Math 130

HOMEWORK #3 - SOLUTIONS

SECTION 2.4

44 (a) First, we find the slope of the line $2x-3y=4$

$3y=2x-4$

$y=\frac{2}{3}x-\frac{4}{3} \Rightarrow m=\frac{2}{3}$

A line parallel to $2x-3y=4$ also has slope $m=\frac{2}{3}$.

We also know that this parallel line passes through $(2,6)$ and $(-4,r)$

$m=\frac{\Delta y}{\Delta x}=\frac{r-6}{-4-2}=\frac{r-6}{-6}$

$m=\frac{r-6}{-6}$

Therefore, $\frac{r-6}{-6}=\frac{2}{3}$

$3(r-6)=2(-6)$

$3r-18=-12$

$3r=6 \Rightarrow \boxed{r=2}$

(b) We find first the slope of the line $x+2y=1$

$2y=-x+1$

$y=-\frac{1}{2}x+\frac{1}{2} \Rightarrow m=-\frac{1}{2}$

A line perpendicular to $x+2y=1$ must have a slope of 2.

Also, this perpendicular line passes through $(2,6)$ and $(-4,r)$

so $m=2=\frac{r-6}{-6}$

$\frac{2}{1}=\frac{r-6}{-6}$

$-12=r-6$

$\boxed{r=-6}$

50 Variables: p = pressure (dependent variable), x = depth (independent variable).

x	p
0	1
100	3.92

(a) Find the linear function that relates p to x $\Leftrightarrow$ Find an equation of the line that passes through $(0,1)$ and $(100,3.92)$

$m=\frac{3.92-1}{100-0}=\frac{2.92}{100}=0.0292$

Use $(0,1)$ = p-intercept

$m=0.0292$

$y=mx+b$ (slope-intercept)

$p=mx+b$

$\boxed{p=0.0292x+1}$

(b) $p=?$ if $x=60$ ft

$p=0.0292x+1$

$p=0.0292(60)+1=2.752$

The pressure at 60 ft is 2.752 atmospheres

SECTION 5.1

(61) Solve in terms of x.

$$\begin{cases} 5x - 4y + z = 9 & (1) \\ x + y = 15 & (2) \end{cases}$$

(2) $x + y = 15 \Rightarrow \boxed{y = -x + 15}$

Substitute y in (1):

$5x - 4(-x+15) + z = 9$

$5x + 4x - 60 + z = 9$

$9x - 60 + z = 9$

$\boxed{z = -9x + 69}$

We have an infinite number of solutions: $(x, -x+15, -9x+69)$ where $x \in \mathbb{R}$.

(63) $\begin{cases} (1)\ 3x + 4y - z = 13 \\ (2)\ x + y + 2z = 15 \end{cases} \Big/ 2$

Eliminate z by multiplying eq. (1) by 2:

$$\begin{cases} 6x + 8y - 2z = 26 \\ x + y + 2z = 15 \end{cases}$$

$7x + 9y = 41$ and solve for y:

$9y = -7x + 41$

$\boxed{y = \frac{-7x+41}{9}}$

Substitute y into one of the equations, for example (2):

$x + \frac{-7x+41}{9} + 2z = 15$ and solve for z.

$9x - 7x + 41 + 18z = 135$

$2x + 41 + 18z = 135$

$18z = -2x + 94$

$z = \frac{-2x+94}{18} = \frac{2(-x+47)}{18} = \frac{-x+47}{9}$

So $\boxed{z = \frac{-x+47}{9}}$

The system has an infinite number of solutions $\left(x, \frac{-7x+41}{9}, \frac{-x+47}{9}\right)$, with $x \in \mathbb{R}$.

(65) $\begin{cases} (1)\ 3x + 5y - z = -2 \\ (2)\ 4x - y + 2z = 1 \\ (3)\ -6x - 10y + 2z = 0 \end{cases} \Big| \div 2$

Divide eq. (3) by the common factor 2:

(3) $-3x - 5y + z = 0$

Also,

(1) $3x + 5y - z = -2$

(+) $0 = -2$ Contradiction

Therefore, the system has NO SOLUTIONS.

OR you can multiply eq (1) by 2 and add the result to equation (3).

(72) $\begin{cases} \frac{2}{x} + \frac{3}{y} = 18 \\ \frac{4}{x} - \frac{5}{y} = -8 \end{cases}$ Note that $x \neq 0$, $y \neq 0$

Let $\begin{cases} \frac{1}{x} = a \\ \frac{1}{y} = b \end{cases}$

Then the system is:

$$\begin{cases} 2a + 3b = 18 \\ 4a - 5b = -8 \end{cases} \Big/ -2$$

$$\begin{cases} -4a - 6b = -36 \\ 4a - 5b = -8 \end{cases}$$

(+) $-11b = -44 \Rightarrow \boxed{b = 4}$

Substitute b in $4a-5b=-8$

$4a-5(4)=-8$

$4a-20=-8$

$4a=12 \Rightarrow \boxed{a=3}$

So, $\begin{cases} a=3 \\ b=4 \end{cases} \Rightarrow \begin{cases} \frac{1}{x}=3 \\ \frac{1}{y}=4 \end{cases} \Rightarrow$

$x=\frac{1}{3}$, $y=\frac{1}{4}$

So, the solution of the system is $\boxed{\left(\frac{1}{3},\frac{1}{4}\right)}$.

(73) $\begin{cases} \frac{2}{x}+\frac{3}{y}-\frac{2}{z}=-1 \\ \frac{8}{x}-\frac{12}{y}+\frac{5}{z}=5 \\ \frac{6}{x}+\frac{3}{y}-\frac{1}{z}=1 \end{cases}$ Note that $x\neq 0$, $y\neq 0$, $z\neq 0$

Let $\frac{1}{x}=a$, $\frac{1}{y}=b$, $\frac{1}{z}=c$

The system becomes:

$\begin{cases} (1)\ 2a+3b-2c=-1 \\ (2)\ 8a-12b+5c=5 \\ (3)\ 6a+3b-c=1 \end{cases}$ $|4$

Eliminate b:

(3)-(1): $\boxed{4a+c=2}$ (4)

$\begin{cases} (2)\ 8a-12b+5c=5 \\ (3)\ 24a+12b-4c=4 \end{cases}$

(+) $\boxed{32a+c=9}$ (5)

$\begin{cases} (4)\ 4a+c=2 \\ (5)\ 32a+c=9 \end{cases}$

(−) $28a=7 \Rightarrow a=\frac{7}{28}$

$a=\frac{1}{4}$

$a=\frac{1}{4}$ substitute in (4):

$4a+c=2$

$4\cdot\frac{1}{4}+c=2$

$1+c=2 \Rightarrow c=1$

$a=\frac{1}{4}$; $c=1$ substitute in (1):

$2a+3b-2c=-1$

$2\cdot\frac{1}{4}+3b-2(1)=-1$

$\frac{1}{2}+3b-2=-1$

$3b=1-\frac{1}{2}$

$3b=\frac{1}{2}$

$b=\frac{1}{6}$

$a=\frac{1}{4} \Rightarrow \frac{1}{x}=\frac{1}{4} \Rightarrow \boxed{x=4}$

$b=\frac{1}{6} \Rightarrow \frac{1}{y}=\frac{1}{6} \Rightarrow \boxed{y=6}$

$c=1 \Rightarrow \frac{1}{z}=1 \Rightarrow \boxed{z=1}$

The solution of the system is $\boxed{(4,6,1)}$.

SECTION 2.5

(20) $f(x) = \begin{cases} -2x, & \text{if } x < -3 \\ 3x-1, & \text{if } -3 \le x \le 2 \\ -4x, & \text{if } x > 2 \end{cases}$

$f(-5) = -2(-5) = -10$ because $x = -5$ and $-5 < -3$

$f(-1) = 3(-1) - 1 = -4$ because $x = -1$ and $-1 \in [-3, 2]$ (or $-3 \le -1 \le 2$)

$f(0) = 3(0) - 1 = -1$ because $x = 0$ and $0 \in [-3, 2]$

$f(3) = -4(3) = -12$ because $x = 3$ and $3 > 2$.

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(22) Graph $f(x) = \begin{cases} 6-x, & x \le 3 \\ 3x-6, & x > 3 \end{cases}$

if $x \le 3$, $f(x) = 6-x$

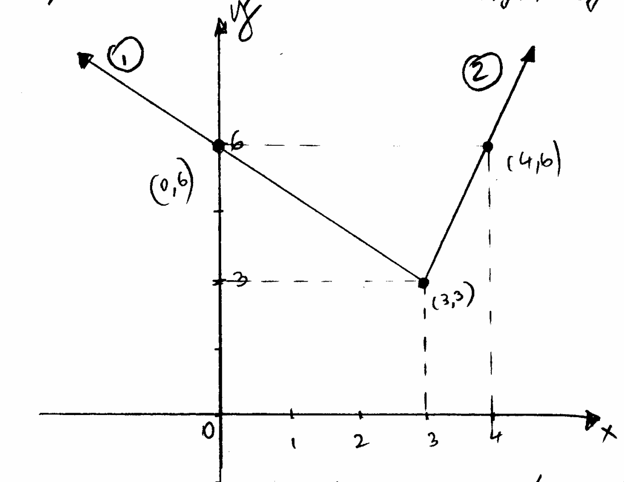
x	y	
3	3	(3,3)
0	6	(0,6)

Graph the line $y = 6-x$ to the left of $x = 3$. (1)

if $x > 3$, $f(x) = 3x-6$

x	y	
3	3	(3,3)
4	6	(4,6)

Graph the line $y = 3x-6$ to the right of $x = 3$. (2)



Note that the two rays have the same endpoint (3,3).

(28) Graph $f(x) = \begin{cases} -2x, & x < -3 \\ 3x-1, & -3 \le x \le 2 \\ -4x, & x > 2 \end{cases}$

if $x < -3$, $f(x) = -2x$

x	y
-3	6
-4	8

Graph the line $y = -2x$ to the left of $x = -3$, but do not include the endpoint (b/c $x < -3$). (1)

if $-3 \le x \le 2$, $f(x) = 3x-1$

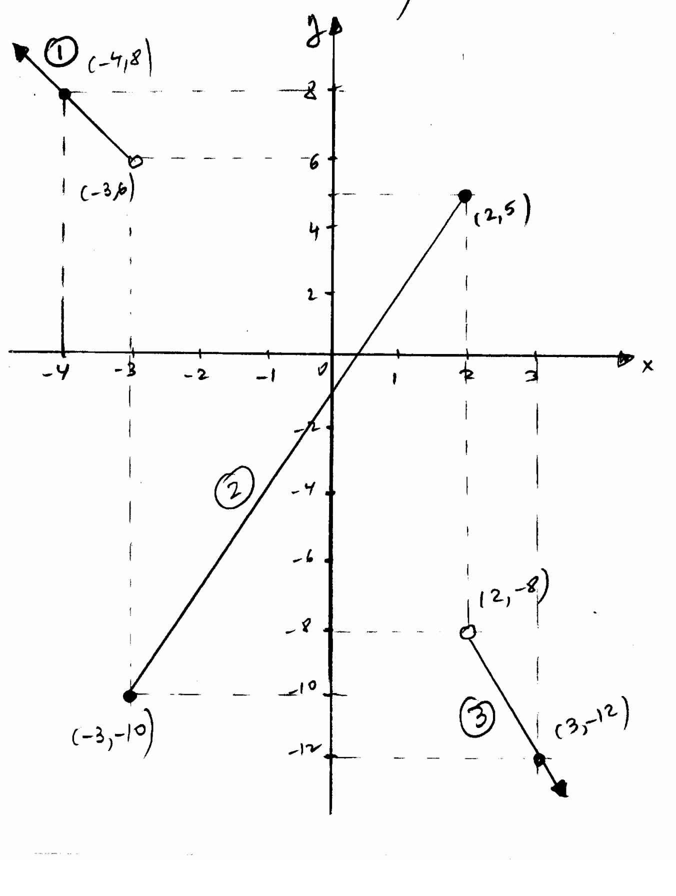
x	y
-3	-10
2	5

Graph the line $y = 3x-1$ between $x = -3$ and $x = 2$ and include both endpoints. (2)

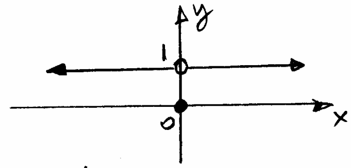
if $x > 2$, $f(x) = -4x$

x	y
2	-8
3	-12

Graph the line $y = -4x$ to the right of $x = 2$, but do not include the endpoint. (3)



(30) Give a rule for the function. Give the domain and the range.



$$f(x)=\begin{cases}1, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$$

Domain: $x\in\mathbb{R}$
Range: $y\in\{0,1\}$

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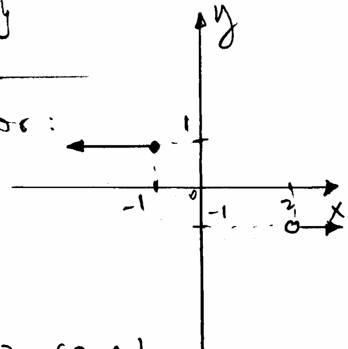
(32) Same as above for:



$$f(x)=\begin{cases}+1 & \text{if } x\leq -1\\ -1 & \text{if } x>2\end{cases}$$

Domain: $x\in(-\infty,-1]\cup(2,\infty)$
Range: $y\in\{1,-1\}$

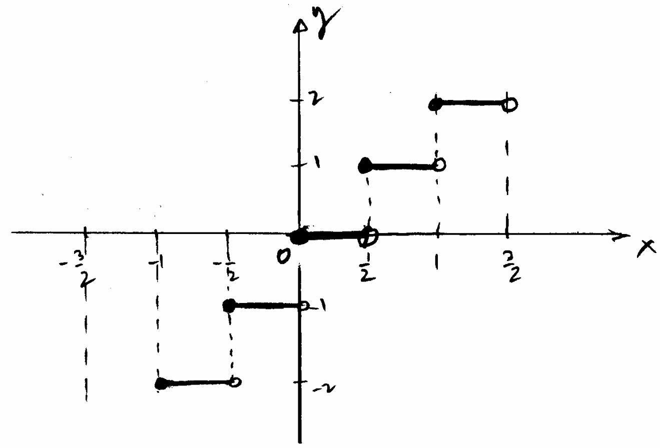
(34) Graph $f(x)=[[2x]]$

From the definition of the greatest-integer function,

$$[[2x]]=\begin{cases}\vdots\\ -2 & \text{if } -2\leq 2x<-1\\ -1 & \text{if } -1\leq 2x<0\\ 0 & \text{if } 0\leq 2x<1\\ 1 & \text{if } 1\leq x<2\\ 2 & \text{if } 2\leq x<3\\ \vdots\end{cases}$$

therefore,

$$[[2x]]=\begin{cases}\vdots\\ -2 & \text{if } -1\leq x<-\frac{1}{2}\\ -1 & \text{if } -\frac{1}{2}\leq x<0\\ 0 & \text{if } 0\leq x<\frac{1}{2}\\ 1 & \text{if } \frac{1}{2}\leq x<1\\ 2 & \text{if } 1\leq x<\frac{3}{2}\\ \vdots\end{cases}$$



(33) Graph $f(x)=[[-x]]$

From the definition of the greatest-integer function:

$$[[-x]]=\begin{cases}\vdots\\ -2 & \text{if } -2\leq -x<-1\\ -1 & \text{if } -1\leq -x<0\\ 0, & \text{if } 0\leq -x<1\\ 1 & \text{if } 1\leq -x<2\\ 2 & \text{if } 2\leq -x<3\\ \vdots\end{cases}$$

$$[[-x]]=\begin{cases}\vdots\\ -2 & \text{if } 2\geq x>1\\ -1 & \text{if } 1\geq x>0\\ 0 & \text{if } 0\geq x>-1\\ 1 & \text{if } -1\geq x>-2\\ 2 & \text{if } -2\geq x>-3\\ \vdots\end{cases}$$

$$=\begin{cases}\vdots\\ -2 & \text{if } 1<x\leq 2\\ -1 & \text{if } 0<x\leq 1\\ 0 & \text{if } -1<x\leq 0\\ 1 & \text{if } -2<x\leq -1.\\ 2 & \text{if } -3<x\leq -2\\ \vdots\end{cases}$$