Sections 9.1 and 9.6
Exponential Functions
Exponential Growth and Decay

The story of chess . It seems that chess had its beginnings in India around 600AD - 1,400 years ago. There was a King in India who loved to play games. He commissioned a poor mathematician who lived in his kingdom to come up with a new game. After months of struggling with all kinds of ideas the mathematician came up with the game of Chaturanga. The game had two armies each lead by a King who commanded the army to defeat the other by capturing the enemy King. It was played on a simple 8x8 square board. The King loved this game so much that he offered to give the mathematician anything he wished for. ***"I would like one grain of rice for the first square of the board, two grains for the second, four grains for the third and so on doubled for each of the 64 squares of the game board"*** said the mathematician. "Is that all?" asked the King, "Why don't you ask for gold or silver coins instead of rice grains". "The rice should be sufficient for me." replied the mathematician. The King ordered his staff to lay down the grains of rice and soon learned that all the wealth in his kingdom would not be enough to buy the amount of rice needed on the 64th square. In fact the whole kingdoms supply of rice was exha
square was reached. "You have provided me with such a great game and yet I cannot fulfill yo
are indeed a genius." said the King and offered to make the mathematician his top most adviso

Question #1: Can you find exactly how many grains of rice would be needed on the 64th square and how much total rice would be needed for all 64 squares?

1st square	1
2nd	2
3rd	2^2
4th	2^3
⋮	
64th	2^{63}

Total # grains of rice $= 1+2+2^2+2^3+\dots+2^{63}$

let S = total # grains

$$S = 1+2+2^2+\dots+2^{63} \quad /\cdot 2$$
$$2S = 2+2^2+2^3+\dots+2^{64}$$
$$(-)\ 2S - S = 2^{64}-1$$

$$\boxed{S = 2^{64}-1}$$

9,223,372,036,854,775,808 on the 64th square and 18,446,744,073,709,551,615 total for the whole board. That's about 18 billion billions. So if a bag of rice contained a billion grains, you would need 18 billion such bags.

Question #2: Suppose that your mathematics instructor, in an effort to improve classroom attendance, offers to pay you each day for attending class! Suppose you are to receive 2 cents the first day you attend class, 4 cents the second day, 8 cents the third day, and so on for 3(
you rather have: $1 million dollars or the above offer?

1st day	2
2nd	2^2
3rd	2^3
⋮	
30th	2^{29}

$$2+\dots+2^{29} = 2^{30}-2$$

$$2^{30} = (2^{10})^3 \approx (10^3)^3 = 10^9 = 1{,}000{,}000{,}000 \text{ cents}$$
$$= 10{,}000{,}000 \$$$

10 million dollars

Note: Simple method for quickly estimating powers of two

$$2^{10} \approx 10^3$$

Review

Complete the following:

$$\boxed{a^{\frac{m}{n}} = \sqrt[n]{a^m}}$$

1) Write using radical notation:

a) $10^{\frac{4}{5}} = \boxed{\sqrt[5]{10^4}}$

b) $x^{\frac{3}{7}} = \boxed{\sqrt[7]{x^3}}$

2) Use exponent notation:

a) $\sqrt[7]{(1+n)^4} = \boxed{(1+n)^{\frac{4}{7}}}$

b) $\sqrt[3]{x^5} = \boxed{x^{\frac{5}{3}}}$

3) Simplify :

a) $16^{\frac{1}{2}} = \sqrt{16} = \boxed{4}$

b) $(-32)^{\frac{1}{5}} = \sqrt[5]{-32}$
$= \boxed{-2}$

c) $64^{-\frac{1}{2}} = \frac{1}{64^{\frac{1}{2}}}$
$= \frac{1}{\sqrt{64}} = \boxed{\frac{1}{8}}$

4) If $f(x) = 3^x$, find each of the following:

a) $f(2) = 3^2 = \boxed{9}$

b) $f(-3) = 3^{-3} = \frac{1}{3^3}$
$= \boxed{\frac{1}{27}}$

5) Solve the following equations:

a) $3^x = 27$

$3^x = 3^3$

$\boxed{x = 3}$

b) $2^{3y+1} = \sqrt{2}$

$2^{3y+1} = 2^{\frac{1}{2}}$

$3y+1 = \frac{1}{2}$

$3y = \frac{-1}{2}$

$\boxed{y = \frac{-1}{6}}$

c) $\left(\frac{1}{2}\right)^k = 4$

$(2^{-1})^k = 2^2$

$2^{-k} = 2^2$

$-k = 2$

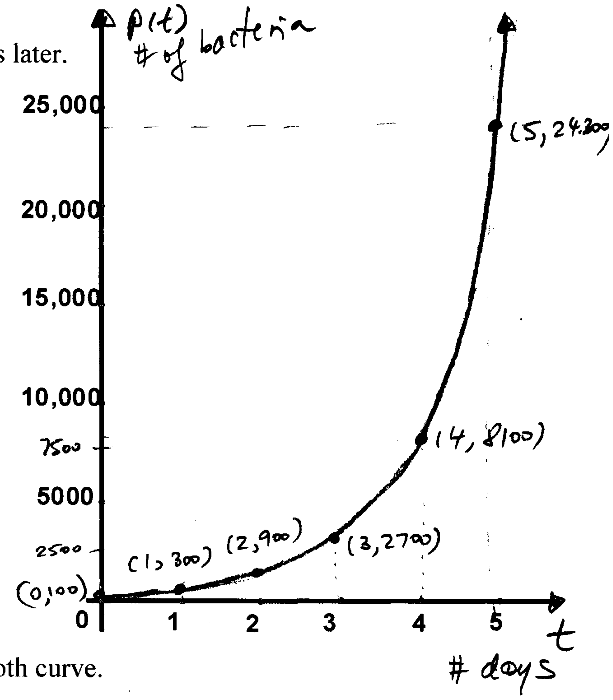
$\boxed{k = -2}$

Exponential Growth and Decay

Example 1 In a laboratory experiment, researchers establish a colony of 100 bacteria and monitor its growth. The experiments discover that the colony triples in population every day.

1. Fill in Table 1. showing the population, $P(t)$, of bacteria t days later.

t	P(t)	
0	100	
1	300	$= 100(3)^1$
2	900	$= 100(3)^2$
3	2700	$= 100(3)^3$
4	8100	$= 100(3)^4$
5	24,300	$= 100(3)^5$



2. Plot the data points from Table 1. and connect them with a smooth curve.

3. Write a function that gives the population of the colony at any time t in days. (Express the values you calculated in part (1) using powers of 3. Do you see a connection between the value of t and the exponent on 3?)

$$P(t) = 100(3)^t$$

4. Evaluate your function to find the number of bacteria present after 8 days. How many bacteria are present after 36 hours?

$t = 8$ days; $P(8) = 100(3)^8 = 656{,}100$ bacteria after 8 days

$t = 36 \text{ hrs} = \frac{36}{24} \text{ days} = \frac{3}{2} \text{ days} = 1.5 \text{ days}$

$P(1.5) = 100(3)^{1.5} \approx 520$ bacteria after 36 hrs

Growth Factors

The function in Example 1 describes **exponential growth.** During each time interval of a fixed length the population is multiplied by a certain constant amount. The bacteria population grows by a factor of 3 every day. For this reason we say that 3 is the **growth factor** for the function. Functions that describe exponential growth can be expressed in the standard form

$$\boxed{P(t) = P_0 a^t}$$

where $P_0 = P(0)$ is the initial value of the function and a is the growth factor.

For the bacteria population we have $P(t) = 100 \cdot 3^t$ so $P_0 = 100$ and $a = 3$

Exercise #1 A lab technician compares the growth of two species of bacteria. She starts two colonies of 50 bacteria each. Species A doubles in population every 2 days, and species B triples every 3 days. Find the growth factor for each species. Which species grows faster?

Species A

t	$P(t)$
0	50
2	$50(2)$
4	$50(2)^2$
6	$50(2)^3$
8	$50(2)^4$
t	$50(2)^{t/2}$

$P(t) = 50\cdot 2^{t/2}$

Species B

t	$P(t)$
0	50
3	$50(3)$
6	$50(3)^2$
9	$50(3)^3$
12	$50(3)^4$
t	$50(3)^{t/3}$

$P(t) = 50\cdot 3^{t/3}$

- A function describing the growth of A is $P(t) = 50\cdot 2^{t/2} = 50\cdot\left(2^{\frac{1}{2}}\right)^t$
  The growth factor for A is $2^{\frac{1}{2}} \approx 1.41$
  Species A grows by a factor of 1.41 every day
- A function describing the growth of B is $P(t) = 50\cdot 3^{t/3} = 50\cdot\left(3^{\frac{1}{3}}\right)^t$
  The growth factor for B is $3^{\frac{1}{3}} \approx 1.44$
  Species B grows by a factor of 1.44 every day
  Species B grows faster than A.

Example 2 **Percent Increase**

Exponential growth occurs in other circumstances, too. For example, of the interest on a savings account is compounded annually, the amount of money in the account grows exponentially.

Consider a principal of \$100 invested at 5% interest compounded annually.

t	A(t)
0	100
1	$100(1.05) = 105$
2	$100(1.05)^2 = 110.25$
3	$100(1.05)^3 = 115.76$

Write the formula for the amount in factored form.

Amount = Principal + interest

$A = P + Prt$

$A = P(1+rt)$ after one year.

What is the amount in the account at the end of 1 year?

$A = 100(1+0.05(1)) = 100(1.05) = 105$

The amount, \$105, becomes the new principal for the second year.
To find the amount at the end of the second year, we apply the formula again, with $P = 105$.
Observe that to find the amount at the end of each year we multiply the principal by a factor of $1+r = 1.05$.

$A = P(1+rt) = 105(1+0.05(1)) = 105(1.05) = 100(1.05)^2$

A formula for the amount after t years is $A(t) = 100(1.05)^t$.

In general, for an initial investment of P dollars at an interest rate , r, compounded annually, the amount accumulated after t years is

$$\boxed{A(t) = P(1+r)^t}$$

This function describes exponential growth with an initial value of P and a growth factor of $a = 1+r$.
The interest rate, r, which indicates the *percent increase* in the account each year, corresponds to a *growth factor* of $a = 1+r$.

Example 3 A small coal-mining town has been losing population since 1940, when 5000 people lived there. At each census thereafter (taken at 10-year intervals) the population has declined to approximately 0.90 of its earlier figure.

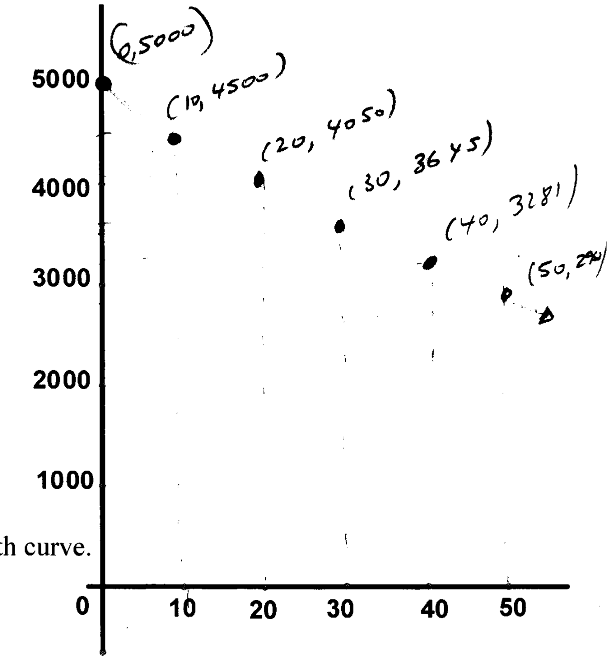
1. Fill in Table 2. showing the population, $P(t)$, of the town t years after 1940.

t	P(t)
0	5000
10	4500
20	4050
30	3645
40	3281
50	2952

$P(10) = 5000(0.90)$

$P(20) = 5000(0.90)^2$

$P(30) = 5000(0.90)^3$



2. Plot the data points from Table 2. and connect them with a smooth curve.

3. Write a function that gives the population of the town at any time t in years after 1940. (Express the values you calculated in part (1) using powers of 0.90. Do you see a connection between the value of t and the exponent on 0.90?)

$$P(t) = 5000 \cdot (0.9)^{0.1t}$$

4. Evaluate your function to find the population of the town in 1995. What was the population in 2000?

1995: $t = 55$ $\quad P(55) = 5000(0.9)^{0.1(55)} \approx 2801$ people in 1995

2000: $t = 60$ $\quad P(60) = 5000(0.9)^{0.1(60)} \approx 2657$ people in 2000

Exponential Growth and Decay Functions

The function

$$P(t) = P_0 a^t$$

models exponential growth and decay.

$P_0 = P(0)$ is the **initial value** of P;

a is the **growth** or **decay factor**.

1. If $a > 1$, $P(t)$ is increasing, and $a = 1 + r$, where r represents percent increase.
2. If $0 < a < 1$, $P(t)$ is decreasing, and $a = 1 - r$, where r represents percent decrease.

Exercise #3 At a large university three students start a rumor that the final exams have been canceled. After two hours six students (including the first three) have heard the rumor.

a) Assuming that the rumor grows linearly, complete the table below for $L(t)$, the number of students who have heard the rumor after t hours. Then write a formula for the function $L(t)$. Graph the function.

$L(t) = mt + b$ with

$L(0) = 3$, so $b = 3$

t	0	2	4	6	8
$L(t)$	3	6	9	12	15

$m = \frac{\Delta L}{\Delta t} = \frac{6-3}{2} = 1.5$ students/h

$\boxed{L(t) = 1.5t + 3}$

b) Complete the table below assuming that the rumor grows exponentially. Write a formula for the function $E(t)$, and graph it on the same set of axes with $L(t)$.

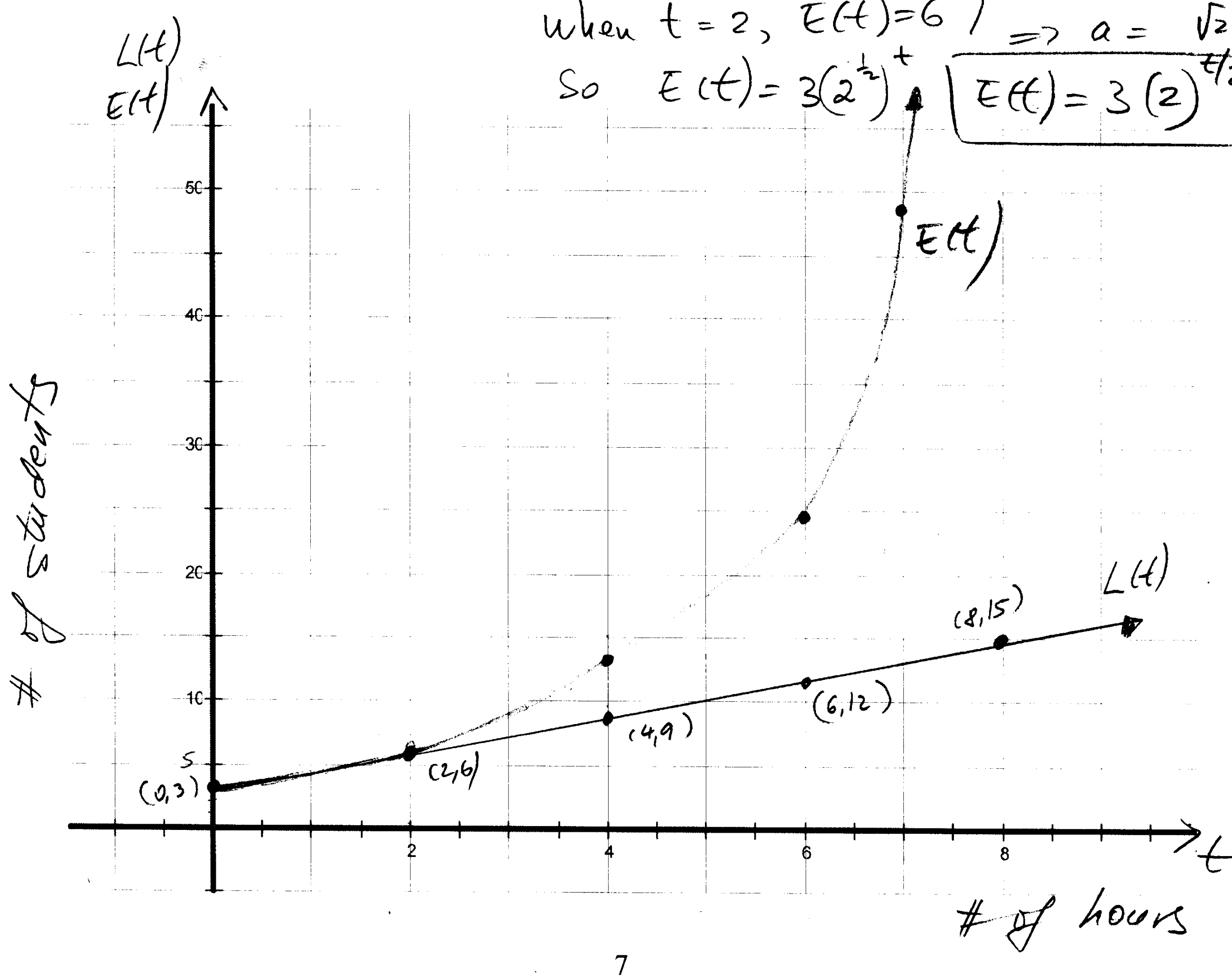
$E(t) = E_0 a^t$ with

$E(0) = 3$, so $E_0 = 3$

t	0	2	4	6	8
$E(t)$	3	6	12	24	48

$\left.\begin{array}{l} E(t) = 3a^t \\ \text{when } t = 2,\ E(t) = 6 \end{array}\right\} \Rightarrow 6 = 3a^2 \Rightarrow a = \sqrt{2} = 2^{\frac{1}{2}}$

So $E(t) = 3(2^{\frac{1}{2}})^t$ $\boxed{E(t) = 3(2)^{t/2}}$



Comparing Linear Growth and Exponential Growth

Exercise #2 A solar energy company sold \$80,000 worth of solar collectors last year, its first year of operation. This year its sales rose to \$88,000, an increase of 10%. The marketing department must estimate its projected sales for the next 3 years.

a) If the marketing department predicts that sales will grow linearly, what should it expect the sales total , $L(t)$, to be next year? Graph the projected sale figures over the next 3 years, assuming that sales grow linearly.

b) If the marketing deparmtnet predicts that sales will grow exponentially. What should it expect the sales total, $E(t)$, to be next year? Graph the projected sales figure over the next 3 years, assuming that sales will grow exponentially.

t	L(t)	E(t)
0	80,000	80,000
1	88,000	88,000
2	96,000	96,800
3	104,000	106,480
5	120,000	128,841
10	160,000	207,499

a) at $t=0$ 1st year of operation

Sales grow linearly =>

$L(t) = mt + b$ with $L(0) = 80,000$

So $b = 80,000$

$m = \frac{\Delta L}{\Delta t} = \frac{88,000 - 80,000}{1-0} = 8000$ \$/year

$\boxed{L(t) = 8000t + 80,000}$

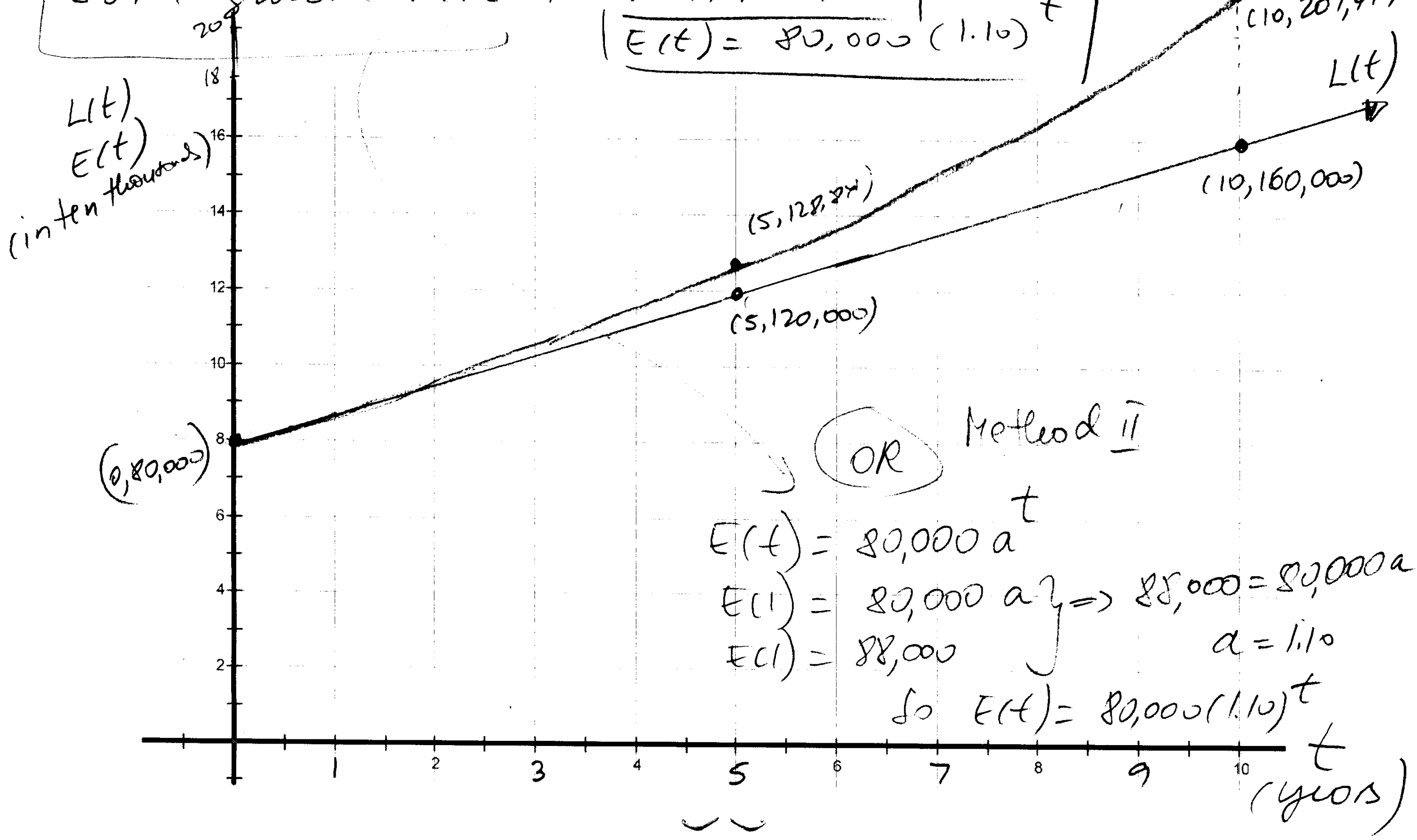
Method I b) Sales grow exponentially =>

$E(t) = E_0 a^t$ with $E(0) = E_0 = 80,000$

The percent increase in sales over the 1st year was $r = 0.10$

So the growth rate is $a = 1 + r = 1.10$

$\boxed{E(t) = 80,000(1.10)^t}$



OR Method II

$E(t) = 80,000a^t$

$E(1) = 80,000a$

$E(1) = 88,000$

$\Rightarrow 88,000 = 80,000a$

$a = 1.10$

So $E(t) = 80,000(1.10)^t$

The Exponential Function: $\boxed{f(x) = ab^x \text{, where } b > 0 \text{ and } b \neq 1, a \neq 0 .}$

a = the coefficient, b = base.

Note: If $b = 1$, then $b^x = 1^x = 1$ for any x – trivial.
If $b = 0$, then $b^x = 0^x$ which is undefined when $x = 0$
If $b < 0$, let $f(x) = (-2)^x$. If $x = \frac{1}{2}$, $f(x) = (-2)^{\frac{1}{2}} = \sqrt{-2} \notin \mathbb{R}$

Graphs of Exponential Functions

$$f(x) = 2^x$$

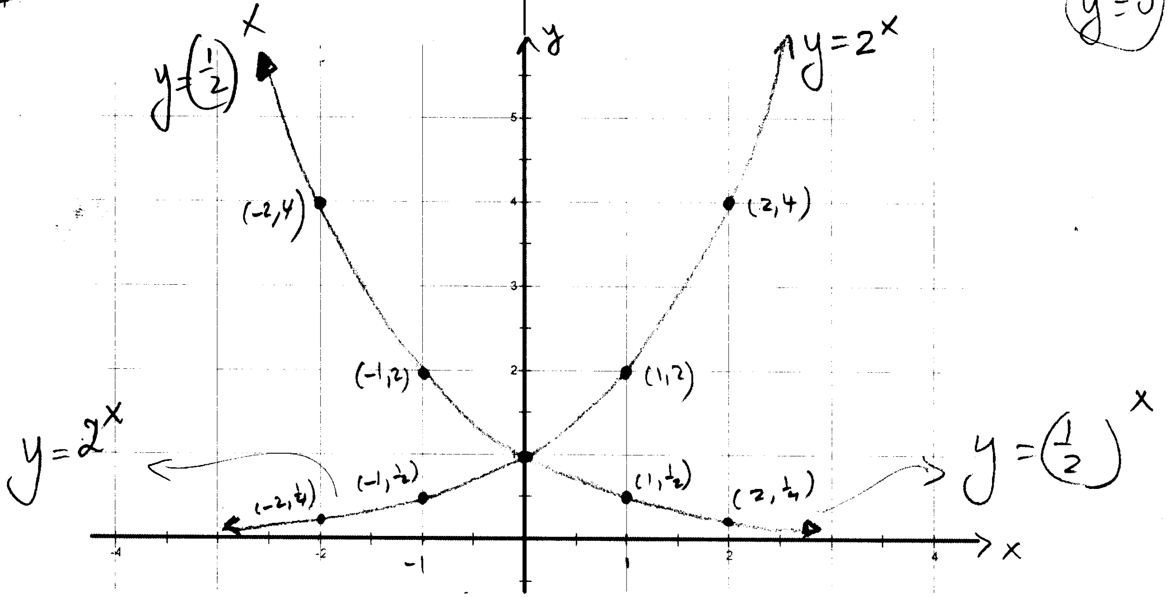
x	$-\infty$	-2	-1	0	1	2	∞
y	$\to 0$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	$\to \infty$

$y = 0$ H.A

$$g(x) = \left(\frac{1}{2}\right)^x$$

x	$-\infty$	-2	-1	0	1	2	∞
y	∞	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\to 0$

$y = 0$ H.A



Domain: $x \in \mathbb{R}$

Range: $y \in (0, \infty)$

Horizontal Asymptote: $y = 0$ (x-axis)

If $b > 1$, the function is **increasing**.
The function is one-to-one.

Domain: $x \in \mathbb{R}$

Range: $y \in (0, \infty)$

Horizontal Asymptote: $y = 0$ (x-axis)

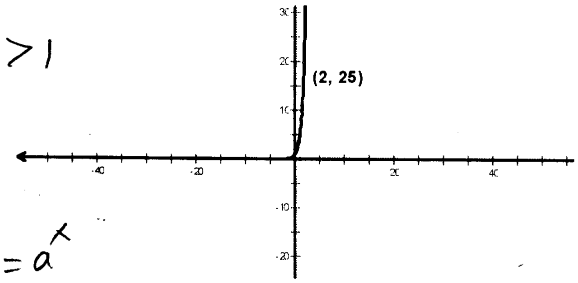
If $0 < b < 1$, the function is **decreasing.**
The function is one-to-one.

Question: Which function grows more rapidly: $y = 3^x$ or $y = 4^x$?
When $b > 1$, the greater the value of b is, the more rapidly the graph rises

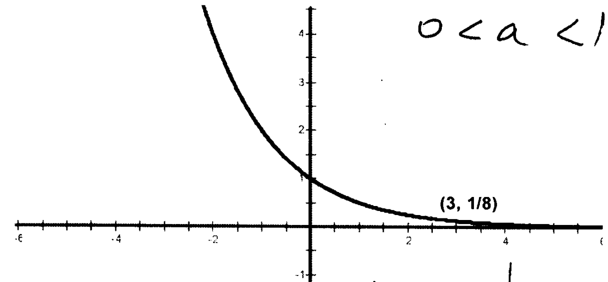
Which function decreases more rapidly: $y = (0.5)^x$ or $y = (0.8)^x$?
When $0 < b < 1$, the smaller the value of b is, the more rapidly the graph falls

Exercise #1 Find the function $f(x) = a^x$ whose graph is given.

$a > 1$



$0 < a < 1$



$y = a^x$

$(2,25) \in$ graph $\Rightarrow$ when $x=2, y=25$

$25 = a^2 \Rightarrow a = \pm 5$ but $a > 0$ $\Rightarrow \boxed{a = 5}$ $\boxed{f(x) = 5^x}$

$(3, \frac{1}{8}) \in$ graph $\Rightarrow$ when $x=3, y=\frac{1}{8}$

$\frac{1}{8} = a^3 \Rightarrow a = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$

$\boxed{a = \frac{1}{2}}$ $\boxed{f(x) = \left(\frac{1}{2}\right)^x}$

Exercise #2 Use the graph of $f(x) = 2^x$ to obtain the graph of each. Specify the domain, range, asymptote and intercept(s).

a) $g(x) = 1 + 2^x$

shift the graph of $y = 2^x$ up 1 unit

x	$-\infty$	-1	0	1	2	∞
$y=2^x$	$y \to 0$	$\frac{1}{2}$	1	2	4	∞
$y=1+2^x$	$y \to 1$	$\frac{3}{2}$	2	3	5	∞

$x \in \mathbb{R}$, $y \in (1, \infty)$

H.A. $y = 1$

b) $G(x) = 2^x - 1$

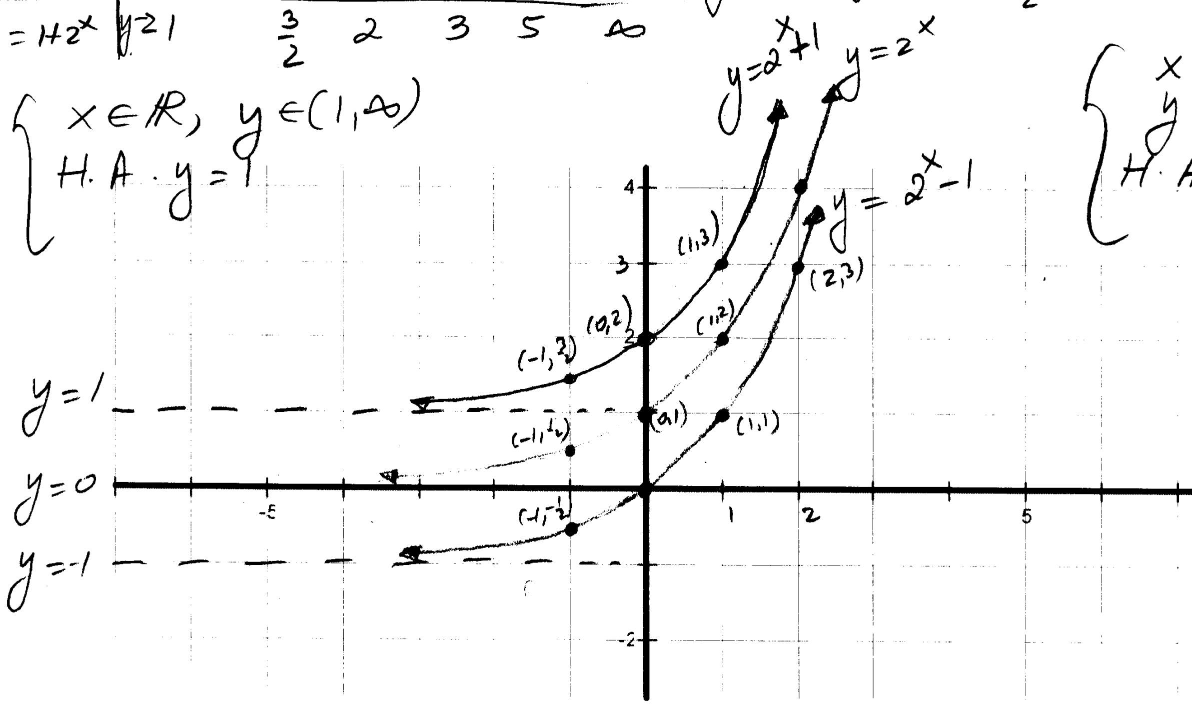
shift the graph of $y = 2^x$ down 1 unit

x	$-\infty$	-1	0	1	2	∞
$y=2^x$	$y \to 0$	$\frac{1}{2}$	1	2	4	∞
$y=2^x-1$	$y \to -1$	$-\frac{1}{2}$	0	1	3	∞

$x \in \mathbb{R}$

$y \in (-1, \infty)$

H.A. $y = -1$



c) $l(x) = 2^{x+1}$ shift the graph of $y=2^x$ left 1 unit

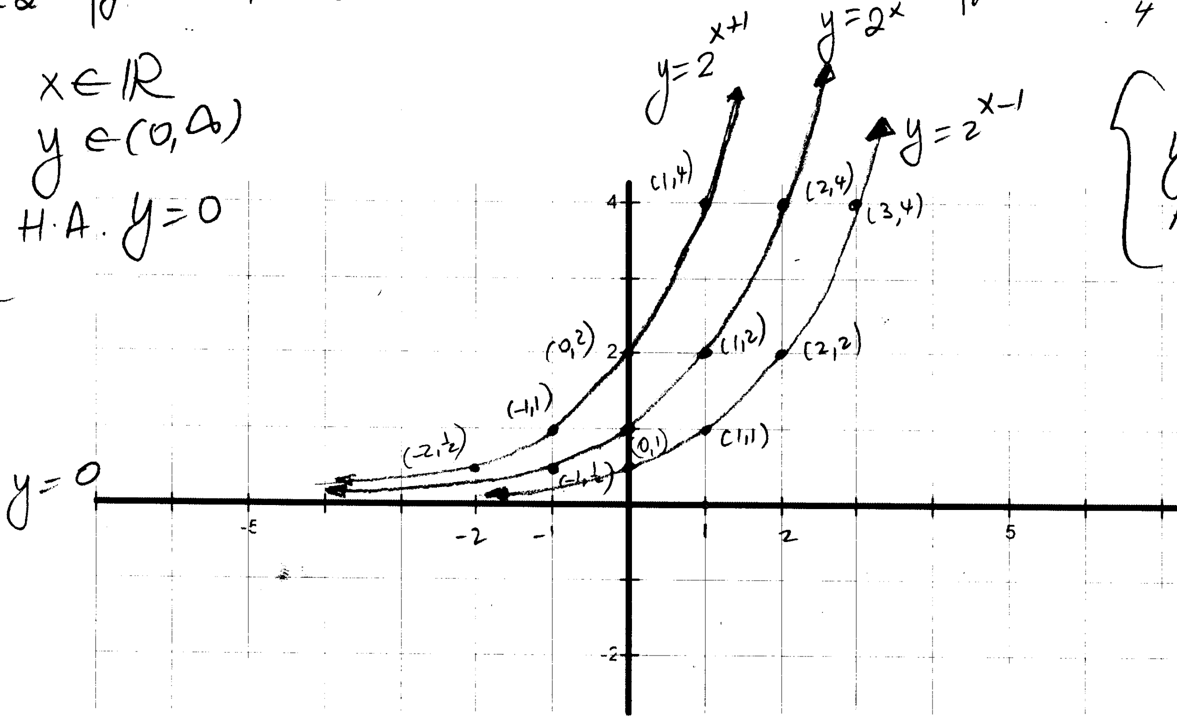
d) $L(x) = 2^{x-1}$ shift the graph of $y = 2^x$ right 1 unit

x	$-\infty$	-1	0	1	2	∞
$y=2^x$	$y \to 0$	$\frac{1}{2}$	1	2	4	∞
$y=2^{x+1}$	$y \to 0$	1	2	4		∞

x	$-\infty$	-1	0	1	∞
$y=2^x$	$y \to 0$	$\frac{1}{2}$	1	2	∞
$y=2^{x-1}$	$y \to 0$	$\frac{1}{4}$	$\frac{1}{2}$	1	∞

$x \in \mathbb{R}$
$y \in (0, \infty)$
H.A. $y = 0$

$x \in \mathbb{R}$
$y \in (0, \infty)$
H.A. $y = 0$

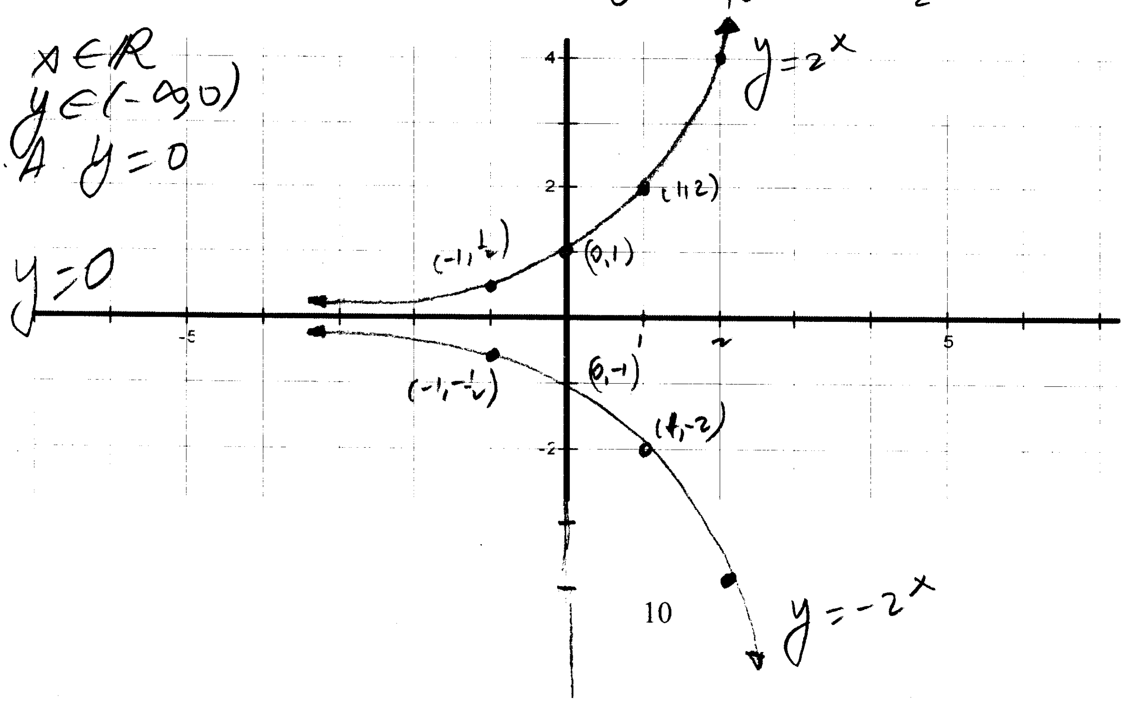


e) $h(x) = -2^x$

reflects the graph of $y=2^x$ about the x-axis

x	$-\infty$	-1	0	1	2	∞
$y=2^x$	$y \to 0$	$\frac{1}{2}$	1	2	4	∞
$y=-2^x$	$y \to 0$	$-\frac{1}{2}$	-1	-2	-4	$-\infty$

$x \in \mathbb{R}$
$y \in (-\infty, 0)$
H.A. $y = 0$



Compound Interest

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

A= amount in the account after t years
P = principal (amount invested)
r = annual interest rate
n = number of times interest is compounded per year
t = number of years

Exercise #4 Assume we invest $1000 in an account that pays 6% interest rate per year. How much is in the account at the end of one year if

a) interest is compounded once a year?

$$A = 1000\left(1+\frac{0.06}{1}\right)^{1}$$
$$= 1000(1.06)$$
$$A = 1060 \text{ dollars}$$

b) interest is compounded quarterly?

$$A = 1000\left(1+\frac{0.06}{4}\right)^{4\cdot t}$$
$$= 1000(1.0614)$$
$$A = 1061.4 \text{ dollars}$$

How much interest was paid in one year under the quarterly compounding?

1061.40 − 1000 = 61.40 $ interest paid

What percentage of $1000 does this represent?

61.40 interest paid is 6.14 % of $1000 .

This is called the **effective yield** (effective annual rate of interest).
The **effective yield** of an interest is the simple interest rate that would yield the same amount in 1 year.

The Number e

An interesting situation occurs if we consider the compound interest formula for $P = \$1$, $r = 100\%$, $t = 1$ year.

The formula becomes $A = \left(1 + \frac{1}{n}\right)^n$.

The following table shows some values, rounded to eight decimal places, of $\left(1+\frac{1}{n}\right)^n$

n	$\left(1+\frac{1}{n}\right)^n$
1	2.00000000
10	2.5937246
100	2.70481383
1000	2.71692393
10,000	2.71814593
100,000	2.71826824
1,000,000	2.71828047
10,000,000	2.71828169
100,000,000	2.71828181
1,000,000,000	2.71828183

a) For a fixed period of time (say one year), does more and more frequent compounding of interest continue to yield greater and greater amounts?

yes

b) Is there a limit on how much money can accumulate in a year when interest is compounded more and more frequently?

yes

The table suggests that as n increases, the value of $\left(1+\frac{1}{n}\right)^n$ gets closer and closer to some fixed number. The fixed number is called e. To five decimal places, $e = 2.71828$.

When $n \to \infty$, $\left(1+\frac{1}{n}\right)^n \to e$ and the formula for

continuously compounded interest is $\boxed{A = Pe^{rt}}$

Exercise #5 Assume we invest \$1000 in an account that pays 6% interest rate per year compounded continuously.

a) How much is in the account at the end of one year?

$A = 1000 \cdot e^{0.06} \approx 1061.84$ dollars

b) How much interest was paid in one year under the continuous compounding?

$1061.84 - 1000 = 61.84$ dollars

c) What is the effective yield?

$61.84 = 6.184\%$ the simple interest that would yield \$1061.84 in the account after 1 year.

Exercise #6 What investment yields the greater return: 7% compounded monthly or 6.85% compounded continuously? (Hint: To answer such a question, we need to compare the effective yield of the accounts). $P =$ amount invested

For the 7% compounded monthly

Let $R =$ effective yield

$\underbrace{\cancel{P}(1+R)}_{\text{amount at simple interest}} = \underbrace{\cancel{P}\left(1+\frac{r}{n}\right)^{nt}}_{\text{amount at compounded interest}}$

$1+R = \left(1+\frac{0.07}{12}\right)^{12 \cdot 1}$

$R = 1.07186 - 1 = 0.07186$

$R = 7.186\%$

For the 6.85 compounded continuously

Let $R =$ effective yield.

$\underbrace{\cancel{P}(1+R)}_{\text{amount at simple interest}} = \underbrace{\cancel{P}e^{rt}}_{\text{amount at continuously compounded interest}}$

$1+R = e^{0.0685(1)}$

$1+R = 1.07090$

$R = 0.07090 = 7.09\%$

$R = 7.09\%$

It's better to invest at 7% compounded monthly

Exercise #7 The exponential growth of the deer population in Massachusetts can be calculated using the model $T = 50,000(1+0.06)^n$, where 50,000 is the initial deer population and 0.06 is the rate of growth. T is the total population after n years have passed.

a) Predict the total population after 4 years.

if $n=4$, $T = 50,000(1+0.06)^4 \approx 63,000$ deer

b) If the initial population was 30,000 and the growth rate was 0.12, approximately how many deer would be present after 3 years?

$T = 30,000(1+0.12)^n$

if $n=3$, $T = 30,000(1+0.12)^3 \approx 42147$ deer

Exercise #8 A mobile home loses 20% of its value every 3 years.

a) A certain mobile home costs $20,000. Write a function for its value after t years.

t	$V(t)$
0	20,000
3	$20,000(0.8)$
6	$20,000(0.8)^2$

$V(t) = 20,000(0.8)^{t/3}$

b) What is the value of the mobile home after 54 months?

$t = 54$ months $= 4.5$ years

$V(4.5) = 20,000(0.8)^{4.5/3} \approx 14,310$ dollars

Exercise #9
a) Complete the table of values.
b) Write a function that describes the exponential growth.
c) Graph the function.
d) Evaluate the function at the given values.

A colony of bacteria starts with 300 organisms and doubles every week. How many bacteria will there be after 8 weeks?
After 5 days?

Weeks	0	1	2	3	4
Bacteria	300	600	1200	2400	4800

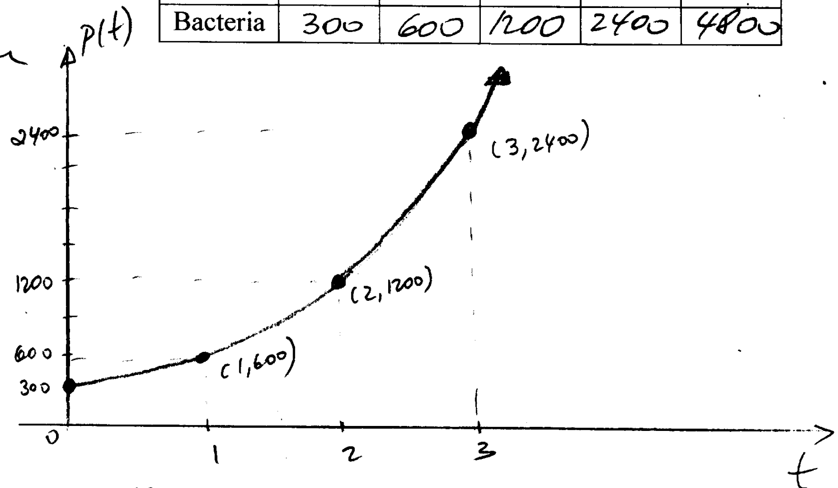
Let $P(t)$ = population

$P(1) = 300(2)$

$P(2) = 300(2)^2$

$P(3) = 300(3)^3$

$\boxed{P(t) = 300(3)^t}$



Exercise #10 (9.1 - #53) India is currently one of the world's fastest-growing countries. By 2040, the population of India will be larger than the population of China; by 2050, nearly one-third of the world's population will live in these two countries alone. The exponential function $f(x) = 574(1.026)^x$ models the population of India, $f(x)$, in millions, x years after 1974.

a) What was India's population in 1974?

when $x=0$, $f(0) = 574(1.026)^0 = 574$ million people in 1974

b) Find $f(27)$ and its meaning.

$f(27) = 574(1.026)^{27} \approx 1148$ million people in 2001

c) Find India's population, to the nearest million, in the year 2028 as predicted by this function.

when $x = 54$, $f(54) = 574(1.026)^{54} \approx 2295$ million people in 2028.

Exercise #11 (9.1 - #63) The function $N(t) = \dfrac{30{,}000}{1+20e^{-1.5t}}$ describes the number of people, $N(t)$, who become ill with influenza t weeks after its initial outbreak in a town with 30,000 inhabitants.

a) How many people became ill with the flu when the epidemic began?

$t=0$, $N(0) = \dfrac{30{,}000}{1+20e^0} = \dfrac{30{,}000}{1+20} = \dfrac{30{,}000}{21} \approx 1429$ people

b) How many people were ill by the end of the third week? Identify the point on the graph.

$t=3$, $N(3) = \dfrac{30{,}000}{1+20e^{-1.5(3)}} \approx 24{,}546$ people

c) What is the horizontal asymptote of the graph? What does it indicate?

$y = 30{,}000$ It shows that there is a limit to the ill epidemic's growth (the number of people will not exceed 30,000, the population of the town)

d) Find the domain and range of the function.

$t \in [0, \infty)$

$N(t) \in [1429, 30{,}000)$