M130

Homework #1 – Solutions

SECTION 1.4

(58) $3 - \frac{4}{x} - \frac{2}{x^2} = 0 \quad / \cdot x^2$

Condition: $x \neq 0$

Eliminate fractions by multiplying each side by x^2 ($x^2 \neq 0$).

$3x^2 - 4x - 2 = 0 \qquad \begin{cases} a = 3 \\ b = -4 \\ c = -2 \end{cases}$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{16 - 4 \cdot 3(-2)}}{2 \cdot 2} = \frac{4 \pm \sqrt{40}}{6} =$$

$$= \frac{4 \pm 2\sqrt{10}}{6} = \frac{2(2 \pm \sqrt{10})}{6} = \boxed{\frac{2 \pm \sqrt{10}}{3}}$$

(59) $x^3 - 8 = 0$ — Note that $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$x^3 - 2^3 = 0$

$(x-2)(x^2 + 2x + 4) = 0$

$\boxed{x_1 = 2}$ OR $x^2 + 2x + 4 = 0$

$$x_{2,3} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 16}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2} = \frac{-2 \pm 2\sqrt{3}\,i}{2} = \boxed{-1 \pm \sqrt{3}\,i}$$

(85) $\begin{cases} x_1 = 1 + \sqrt{2} \\ x_2 = 1 - \sqrt{2} \end{cases}$ solutions

then $(x - x_1)(x - x_2) = 0$

$(x - (1 + \sqrt{2}))(x - (1 - \sqrt{2})) = 0$

$(x - 1 - \sqrt{2})(x - 1 + \sqrt{2}) = 0$

$(x-1)^2 - (\sqrt{2})^2 = 0$

$x^2 - 2x + 1 - 2 = 0$

$\boxed{x^2 - 2x - 1 = 0} \qquad \begin{cases} a = 1 \\ b = -2 \\ c = -1 \end{cases}$

(86) $\begin{cases} x_1 = i \\ x_2 = -i \end{cases}$ – solutions

then $(x - x_1)(x - x_2) = 0$

$(x - i)(x + i) = 0$

$x^2 - i^2 = 0$

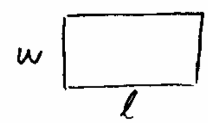
$\boxed{x^2 + 1 = 0} \qquad \begin{cases} a = 1 \\ b = 0 \\ c = 1 \end{cases}$

SECTION 1.5

(6) Given: 300 m fencing
5000 m^2 area

Find: length = ?
width = ?

Solution



300 m of fencing ⇒

$2l + 2w = 300 \quad / \div 2$

$\boxed{l + w = 150} \quad (1)$

Area = 5000 m^2

$\boxed{l \cdot w = 5000} \quad (2)$

$\begin{cases} l + w = 150 \Rightarrow l = 150 - w \\ lw = 5000 \end{cases}$

$(150 - w)w = 5000$

$150w - w^2 = 5000$

$w^2 - 150w + 5000 = 0$

$(w - 100)(w - 50) = 0 \quad < \begin{matrix} w = 100 \\ \text{OR} \\ w = 50 \end{matrix}$

If $w = 100$, then $l = 50$

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The dimensions of the garden are 50 m by 100 m.

(18) $x^2+(x+10)^2=50^2$
(Pythagorean theorem)

$x^2+x^2+20x+100=2500$

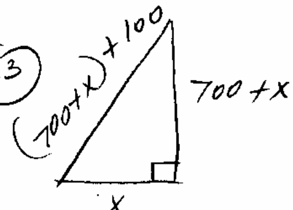
$2x^2+20x-2400=0 \quad /\div 2$

$x^2+10x-1200=0$

$(x+40)(x-30)=0 \quad \left\langle \begin{array}{l} \cancel{x=-40} \\ \text{or} \\ x=30 \end{array}\right.$

$\boxed{x=30}$ The horizontal distance is 30 ft

(23)



$x^2+(700+x)^2=(800+x)^2$

$x^2+490000+1400x+\cancel{x^2}=$

$=640000+1600x+\cancel{x^2}$

$x^2-200x-150,000=0$

$x_{1,2}=\dfrac{200\pm\sqrt{40000+4\cdot150,000}}{2}$

$=\dfrac{200\pm\sqrt{640,000}}{2}=\dfrac{200\pm800}{2} \quad \left\langle \begin{array}{l} \cancel{-300} \\ \text{or} \\ 500 \end{array}\right.$

$x=600$

Therefore, the length of the walkway will be
$500+(700+500)+(800+500)=$
$=1500+1500=3000$ ft.

SECTION 16

(47) $3-\sqrt{x}=\sqrt{2\sqrt{x}-3} \quad /^2$

$(3-\sqrt{x})^2=2\sqrt{x}-3$

$\cancel{9}-6\sqrt{x}+x=4x-12\sqrt{x}+\cancel{9}$

$12\sqrt{x}-6\sqrt{x}=4x-x$

$6\sqrt{x}=3x$

$2\sqrt{x}=x \quad /^2$

$4x=x^2$

$x^2-4x=0$

$x(x-4)=0 \quad \left\langle \begin{array}{l} \cancel{x=0} \\ \text{or} \\ x=4 \end{array}\right.$

check: $x=0$
$3-0=\sqrt{-3}$ false $\Rightarrow$ $x=0$ is not a solution

check: $x=4$
$3-\sqrt{4}=\sqrt{2\sqrt{4}-3}$
$3-2=\sqrt{4-3}$ true

The solution is $\boxed{x=4}$

(48) $\sqrt{x}+2=\sqrt{4+7\sqrt{x}} \quad /^2$

$(\sqrt{x}+2)^2=4+7\sqrt{x}$

$x+4\sqrt{x}+\cancel{4}=\cancel{4}+7\sqrt{x}$

$x=3\sqrt{x} \quad /^2$

$x^2=9x$

$x^2-9x=0$

$x(x-9)=0 \quad \left\langle \begin{array}{l} x=0 \\ \text{or} \\ x=9 \end{array}\right.$

Check $x=0$
$0+2=\sqrt{4}$ true

Check $x=9$
$\sqrt{9}+2=\sqrt{4+7\cdot3}$
$5=\sqrt{25}$ true

The solution set is $\boxed{\{0,9\}}$

(70) $(2x-1)^{2/3}+2(2x-1)^{1/3}-3=0$

Let $\boxed{(2x-1)^{1/3}=t}$

$t^2+2t-3=0$

$(t+3)(t-1)=0$

$t=-3$ OR $t=1$

if $t=-3$, then

$(2x-1)^{1/3}=-3 \quad /^3$

$\left[(2x-1)^{\frac{1}{3}}\right]^3=(-3)^3$

$2x-1=-27$

$2x=-26$

$\boxed{x=-13}$

if $t=1$, then $(2x-1)^{1/3}=1 \quad /^3$

$\left((2x-1)^{1/3}\right)^3=1^3$

$2x-1=1$

$2x=2$

$\boxed{x=1}$

The solution set is $\{-13, 1\}$

(74) $8(x-4)^4-10(x-4)^2=-3$

Let $\boxed{(x-4)^2=t}$

$8t^2-10t+3=0$

$t_{1,2}=\dfrac{10\pm\sqrt{100-96}}{16}=\dfrac{10\pm 2}{16}$

$t_1=\frac{12}{16}=\frac{3}{4}$

$t_2=\frac{8}{16}=\frac{1}{2}$

$t=\frac{3}{4}$ OR $t=\frac{1}{2}$

if $t=\frac{3}{4}$, then

$(x-4)^2=\frac{3}{4} \quad /\sqrt{\ }$

$\sqrt{(x-4)^2}=\sqrt{\frac{3}{4}}$

$|x-4|=\frac{\sqrt{3}}{2}$

$x-4=\pm\frac{\sqrt{3}}{2} \implies \boxed{x=4\pm\frac{\sqrt{3}}{2}}$

if $t=\frac{1}{2}$, then

$(x-4)^2=\frac{1}{2}$

$\sqrt{(x-4)^2}=\sqrt{\frac{1}{2}}$

$|x-4|=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

$x-4=\pm\frac{\sqrt{2}}{2} \implies \boxed{x=4\pm\frac{\sqrt{2}}{2}}$