Math 130 Spring 2006
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Date assigned: 01/30

Activity Lab #1 (Group set)

25 points – Due Monday, February 13

Each group must have at least 2 students and may not be more than 3 students. Individual efforts will not be accepted. Make sure you SHOW AND JUSTIFY YOUR WORK in order to get credit.

Names: ______________________

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1) Let $A(-7,-4)$ and $B(4,-1)$ be two points in a plane. Find the following and sketch an appropriate figure:

a) An equation of the circle with diameter AB. Show how you obtain the equation. + graph

Center $M(x_M, y_M)$

$x_M = \frac{x_A + x_B}{2} = \frac{-7+4}{2} = \frac{-3}{2}$

$y_M = \frac{y_A + y_B}{2} = \frac{-4+(-1)}{2} = \frac{-5}{2}$

$\boxed{M\left(\frac{-3}{2}, \frac{-5}{2}\right)}$ the center

Radius: $r = \frac{AB}{2}$

$AB^2 = (\Delta x)^2 + (\Delta y)^2$

$= (-7-4)^2 + (-4-(-1))^2$

$= (-11)^2 + (-3)^2$

$= 121 + 9 = 130$

$\to AB^2 = 130$

$AB = \sqrt{130}$

$\boxed{r = \frac{\sqrt{130}}{2}}$ the radius

Circle: $(x-h)^2 + (y-k)^2 = r^2$ where $(h,k) =$ center

$\left(x - \frac{-3}{2}\right)^2 + \left(y - \frac{-5}{2}\right)^2 = \left(\frac{\sqrt{130}}{2}\right)^2$

$\left(x + \frac{3}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{130}{4}$

$\boxed{\left(x + \frac{3}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{65}{2}}$ equation of the circle

b) Which point "A" or "B" is closer to the point $\left(\frac{1}{2}, 1\right)$? Justify your reasoning with appropriate work.

Calculate the distance between $A(-7,-4)$ and $\left(\frac{1}{2}, 1\right)$

$\sqrt{(\Delta x)^2 + (\Delta y)^2} =$

$= \sqrt{\left(-7-\frac{1}{2}\right)^2 + (-4-1)^2} =$

$= \sqrt{\left(\frac{15}{2}\right)^2 + 5^2} = \sqrt{\frac{225}{4} + \frac{25}{1}}$

$= \sqrt{\frac{325}{4}} = \frac{5}{2}\sqrt{13} \simeq 9.01$

Calculate the distance between $B(4,-1)$ and $\left(\frac{1}{2}, 1\right)$

$\sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{\left(4-\frac{1}{2}\right)^2 + (-1-1)^2}$

$= \sqrt{\left(\frac{7}{2}\right)^2 + 4} = \sqrt{\frac{49}{4} + 4}$

$= \sqrt{\frac{65}{4}} = \frac{\sqrt{65}}{2} \simeq 4.03$

Therefore, B is closer to $\left(\frac{1}{2}, 1\right)$

$$\left(x+\frac{3}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=\frac{65}{2}$$

c) Find the exact x-intercepts (if any).

$y=0$ $\quad\left(x+\frac{3}{2}\right)^2+\left(\frac{5}{2}\right)^2=\frac{65}{2}$

$$\left(x+\frac{3}{2}\right)^2+\frac{25}{4}=\frac{65}{2}$$

$$\left(x+\frac{3}{2}\right)^2=\frac{65}{2}-\frac{25}{4}$$

$$\left(x+\frac{3}{2}\right)^2=\frac{130-25}{4}$$

$$\left(x+\frac{3}{2}\right)^2=\frac{105}{4}$$

$$\sqrt{\left(x+\frac{3}{2}\right)^2}=\sqrt{\frac{105}{4}}$$

$$\left|x+\frac{3}{2}\right|=\frac{\sqrt{105}}{2}$$

$$x+\frac{3}{2}=\pm\frac{\sqrt{105}}{2}$$

$$x=-\frac{3}{2}\pm\frac{\sqrt{105}}{2}$$

The x-intercepts are $\left(-\frac{3}{2}-\frac{\sqrt{105}}{2},0\right)$ and $\left(-\frac{3}{2}+\frac{\sqrt{105}}{2},0\right)$

$(-6.6,0)$

$(3.62,0)$

d) Find the exact y-intercepts (if any).

$x=0$ $\quad\left(\frac{3}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=\frac{65}{2}$

$$\frac{9}{4}+\left(y+\frac{5}{2}\right)^2=\frac{65}{2}$$

$$\left(y+\frac{5}{2}\right)^2=\frac{65}{2}-\frac{9}{4}$$

$$\left(y+\frac{5}{2}\right)^2=\frac{121}{4}$$

$$\sqrt{\left(y+\frac{5}{2}\right)^2}=\sqrt{\frac{121}{4}}$$

$$\left|y+\frac{5}{2}\right|=\frac{11}{2}$$

$$y+\frac{5}{2}=\pm\frac{11}{2}$$

$$y=-\frac{5}{2}\pm\frac{11}{2}$$

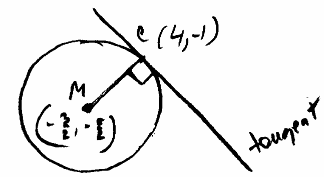
$$y=-\frac{5}{2}-\frac{11}{2}=\frac{-16}{2}=-8$$

OR

$$y=-\frac{5}{2}+\frac{11}{2}=\frac{6}{2}=3$$

The y-intercepts are $(0,-8)$ and $(0,3)$

e) Find the equation of the line tangent to the circle at the point (4, -1). (Hint: the tangent to the circle is perpendicular to the radius at the point (4, -1))



Line tangent $\begin{cases}\text{slope } m=?\\ \text{point } (4,-1)\end{cases}$

Find the slope of the radius MC, $M\left(-\frac{3}{2},-\frac{5}{2}\right)$, $C(4,-1)$

$$m=\frac{\Delta y}{\Delta x}=\frac{-1-\left(-\frac{5}{2}\right)}{4-\left(-\frac{3}{2}\right)}=\frac{-1+\frac{5}{2}}{4+\frac{3}{2}}$$

$$=\frac{\frac{3}{2}}{\frac{11}{2}}=\frac{3}{11}$$

$m=\frac{3}{11}$, therefore the slope of the tangent is $m_\perp=-\frac{11}{3}$

→ The equation of the tangent:

$$y-(-1)=\frac{-11}{3}(x-4)$$

$$\boxed{y+1=\frac{-11}{3}(x-4)}$$

2. Astronomers use a numerical scale called magnitude to measure the brightness of a star, with brighter stars assigned smaller magnitudes. When we view a start from earth, molecules and dust in the air scatter and absorb some of the light, making the star appear fainter than it really is. Thus, the observed magnitude of a star depends on the distance its light rays must travel through the earth's atmosphere. The observed magnitude, *m*, is given by

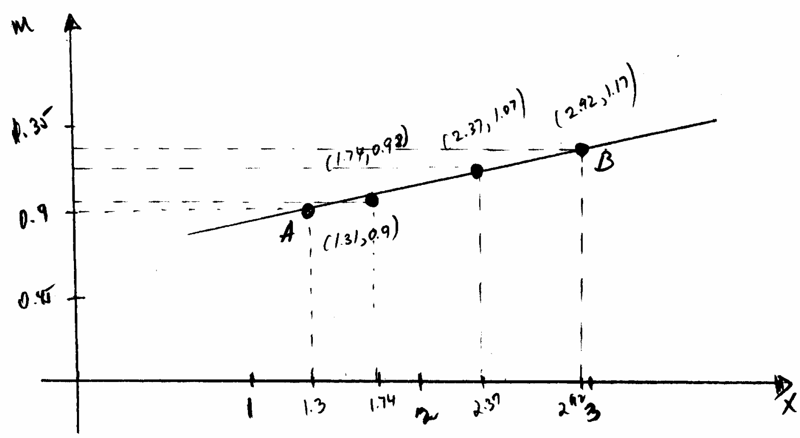
$$m = m_0 + kx,$$

where m_0 is the apparent magnitude of the start outside the atmosphere, x is the air mass (a measure of the distance through the atmosphere), and k is called the extinction coefficient. To calculate m_0, astronomers observe the same object several times during the night at different positions in the sky, and hence different values of x. Here are data from such observations.

Altitude	Air mass, x	Magnitude, m
50°	1.31	0.90
35°	1.74	0.98
25°	2.37	1.07
20°	2.92	1.17

x = air mass - independent variable
m = magnitude - dependent variable

a) Plot magnitude against air mass, and draw a line of best fit through the data.



b) Find the equation of your line of best fit. Use $A(1.31, 0.9)$ and $B(2.92, 1.17)$

slope $= k = \frac{\Delta m}{\Delta x} = \frac{1.17-0.9}{2.92-1.31} = \frac{0.27}{1.64} \simeq 0.165$

$m - 0.9 = 0.165(x - 1.31)$

$\boxed{m = 0.165x + 0.68}$

c) What is the value of the extinction coefficient? What is the apparent magnitude of the star outside the earth's atmosphere?

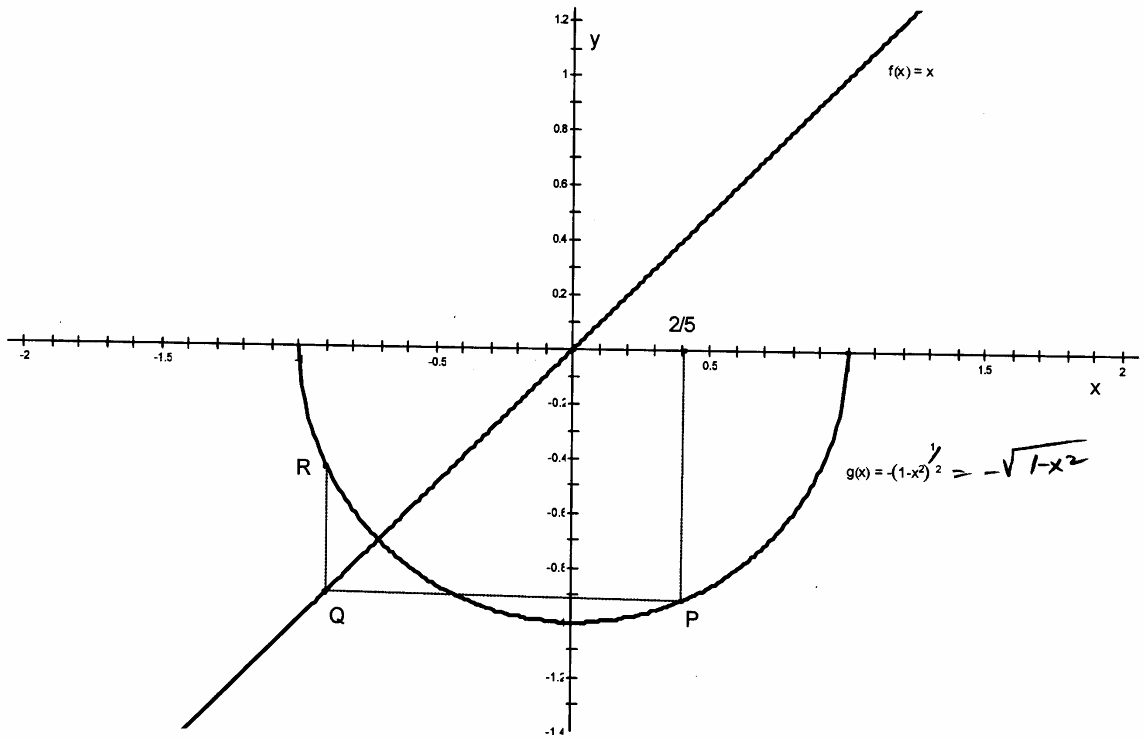
$m = 0.165x + 0.68$

extinction coefficient $= k$ = the slope of the line

$k = 0.165$

apparent magnitude $= m_0 = 0.68$

3. Determine the coordinates of the points P, Q, and R in the figure; give an exact expression and also a calculator approximation rounded to three decimal places. Assume that each dashed line is parallel to one of the coordinate axes.



$P(x_P, y_P)$ $\quad x_P = \frac{2}{5} \Rightarrow y_P = -\sqrt{1-\left(\frac{2}{5}\right)^2} = -\sqrt{1-\frac{4}{25}} = -\sqrt{\frac{21}{2}} = \frac{-\sqrt{21}}{5}$

$\boxed{P\left(\frac{2}{5}, -\frac{\sqrt{21}}{5}\right)}$ $\quad P \in$ graph of $g(x)$

$Q(x_Q, y_Q)$ $\quad y_Q = y_P = \frac{-\sqrt{21}}{5} \Rightarrow x_Q = y_Q = \frac{-\sqrt{21}}{5}$

$\boxed{Q\left(\frac{-\sqrt{21}}{5}, \frac{-\sqrt{21}}{5}\right)}$ $\quad Q \in$ graph of $f(x)$

$R(x_R, y_R)$ $\quad x_R = x_Q = \frac{-\sqrt{21}}{5} \Rightarrow y_R = -\sqrt{1-\left(\frac{-\sqrt{21}}{5}\right)^2} = -\sqrt{1-\frac{21}{25}}$

$R \in$ graph of $g(x)$ $\quad = -\sqrt{\frac{4}{25}} = \frac{-2}{5}$

$\boxed{R\left(\frac{-\sqrt{21}}{5}, \frac{-2}{5}\right)}$

4. Sketch the graph of the following piece-defined function. Show all work.

$$f(x)=\begin{cases} |x+1|, -2 \le x < 0 \\ \sqrt{x}, 0 \le x \le 1 \\ x^3, 1 < x < 2 \end{cases}$$

$\boxed{f(x) = |x+1| \text{ if } -2 \le x < 0}$

$$|x+1| = \begin{cases} x+1 & \text{if } x+1 \geq 0 \\ -x-1 & \text{if } x+1 < 0 \end{cases}$$

$$|x+1| = \begin{cases} x+1 & \text{if } -1 \le x < 0 \\ -x-1 & \text{if } -2 \le x < -1 \end{cases}$$

x	y
-1	0
0	1

1

x	y
-2	1
-1	0

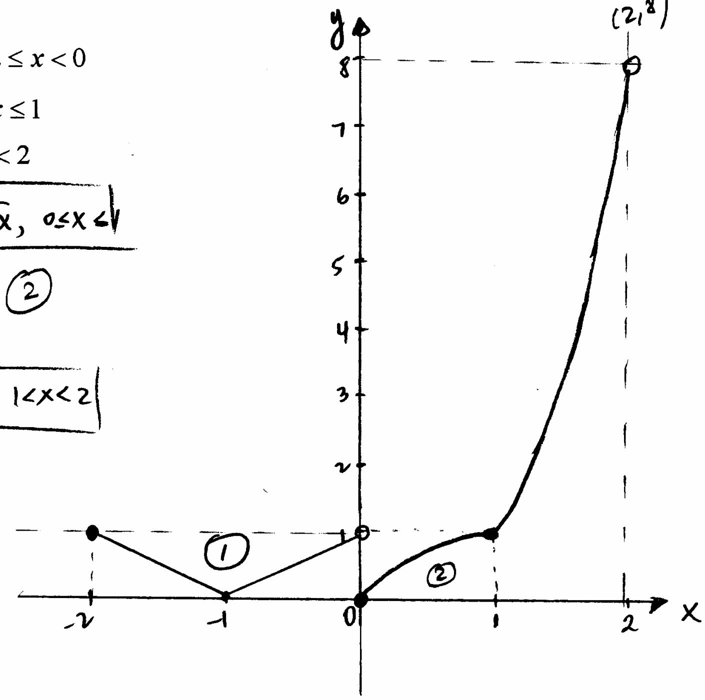
$\boxed{f(x) = \sqrt{x},\ 0 \le x \le 1}$

x	y
0	0
1	1

2

$\boxed{f(x) = x^3,\ 1 < x < 2}$

x	y
1	1
2	8



Answer the following questions:

a) What is the domain of the function?

$x \in [-2, 2)$

b) What is the range of the function?

$y \in [0, 8)$

c) Find $f\left(\frac{1}{2}\right)$, $f\left(-\frac{1}{2}\right)$, and $f\left(\frac{3}{2}\right)$.

$f(\frac{1}{2}) = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ b/c $x = \frac{1}{2} \in [0,1]$

$f(-\frac{1}{2}) = |\frac{-1}{2}+1| = |\frac{1}{2}| = \frac{1}{2}$ b/c $x = \frac{-1}{2} \in [-2,0)$

$f(\frac{3}{2}) = (\frac{3}{2})^3 = \frac{27}{8}$ b/c $x = \frac{3}{2} \in (1,2)$

d) On what intervals is the function increasing ?

$x \in [-1, 2)$

e) On what intervals is the function decreasing ?

$x \in [-2, -1)$