
Activity Lab #2 (Group set)
25 points – Due Wednesday, March 22

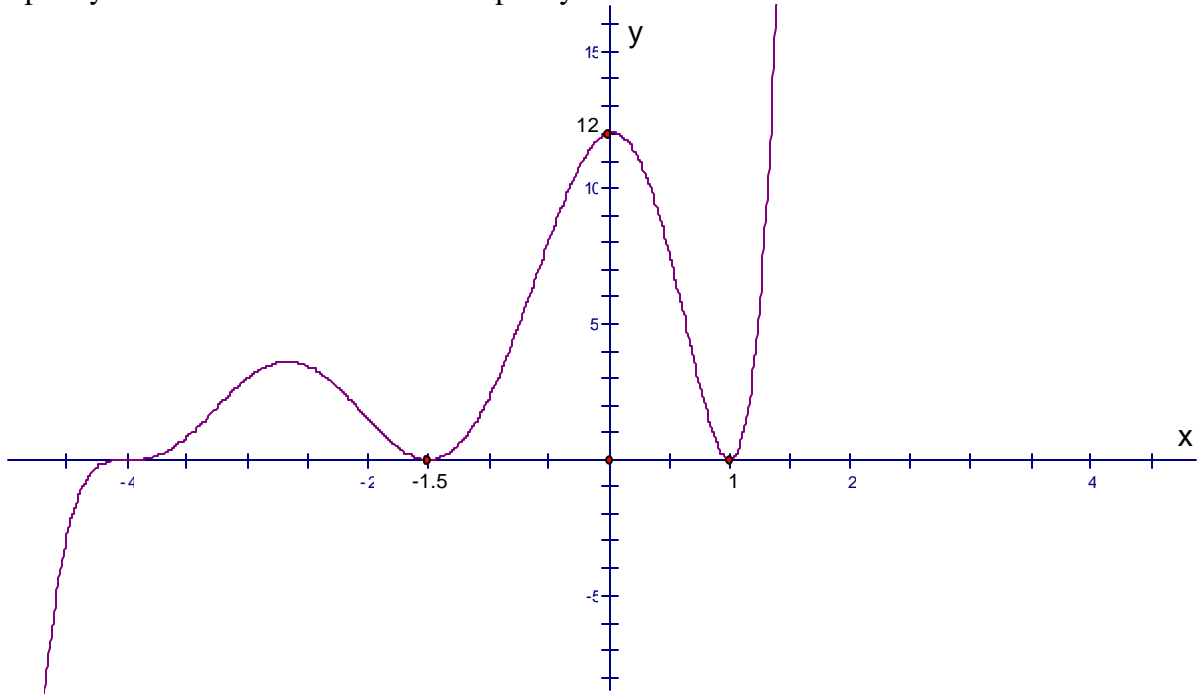
Each group must have at least 2 students and may not be more than 3 students. Individual efforts will not be accepted. Make sure you **SHOW AND JUSTIFY YOUR WORK** in order to get credit. Turn in one assignment per group. Do not just write down an answer. No proof, no credit given.

Names: _____

1. Consider the following polynomial function $f(x) = -6x^4 - x^3 + 39x^2 - 21x - 20$. Questions a-i below relate to this polynomial function.

- a) Use the leading term to describe the long-term behavior of this function; that is, what happens as $x \rightarrow \pm\infty$.
- b) Use synthetic division to divide $f(x)$ by $x - \frac{4}{3}$ and relate dividend, divisor, quotient and remainder in an equation.
- c) Compute and compare the values of $f(-1)$ and $f(0)$. What can you conclude using the intermediate value theorem?
- d) State why the condition for the theorem on rational zeros is satisfied and use the theorem on rational zeros to list all possible rational zeros for $f(x)$.
- e) Use synthetic division to divide $f(x)$ by $x + \frac{2}{3}$ and relate the dividend, divisor, quotient and remainder in an equation.
- f) Use synthetic division to divide $f(x)$ by $x - \frac{5}{2}$ and then the theorem on bounds to show that $\frac{5}{2}$ is an upper bound on the roots of $f(x)$.
- g) Use long division to show that $x^2 + x - 5$ is a divisor of $f(x)$ and relate the dividend, divisor, quotient and remainder in an equation.
- h) Write $f(x)$ in completely factored form.
- i) Sketch a graph of $f(x)$ showing how it passes through its intercepts.

2. Find a formula for the 7th degree polynomial whose graph is shown. Hint: it has a root of multiplicity 3 at $x = -4$ and roots of multiplicity 2 at -1.5 and 1 .



3. Find the values of a and b such that $x - 1$ is a factor of both $x^3 + x^2 + ax + b$ and $x^3 - x^2 - ax + b$.
Hint: Use the factor theorem and/or synthetic division.
4. Find all prime numbers p for which the equation $x^3 + x^2 + x - p = 0$ has at least one rational root. For each value of p that you find, find all the corresponding roots of the equation.
5. Show that the following equation has at least one real root. Locate the root between successive tenths:
 $x^3 - 3x^2 + 3x - 26 = 0$
6. Consider $y = \frac{x^3 + 6x^2 + 9x}{2x^3 - 2}$.
- Factor the numerator and denominator.
 - What are the intercepts for this function?
 - What is the vertical asymptote?
 - What is the horizontal asymptote?
 - Plot additional points, as necessary, to get the shape of this function and sketch a graph.
7. Sketch the graph of the following showing all intercepts, asymptotes and additional points, as necessary, to get the shape.

$$y = \frac{(x^2 - 4)(2x^2 - 1)}{(x - 1)^2(x^2 + x + 1)}$$