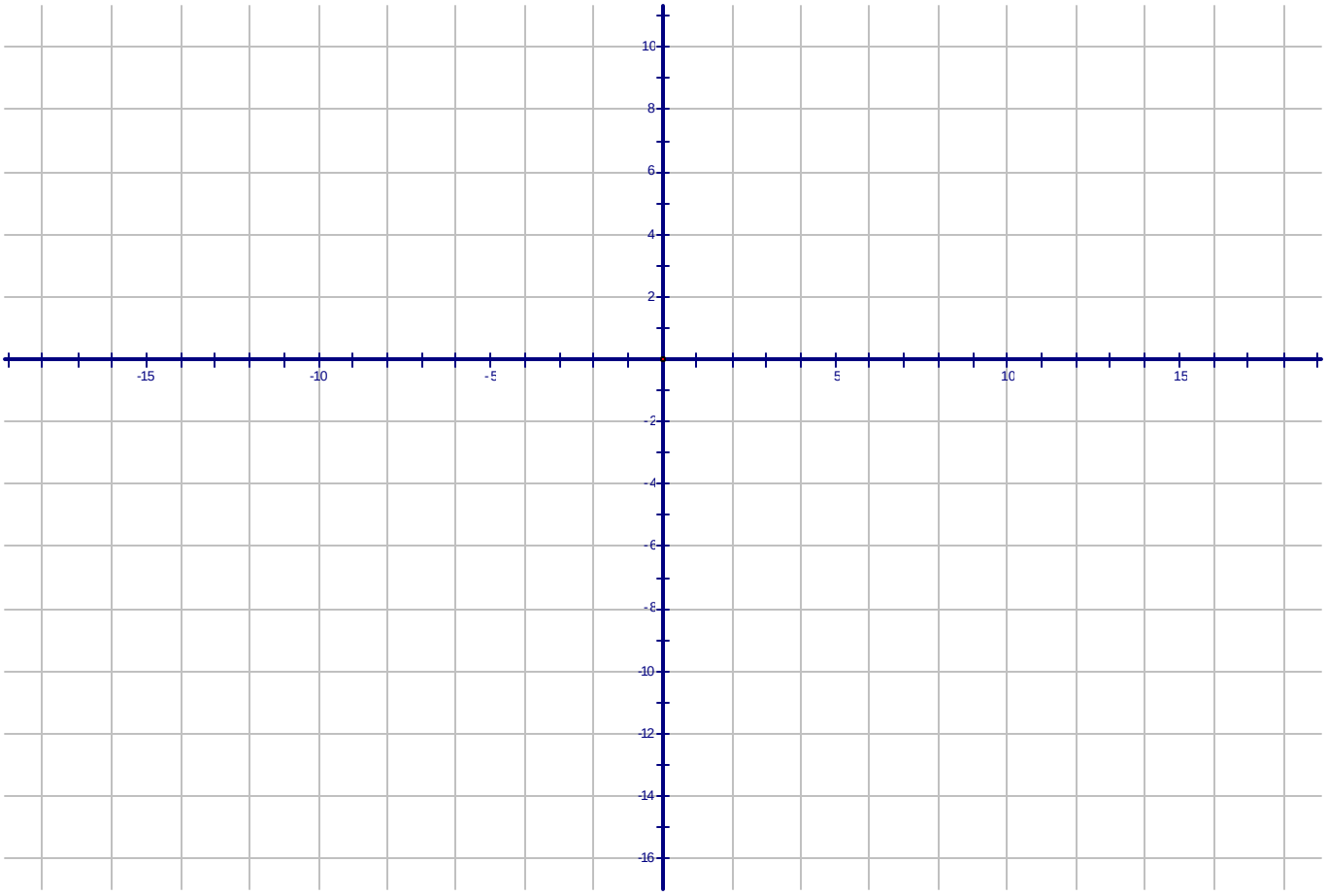
Exercise #6 Sketch the graph of $f(x)=\dfrac{x-2}{x^2-1}$. Find the domain, all the asymptotes, the *x*- and *y*-intercepts Determine if the graph intersects its nonvertical asymptote. Plot additional test points, as needed.



Exercise #7 Sketch the graph of $f(x)=\dfrac{x^2-2x-8}{x^2-4x+3}$. Find the domain, all the asymptotes, the *x*- and *y*-intercepts Determine if the graph intersects its nonvertical asymptote. Plot additional test points, as needed.



Exercise #8 Sketch the graph of $f(x)=\dfrac{x^2+1}{x+3}$. Find the domain, all the asymptotes, the x- and y-intercepts Determine if the graph intersects its nonvertical asymptote. Plot additional test points, as needed.



Exercise #9 Graph the following functions: $f(x)=\dfrac{x+2}{x+2}$ and $g(x)=\dfrac{x^2-9}{x+3}$.