8.1 & 8.2 Quadratic Equations and Their Applications

The Square Root Property

1. Solve by extracting roots:

a)
$$81\left(x+\frac{1}{3}\right)^2 + 1 = 0$$
 b) $3\left(8x-7\right)^2 - 24 = 0$ c) $48 + 2\left(5x-12\right)^2 = 0$

2. Cyril plans to invest \$5000 in a money market account paying interest compounded annually ($A = P(1+r)^n$, where A is the amount in the account, P is the principal, r is the interest rate, and n is the number of years).

a) Write a formula in terms of the interest rate, r, for the balance, B, in Cyril's account after two years.

b) Complete the table showing Cyril's account balance after two years for the given interest rates.

r	0.02	0.04	0.06	0.08
В				

c) If Cyril would like to have \$6250 in two years, what interest rate must the account pay?

(A:
$$a)B = 5000(1+r)^2$$
; $b)5202,5408,5616,5832; c)11.8\%$)

3. Hortense is investing \$2600 in an account where interest is calculated according to the formula $A = P(1+r)^t$ where P is the original principal, r is the interest rate and t is the time measured in years. If Hortense wants her money to grow to double in two years, what interest rate must the account have? (Approximate the answer to the nearest hundredth of a percent) (A: 41.42%)

4. Earl borrowed \$5500 from his uncle for 2 years with interest compounded annually according to the formula $A = p(1+r)^t$ where "p" is the original principal, "r" is the interest rate and "t" is the time measured in years. At the end of 2 years he owed his uncle \$6474.74. What was the interest rate on the loan? (A: 8.5%)

5. Solve for x in terms of a, b, and c.

i)
$$(x+a)^2 = 36$$
 ii) $(ax-b)^2 = 25$

6. Solve the formula for the specified variable (textbook #65, 66 /571).

Completing the Square

1. Solve the following equations by completing the square:

a)
$$x^2 + \frac{4}{7}x + \frac{3}{49} = 0$$
 (textbook #45 / 571)
b) $8x^2 - 4x + 1 = 0$ (textbook #55/571)

c)
$$9v^2 + 12v - 41 = 0$$
 (A: $v = -\frac{2}{3} \pm \sqrt{5}$) d) $6t^2 + 4t + 18 = 0$

More applications

1. Solve:

a)
$$\frac{19x+14}{3} = x^2$$
 b) Solve for p: $r = 5p - mp^2$ c) Solve for w in terms of A: $A = \frac{1}{2}w(100 - 2w)$
d) $6x - 13 = \frac{5}{x}$ e) $\frac{1}{x^2 - 3x + 2} = \frac{1}{x + 2} + \frac{5}{x^2 - 4}$ (#67/576) f) $\sqrt{2}x^2 + 3x - 2\sqrt{2} = 0$ (#69/586)
g) $|x^2 + 2x| = 3$ (#71/586) (A: c) $w = 25 \pm \sqrt{625 - A}$; d) $-1/3, 5/2$)

2. When Maria serves in volleyball, the ball leaves her hand with an upward velocity of 20 feet per second. The height "h" of the volleyball above the ground after "t" seconds is given by: $h = -16t^2 + 20t + 5$. a) If nobody hits the ball, how long will it take the ball to hit the ground? (A:1.46 sec) b) If nobody hits the ball, how long will it take the ball to reach its initial height again? (A: 5/4 sec)

3. A box with no top is to be made from a rectangular piece of cardboard in which the length of the cardboard is three times the width of the cardboard. The box will be made by cutting 2 inch squares from each corner and holding up the sides.

a) Find a formula for the volume "V" in terms of the width "x" of the cardboard.

b) If the volume of the box is 1222 cubic inches, find the dimensions of the cardboard. (A: 17 in by 51 in)

4. The owners of a day-care center plan to enclose a divided play area against the back wall of their building. They have 300 feet of picket fence and would like the total area of the playground to be 6000 square feet. Can they enclose the playground with the fence they have, and if so what should the dimensions of the playground be? (A: 72.35 ft by 82.95 ft or 27.65 ft by 217.05 ft)

5. A piece of wire is 8 inches long. The wire is cut into two pieces and then each piece is bent into a square. Find the length of each piece if the sum of the areas of these squares is to be 2 square inches (textbook # 82/587) (A: 4in by 4in)

6. The fish population in a certain lake rises and falls according to the formula:

$$F = 2000 \left(15 + \frac{17}{2}t - \frac{1}{2}t^2 \right).$$

Here "F" is the number of fish at the time "t" where "t" is measured in years since January 1,1997 when the fish population was first estimated.

a) On what date will the fish population again be the same as on January 1,1998?

b) By what date will all the fish in the lake have died? (Approximate your answer in years to one decimal place).

7. The number of fatal vehicle crashes per 100 million miles, f(x), for drivers of age x can be modeled by the quadratic function $f(x) = 0.013x^2 - 1.19x + 28.24$.

a) What age groups are expected to be involved in 3 fatal crashes per 100 million miles driven?

b) What age groups are expected to be involved in 10 fatal crashes per 100 million miles driven? (textbook # 73, 74 / 587) (A: a) 33 and 58 year olds; b) 19 and 72 year olds)