M 130

Homework # 6 - Solutions

SECTION 3.2

(36) $f(x) = 6x^3 - 31x^2 - 15x$, $k = -\frac{1}{2}$

find $f(-\frac{1}{2})$

Solution

Use Remainder Theorem:

$$\begin{array}{r|rrrr} & 6 & -31 & -15 & 0 \\ \hline -\frac{1}{2} & 6 & -34 & 2 & \boxed{-1}\ R \end{array}$$

$f(-\frac{1}{2}) = -1$

SECTION 3.3

(31) $f(x) = x^3 - 7x^2 + 17x - 15$

$x = 2 - i$ zero

Find all the zeros

Solution

degree $f(x) = 3 \Rightarrow$ there are 3 zeros

$x_1 = 2 - i$ (given)

$x_2 = 2 + i$ (Conjugate Zeros Theorem)

Therefore, $x - (2-i) \mid f(x)$

$x - (2+i) \mid f(x)$

Therefore,

$(x - 2 + i)(x - 2 - i) \mid f(x)$

$(x-2)^2 - i^2 \mid f(x)$

$(x^2 - 4x + 4 + 1) \mid f(x)$

$(x^2 - 4x + 5) \mid f(x)$

$$\begin{array}{r} x - 3 \\ x^2 - 4x + 5 \overline{) x^3 - 7x^2 + 17x - 15} \\ \underline{-x^3 + 4x^2 - 5x} \\ -3x^2 + 12x - 15 \\ \underline{+3x^2 - 12x + 15} \end{array}$$

So, $f(x) = (x^2 - 4x + 5)(x - 3)$

So, the zeros are:

$$\begin{cases} x_1 = 2 - i \\ x_2 = 2 + i \\ x_3 = 3 \end{cases}$$

(Or, you can use synthetic division instead of long division)

(32) $f(x) = 4x^3 + 6x^2 - 2x - 1$

$x = \frac{1}{2}$ zero

Find all the zeros.

Solution

$x = \frac{1}{2}$ zero $\Rightarrow x - \frac{1}{2} \mid f(x)$

$$\begin{array}{r|rrrr} & 4 & 6 & -2 & -1 \\ \hline \frac{1}{2} & 4 & 8 & 2 & 0 \end{array}$$

$f(x) = (x - \frac{1}{2})(4x^2 + 8x + 2)$

$f(x) = 0 \Leftrightarrow 4x^2 + 8x + 2 = 0 \quad | \div 2$

$2x^2 + 4x + 1 = 0$

$x = \frac{-4 \pm \sqrt{16 - 8}}{4} = \frac{-4 \pm \sqrt{8}}{4}$

$= \frac{-4 \pm 2\sqrt{2}}{4} = \frac{2(-2 \pm \sqrt{2})}{4} = \frac{-2 \pm \sqrt{2}}{2}$

$x = \frac{-2 + \sqrt{2}}{2}$

$x = \frac{-2 - \sqrt{2}}{2}$

$\{ \frac{1}{2}, \frac{-2 \pm \sqrt{2}}{2} \}$

(34) $f(x) = x^4 + 10x^3 + 27x^2 + 10x + 26$

$x = i$ zero

Find all the zeros.

Solution

$x_1 = i$ zero $\Rightarrow$

$x_2 = -i$ zero (The Conjugate Zeros theorem)

	1	10	27	10	26
i	1	$10+i$	$10i+26$	$26i$	0
$-i$	1	10	26	0	

$f(x) = (x-i)(x+i)(x^2+10x+26)$

$f(x) = 0 \Leftrightarrow$ $x - i = 0$, $x + i = 0$ — $x = i$, $x = -i$

$x^2 + 10x + 26 = 0$

$x = \frac{-10 \pm \sqrt{100-104}}{2} = \frac{-10 \pm 2i}{2}$

$x = -5 \pm i$

$\{i, -i, -5 \pm i\}$

(41) $f(x) = 24x^3 + 40x^2 - 2x - 12$

Possible rational zeros:

$\frac{p}{q} = \frac{\text{factors of } 12}{\text{factors of } 24} = \frac{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12}{\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24}$

(a) $\frac{p}{q} \in \{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{8}, \pm\frac{1}{12}, \pm\frac{1}{24}, \pm\frac{2}{3}, \pm\frac{1}{6}, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm\frac{3}{8}\}$

(b)

	24	40	-2	-12
$\frac{1}{2}$	24	52	24	0
$-\frac{3}{2}$	24	16	0	

$f(x) = (x-\frac{1}{2})(24x^2+52x+24)$

$f(x) = (x-\frac{1}{2})(x+\frac{3}{2})(24x+16)$

$x_1 = \frac{1}{2}$

$x_2 = -\frac{3}{2}$

$24x + 16 = 0$

$x_3 = \frac{-16}{24}$

$x_3 = -\frac{2}{3}$

(b) $\{\frac{1}{2}, \frac{-3}{2}, \frac{-2}{3}\}$

c) $f(x) = (x-\frac{1}{2})(x+\frac{3}{2})(24x+16)$

$= 8(x-\frac{1}{2})(x+\frac{3}{2})(3x+2)$

$= 2 \cdot 2(x-\frac{1}{2})\,2(x+\frac{3}{2})(3x+2)$

$\boxed{f(x) = 2(2x-1)(2x+3)(3x+2)}$

OR, knowing the zeros $\{\frac{1}{2}, -\frac{3}{2}, -\frac{2}{3}\}$ you can write

$\boxed{f(x) = 24(x-\frac{1}{2})(x+\frac{3}{2})(x+\frac{2}{3})}$

(42) $f(x) = 24x^3 + 80x^2 + 82x + 24$

Possible rational zeros:

$\frac{p}{q} = \frac{\text{factors of } 24}{\text{factors of } 24} = \frac{\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24}{\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24}$

(a) $\frac{p}{q} \in \{\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{8}, \pm\frac{1}{12}, \pm\frac{1}{24}, \pm\frac{2}{3}, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm\frac{3}{8}, \pm\frac{4}{3}, \pm\frac{8}{3}\}$

(b)

	24	80	82	24
$-\frac{1}{2}$	24	68	48	0

$f(x) = (x+\frac{1}{2})(24x^2+68x+48)$

To find the other zeros, you can continue as in (#41), or you solve

$24x^2 + 68x + 48 = 0 \quad | \div 4.$

$6x^2 + 17x + 12 = 0$

$x = \frac{-17 \pm \sqrt{289-288}}{12} = \frac{-17 \pm 1}{12}$

$x = \frac{-18}{12} = -\frac{3}{2}$ — $x = \frac{-3}{2}$

OR

$x = \frac{-16}{12} = -\frac{4}{3}$ — $x = \frac{-4}{3}$

(b) $\{-\frac{1}{2}, \frac{-3}{2}, \frac{-4}{3}\}$

(c) $\boxed{f(x) = 24(x+\frac{1}{2})(x+\frac{3}{2})(x+\frac{4}{3})}$

For a linear factorization with integer coefficients,

$f(x) = 2 \cdot 2(x+\frac{1}{2})2(x+\frac{3}{2})3(x+\frac{4}{3})$

$\boxed{f(x) = 2(2x+1)(2x+3)(3x+4)}$.

(44) $f(x) = (x+1)^2(x-1)^3(x^2-10)$
Find all the zeros.

Solution

$f(x) = 0 \iff$

$(x+1)^2 = 0 \iff x+1 = 0 \iff x = -1$
zero of multiplicity 2

OR

$(x-1)^3 = 0 \iff x-1 = 0 \iff x = 1$
zero of multiplicity 3

OR

$x^2 - 10 = 0 \iff x^2 = 10 \iff x = \pm\sqrt{10}$
zeros of multiplicity 1

(47) $f(x) = (x^2+x-2)^5(x-1+\sqrt{3})^2$
Find all the zeros.

Solution

$f(x) = 0 \iff$

$(x^2+x-2)^5 = 0 \iff x^2+x-2 = 0$

$(x+2)(x-1) = 0$

$x = -2$ OR $x = 1$
zeros of multiplicity 5

OR

$(x-1+\sqrt{3})^2 = 0 \iff x-1+\sqrt{3} = 0$

$x = 1-\sqrt{3}$
zero of multiplicity 2

(53) Find $f(x)$ such that

degree $f(x) = 3$
$f(x)$ has real coefficients
$x = -3$ zero of multiplicity 3
$f(3) = 36$

Solution

$x = -3$ zero of mult. 3 $\Rightarrow$

$(x+3)^3 \mid f(x)$

But degree $f(x) = 3 \Rightarrow$ there are no other zeros (factors) of $f(x)$

Therefore, $f(x) = a(x+3)^3$

$f(3) = 36$
when $x = 3$, $f(x) = 36$

$36 = a(6)^3 \Rightarrow a = \frac{1}{6}$

Therefore, $\boxed{f(x) = \frac{1}{6}(x+3)^3}$.

(54) Find $f(x)$ such that

degree $f(x) = 3$
$f(x)$ has real coefficients
$x = 4$ zero of multiplicity 2
$x = 2$ zero of multiplicity 1
$f(1) = -18$

Solution

$x = 4$ zero of mult. 2 $\Rightarrow$

$(x-4)^2 \mid f(x)$

$x = 2$ zero of mult 1 $\Rightarrow$

$(x-2) \mid f(x)$

degree $f(x) = 3 \Rightarrow$ there are no other zeros of $f(x)$

$(x-4)^2$ and $(x-2)$ are the only factors

$\Rightarrow f(x) = a(x-4)^2(x-2)$

$f(1) = -18 \Rightarrow$

$-18 = a(-3)^2(-1)$

$-18 = -9a \Rightarrow a = +2$

Therefore, $\boxed{f(x) = 2(x-4)^2(x-2)}$

(59) Find $f(x)$ such that

degree $f(x)$ = minimum

$f(x)$ has real coefficients

$\begin{cases} x = 1+\sqrt{2} \\ x = 1-\sqrt{2} \\ x = 1 \end{cases}$ zeros

Solution

$x = 1+\sqrt{2}$ - zero $\Rightarrow x-(1+\sqrt{2}) \mid f(x)$

$x = 1-\sqrt{2}$ zero $\Rightarrow x-(1-\sqrt{2}) \mid f(x)$

$x = 1$ zero $\Rightarrow x-1 \mid f(x)$

$f(x) = (x-(1+\sqrt{2}))(x-(1-\sqrt{2}))(x-1)$

$f(x) = (x-1-\sqrt{2})(x-1+\sqrt{2})(x-1)$

$f(x) = ((x-1)^2-(\sqrt{2})^2)(x-1)$

$f(x) = (x^2-2x+1-2)(x-1)$

$f(x) = (x^2-2x-1)(x-1)$

$= x^3 - x^2$

$\quad - 2x^2 + 2x$

$\quad - x + 1$

$\boxed{f(x) = x^3 - 3x^2 + x + 1}$

(63) Same as #59 with zeros:

$\begin{cases} x = 2 \\ x = 3+i \end{cases}$

Solution

$x = 2$ - zero $\Rightarrow (x-2) \mid f(x)$

$x = 3+i$ - zero and $f(x)$ has real coefficients $\Rightarrow$

$x = 3-i$ - zero

so, $x-(3+i) \mid f(x)$

and $x-(3-i) \mid f(x)$

$f(x) = (x-2)(x-(3+i))(x-(3-i))$

$f(x) = (x-2)(x-3-i)(x-3+i)$

$f(x) = (x-2)((x-3)^2-i^2)$

$f(x) = (x-2)(x^2-6x+9+1)$

$= (x-2)(x^2-6x+10)$

$= x^3 - 6x^2 + 10x$

$\quad - 2x^2 + 12x - 20$

$\boxed{f(x) = x^3 - 8x^2 + 22x - 20}$