Math 130

HOMEWORK #4 - SOLUTIONS

SECTION 2.6

#24 $y = x^3 x$

Symmetry about x-axis: iff the equation is unchanged when replacing y with -y $(y \mapsto -y)$:

$-y = x^3 - x \quad |(-1)$

$y = -x^3 + x$

$\Rightarrow$ the graph is not symmetric with x-axis

Symmetry about y-axis iff the equation is unchanged when replacing x with -x $(x \mapsto -x)$:

$y = (-x)^3 - (-x)$

$y = -x^3 + x$

$\Rightarrow$ the graph is not symmetric about the y-axis

Symmetry about origin iff the equation is unchanged when replacing | x with -x and | y with -y $\left(\begin{matrix} x \mapsto -x \\ y \mapsto -y \end{matrix}\right)$:

$-y = (-x)^3 - (-x)$

$-y = -x^3 + x \quad |(-1)$

$y = x^3 - x$

$\Rightarrow$ the graph is symmetric about the origin

SECTION 2.7

#14 $f(x) = \sqrt{5x-4}$

$g(x) = -\frac{1}{x}$

$(f+g)(x) = f(x) + g(x)$

$= \sqrt{5x-4} - \frac{1}{x}$

$(f-g)(x) = f(x) - g(x)$

$= \sqrt{5x-4} - \left(-\frac{1}{x}\right) = \sqrt{5x-4} + \frac{1}{x}$

$(fg)(x) = f(x)\,g(x)$

$= \sqrt{5x-4}\left(\frac{-1}{x}\right) = \frac{-\sqrt{5x-4}}{x}$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$= \frac{\sqrt{5x-4}}{-\frac{1}{x}} = -x\sqrt{5x-4}$

Domain of $f+g$, $f-g$, fg, f/g

$\left\{\begin{matrix} 5x-4 \geq 0 \\ \text{and} \\ x \neq 0 \end{matrix}\right. \iff x \geq \frac{4}{5}$

$\boxed{x \in [\frac{4}{5}, \infty)}$

#45 $f(x) = 2x - 3$

$g(x) = -x + 3$

$(g \circ f)(0) = g(f(0)) = {}^{*}/.$

$(\text{but } f(0) = 2(0) - 3 = -3)$

${}^{*}/. = g(-3)$

$= -3 + 3$

$= 0$

$\boxed{(g \circ f)(0) = 0}$

(40) $f(x) = 8 - 3x^2$

(a) $f(x+h) = 8 - 3(x+h)^2$
$= 8 - 3(x^2 + 2xh + h^2)$
$= 8 - 3x^2 - 6xh - 3h^2$

(b)
$f(x+h) - f(x) =$
$= (8 - 3x^2 - 6xh - 3h^2) - (8 - 3x^2)$
$= \cancel{8} - \cancel{3x^2} - 6xh - 3h^2 - \cancel{8} + \cancel{3x^2}$
$= -6xh - 3h^2$

(c) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-6xh - 3h^2}{h} =$
$= \dfrac{-h(6x + 3h)}{h} = -(6x + 3h)$
$= -6x - 3h$

(62) $f(x) = \dfrac{1}{x}$, $g(x) = x^2$

$(f \circ g)(x) = f(g(x))$
$= f(x^2)$
$= \dfrac{1}{x^2}$

$D_{f \circ g} = \mathbb{R} \setminus \{0\}$

$(g \circ f)(x) = g(f(x))$
$= g\left(\dfrac{1}{x}\right)$
$= \left(\dfrac{1}{x}\right)^2$
$= \dfrac{1}{x^2}$

$D_{g \circ f} = \mathbb{R} \setminus \{0\}$

(77) $h(x) = \sqrt{6x} + 12$
Find f and g such that $(f \circ g)(x) = h(x)$

For example,
$g(x) = 6x$
$f(x) = \sqrt{x} + 12$

check: $(f \circ g)(x) = f(g(x))$
$= f(6x)$
$= \sqrt{6x} + 12$
$= h(x)$

(78) $h(x) = \sqrt[3]{2x+3} - 4$

For example,
$g(x) = 2x + 3$
$f(x) = \sqrt[3]{x} - 4$

Check: $(f \circ g)(x) = f(g(x))$
$= f(2x+3)$
$= \sqrt[3]{2x+3} - 4$