Math 130

Activity Lab # 3 - SOLUTIONS

② Solve $2e^{3x} = 4e^{5x}$ $\quad / \div 2$

$e^{3x} = 2e^{5x}$ $\quad / \div e^{3x} \neq 0$

$1 = 2e^{5x-3x}$

$1 = 2e^{2x}$

$e^{2x} = \frac{1}{2}$

$\ln e^{2x} = \ln \frac{1}{2}$

$2x = \ln \frac{1}{2}$

$\boxed{x = \frac{\ln \frac{1}{2}}{2}} \simeq -0.347$

④ Solve $9^x = 2e^{x^2}$

$\ln 9^x = \ln(2e^{x^2})$

$x \ln 9 = \ln 2 + \ln e^{x^2}$

$x \ln 9 = \ln 2 + x^2 \ln e$

$x \ln 9 = \ln 2 + x^2$

$x^2 - \ln 9 \, x + \ln 2 = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $\begin{cases} a = 1 \\ b = -\ln 9 \\ c = \ln 2 \end{cases}$

$$\boxed{x_{1,2} = \frac{\ln 9 \pm \sqrt{(\ln 9)^2 - 4\ln 2}}{2}}$$

$x_1 \simeq 1.81$

$x_2 \approx 0.38$

⑥ $3^{x^2-4} = 27$

$3^{x^2-4} = 3^3$

$x^2 - 4 = 3$

$x^2 = 7$

$\boxed{x = \pm\sqrt{7}}$

⑧ $10^{2x} + 3(10^x) - 10 = 0$

Let $10^x = t$

then $10^{2x} = t^2$

$t^2 + 3t - 10 = 0$

$(t+5)(t-2) = 0$ $\quad \left\langle \begin{array}{l} t = -5 \\ \text{OR} \\ t = 2 \end{array} \right.$

I if $t = -5$

$10^x = -5$ not possible

II if $t = 2$

$10^x = 2$

$\log 10^x = \log 2$

$x \log 10 = \log 2$

$\boxed{x = \log 2}$

(10) Solve $e^x - e^{-x} = 1$

$$e^x - \frac{1}{e^x} = 1 \quad | \cdot e^x \neq 0$$

$$e^{2x} - 1 = e^x$$

$$e^{2x} - e^x - 1 = 0$$

Let $e^x = t$

then $e^{2x} = t^2$

$$t^2 - t - 1 = 0$$

$$t_{1,2} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

I if $t = \frac{1+\sqrt{5}}{2} > 0$

$$e^x = \frac{1+\sqrt{5}}{2}$$

$$\ln e^x = \ln \frac{1+\sqrt{5}}{2}$$

$$\boxed{x = \ln \frac{1+\sqrt{5}}{2}}$$

II if $t = \frac{1-\sqrt{5}}{2} < 0$

$$e^x = \frac{1-\sqrt{5}}{2} < 0$$

not possible

(12) Solve for t: $a > 0$, $(k \neq b)$

$$ae^{kt} = e^{bt}$$

$$a = \frac{e^{bt}}{e^{kt}}$$

$$a = e^{bt-kt}$$

$$a = e^{t(b-k)}$$

$$\ln a = \ln e^{t(b-k)}$$

$$\ln a = t(b-k)$$

$$\boxed{t = \frac{\ln a}{b-k}}$$

(14) Simplify:

$$\ln(e^{2ab}) = \boxed{2ab}$$

(16) Simplify:

$$2 \ln e^A + 3 \ln B^e =$$

$$= 2A \ln e + 3e \ln B$$

$$= \boxed{2A + 3e \ln B}$$

(18) $P = 15e^{0.25t}$

Write it as $P = P_0 a^t$

$P_0 = 15$ $\quad a = ?$

we want $e^{0.25t} = a^t$

$$(e^{0.25})^t = a^t$$

$$\Leftrightarrow$$

$$e^{0.25} = a$$

So $a = e^{0.25} \approx 1.284$ $\quad \boxed{P = 15(1.284)^t}$

increasing.

(20) $P = 4(0.55)^t$

write it as $P = P_0 e^{kt}$

$P_0 = 4$

we want $(0.55)^t = e^{kt}$

$(0.55)^t = (e^k)^t$

$\iff$

$e^k = 0.55$

$\ln e^k = \ln 0.55$

$k = \ln 0.55 \approx -0.6$

so, $\boxed{P = 4e^{-0.6t}}$

(22) Find the inverse of

$f(t) = 1 + \ln t$

1st $y = 1 + \ln t$

and solve for t:

$\ln t = y - 1$

$\iff$

$e^{y-1} = t$

$t = e^{y-1}$

3rd $t \leftrightarrow y$

$y = e^{t-1}$

$\boxed{f^{-1}(t) = e^{t-1}}$

(24) $P = P_0 e^{-kt}$

10% of pollution is removed in the first 5 hours $\Rightarrow$

90% of pollution is left

so, when $t = 5$, $P = 0.9P_0$

$0.9\cancel{P_0} = \cancel{P_0} e^{-k(5)}$

$0.9 = e^{-5k}$

$\ln 0.9 = \ln e^{-5k}$

$\ln 0.9 = -5k \Rightarrow k = \dfrac{\ln 0.9}{-5}$

$k \approx 0.02$

Therefore, $\boxed{P = P_0 e^{-0.02t}}$

a) What percentage of the pollution is left after 10 hrs?

$P = P_0 e^{-0.02(10)}$

$P = P_0 e^{-0.2}$

$\dfrac{P}{P_0} = e^{-0.2} \approx 0.81$

$\dfrac{P}{P_0} = 81\%$,

Therefore, after 10 hrs, 81% of the pollution is left.

b) How long will it take before the pollution is reduced by 50%?

so $P = \dfrac{P_0}{2}$, $t = ?$

$\dfrac{P_0}{2} = P_0 e^{-0.02t}$

$$\frac{\cancel{P_0}}{2} = \cancel{P_0} e^{-0.02t}$$

$$\frac{1}{2} = e^{-0.02t}$$

$$\ln\frac{1}{2} = \ln e^{-0.02t}$$

$$\ln 1 - \ln 2 = -0.02t$$

$$-\ln 2 = -0.02t$$

$$t = \frac{\ln 2}{0.02} \approx 34.6 \text{ hrs}$$

it will take about 34.6 hrs for the pollution to be reduced by 50%.

(26) Let t = the number of years
w = the quantity of radioactive material (kg)

t	w
0	20
5	$\frac{1}{2}(20)$
10	$\left(\frac{1}{2}\right)^2 20$
15	$\left(\frac{1}{2}\right)^3 20$

$$\boxed{w = 20\left(\frac{1}{2}\right)^{\frac{t}{5}}}$$

(a) $t = 10$, $w = 20\left(\frac{1}{2}\right)^2 = \boxed{5 \text{ kg}}$

(b) $w = 0.1$ kg, $t = ?$

$$0.1 = 20\cdot\left(\frac{1}{2}\right)^{\frac{t}{5}}$$

$$\left(\frac{1}{2}\right)^{\frac{t}{5}} = 0.002$$

$$\ln\left(\frac{1}{2}\right)^{\frac{t}{5}} = \ln 0.002$$

$$\frac{t}{5}\ln\frac{1}{2} = \ln 0.002$$

$$t = \frac{5 \ln 0.002}{\ln\frac{1}{2}} \approx 38.12 \text{ years}$$

(28) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

where $P = 800$ \$
$r = 0.04$
$n = 1$
$A = 2000$ \$

$$2000 = 800(1 + 0.04)^t$$

$$20 = 8(1 + 0.04)^t$$

$$5 = 2(1 + 0.04)^t$$

$$\frac{5}{2} = (1 + 0.04)^t$$

$$\ln\frac{5}{2} = \ln(1 + 0.04)^t$$

$$\ln\frac{5}{2} = t\ln(1 + 0.04)$$

$$t = \frac{\ln\frac{5}{2}}{\ln(1 + 0.04)} \approx 23.4 \text{ years}$$